

## **Unit 1: Are These Numbers Real?**

This unit recalls the relationships between sets of real numbers and the rules involved when working with them.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Writing Process Strand**

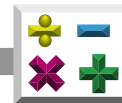
##### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 10: Mathematical Reasoning and Problem Solving**

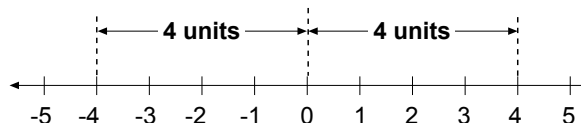
- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.
- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).



## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

**absolute value** .....a number's distance from zero (0) on a number line; distance expressed as a positive value  
*Example:* The absolute value of both 4, written  $|4|$ , and negative 4, written  $|-4|$ , equals 4.



**addend** .....any number being added  
*Example:* In  $14 + 6 = 20$ , the addends are 14 and 6.

**additive identity** .....the number zero (0); when zero (0) is added to another number the sum is the number itself  
*Example:*  $5 + 0 = 5$

**additive inverses** .....a number and its opposite whose sum is zero (0); also called *opposites*  
*Example:* In the equation  $3 + (-3) = 0$ , the additive inverses are 3 and -3.

**algebraic expression** .....an expression containing numbers and variables ( $7x$ ) and operations that involve numbers and variables ( $2x + y$  or  $3a^2 - 4b + 2$ ); however, they do not contain equality ( $=$ ) or inequality symbols ( $<$ ,  $>$ ,  $\leq$ ,  $\geq$ , or  $\neq$ )

**associative property** .....the way in which three or more numbers are grouped for addition or multiplication does *not* change their sum or product, respectively  
*Examples:*  $(5 + 6) + 9 = 5 + (6 + 9)$  or  
 $(2 \times 3) \times 8 = 2 \times (3 \times 8)$



**braces { }** .....grouping symbols used to express sets

**commutative property** .....the order in which two numbers are added or multiplied does *not* change their sum or product, respectively

*Examples:*  $2 + 3 = 3 + 2$  or

$$4 \times 7 = 7 \times 4$$

**counting numbers**

**(natural numbers)** .....the numbers in the set  $\{1, 2, 3, 4, 5, \dots\}$

**cube (power)** .....the third power of a number

*Example:*  $4^3 = 4 \times 4 \times 4 = 64$ ;

64 is the cube of 4

**decimal number** .....any number written with a decimal point in the number

*Examples:* A decimal number falls between two whole numbers, such as 1.5, which falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called *decimal fractions*, such as five-tenths, or  $\frac{5}{10}$ , which is written 0.5.

**difference** .....a number that is the result of subtraction

*Example:* In  $16 - 9 = 7$ , the difference is 7.

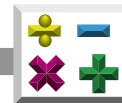
**digit** .....any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9

**element or member** .....one of the objects in a set

**empty set or null set ( $\emptyset$ )** ....a set with no elements or members

**equation** .....a mathematical sentence stating that the two expressions have the same value

*Example:*  $2x = 10$



- even integer** .....any integer divisible by 2; any integer with the digit 0, 2, 4, 6, or 8 in the units place; any integer in the set  $\{\dots, -4, -2, 0, 2, 4, \dots\}$
- exponent**  
**(exponential form)** .....the number of times the base occurs as a factor  
*Example:*  $2^3$  is the exponential form of  $2 \times 2 \times 2$ . The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.
- expression** .....a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables  
*Examples:*  $4r^2$ ;  $3x + 2y$ ;  $\sqrt{25}$   
An expression does *not* contain equal (=) or inequality (<, >,  $\leq$ ,  $\geq$ , or  $\neq$ ) signs.
- finite set** .....a set in which a whole number can be used to represent its number of elements; a set that has bounds and is limited
- fraction** .....any part of a whole  
*Example:* One-half written in fractional form is  $\frac{1}{2}$ .
- grouping symbols** .....parentheses ( ), braces { }, brackets [ ], and fraction bars indicating grouping of terms in an expression
- infinite set** .....a set that is not finite; a set that has no boundaries and no limits
- integers** .....the numbers in the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
- irrational number** .....a real number that cannot be expressed as a ratio of two integers  
*Example:*  $\sqrt{2}$



**member or element** .....one of the objects in a set

**multiples** .....the numbers that result from multiplying a given whole number by the set of whole numbers  
*Example:* The multiples of 15 are 0, 15, 30, 45, 60, 75, etc.

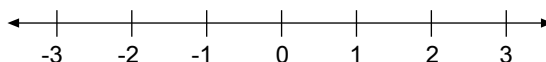
**natural numbers**  
**(counting numbers)** .....the numbers in the set  $\{1, 2, 3, 4, 5, \dots\}$

**negative integers** .....integers less than zero

**negative numbers** .....numbers less than zero

**null set ( $\emptyset$ ) or empty set** ....a set with no elements or members

**number line** .....a line on which ordered numbers can be written or visualized

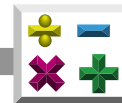


**odd integer** .....any integer *not* divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set  $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$

**opposites** .....two numbers whose sum is zero; also called *additive inverses*

*Examples:*  $-5 + 5 = 0$  or  $\frac{2}{3} + \left(-\frac{2}{3}\right) = 0$

$\uparrow \quad \uparrow$                        $\uparrow \quad \uparrow$   
 opposites                      opposites



**order of operations** .....the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations*

*Example:*  $5 + (12 - 2) \div 2 - 3 \times 2 =$   
 $5 + 10 \div 2 - 3 \times 2 =$   
 $5 + 5 - 6 =$   
 $10 - 6 =$   
 $4$

**pattern (relationship)** .....a predictable or prescribed sequence of numbers, objects, etc.; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions)

*Example:* 2, 5, 8, 11 ... is a pattern. Each number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule,  $3n - 1$ , by using the set of counting numbers for  $n$ .

**pi ( $\pi$ )** .....the symbol designating the ratio of the circumference of a circle to its diameter; an irrational number with common approximations of either 3.14 or  $\frac{22}{7}$

**positive integers** .....integers greater than zero

**positive numbers** .....numbers greater than zero

**power (of a number)** .....an exponent; the number that tells how many times a number is used as a factor  
*Example:* In  $2^3$ , 3 is the power.



**product** .....the result of multiplying numbers together  
*Example:* In  $6 \times 8 = 48$ , the product is 48.

**quotient** .....the result of dividing two numbers  
*Example:* In  $42 \div 7 = 6$ , the quotient is 6.

**ratio** .....the comparison of two quantities  
*Example:* The ratio of  $a$  and  $b$  is  $a:b$  or  $\frac{a}{b}$ , where  $b \neq 0$ .

**rational number** .....a number that can be expressed as a ratio  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$

**real numbers** .....the set of all rational and irrational numbers

**repeating decimal** .....a decimal in which one digit or a series of digits repeat endlessly  
*Examples:*  $0.3333333\ldots$  or  $0.\overline{3}$   
 $24.6666666\ldots$  or  $24.\overline{6}$   
 $5.27272727\ldots$  or  $5.\overline{27}$   
 $6.2835835\ldots$  or  $6.\overline{2835}$

**root** .....an equal factor of a number  
*Examples:*  
In  $\sqrt{144} = 12$ , the square root is 12.  
In  $\sqrt[3]{125} = 5$ , the cube root is 5.

**set** .....a collection of distinct objects or numbers

**simplify an expression** .....to perform as many of the indicated operations as possible





**solve** .....to find all numbers that make an equation or inequality true

**square (of a number)** .....the result when a number is multiplied by itself or used as a factor twice  
*Example:* 25 is the square of 5.

**sum** .....the result of adding numbers together  
*Example:* In  $6 + 8 = 14$ , the sum is 14.

**terminating decimal** .....a decimal that contains a finite (limited) number of digits  
*Example:*  $\frac{3}{8} = 0.375$   
 $\frac{2}{5} = 0.4$

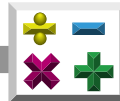
**value (of a variable)** .....any of the numbers represented by the variable

**variable** .....any symbol, usually a letter, which could represent a number

**Venn diagram** .....a diagram which shows the relationships between sets

**whole numbers** .....the numbers in the set  $\{0, 1, 2, 3, 4, \dots\}$





## Unit 1: Are These Numbers Real?

### Introduction

The focus of Algebra I is to introduce and strengthen algebraic skills. These skills are necessary for further study and success in mathematics. Algebra I fosters

- an understanding of the real number system
- an understanding of different sets of numbers
- an understanding of various ways of representing numbers.

Many topics in this unit will be found again in later units. There is an emphasis on problem solving and real-world applications.

### Lesson One Purpose

#### Reading Process Strand

#### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.



## Writing Process Strand

### Standard 3: Prewriting

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

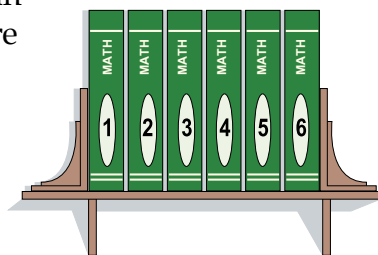
### Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.
- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

## The Set of Real Numbers

A **set** is a collection. It can be a collection of DVDs, books, baseball cards, or even numbers. Each item in the *set* is called an **element** or **member** of the set. In algebra, we are most often interested in sets of numbers.

The first set of numbers you learned when you were younger was the set of **counting numbers**, which are also called the **natural numbers**. These are the **positive numbers** you count with (1, 2, 3, 4, 5, ...). Because this set has *no final number*, we call it an **infinite set**. A set that has a *specific number of elements* is called a **finite set**.



*A set can be a collection of books or numbers.*



Symbols are used to represent sets. **Braces** { } are the symbols we use to show that we are talking about a set.

A set with *no elements* or members is called a **null set ( $\emptyset$ )** or **empty set**. It is often denoted by an *empty set of braces* { }.

The set of *counting numbers* looks like {1, 2, 3, ...}.



**Remember:** The counting numbers can also be called the *natural numbers, naturally!*

The set of natural number **multiples** of 10 is {10, 20, 30, ...}.

The set of integers looks like {..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}.

The set of **integers** that are *multiples* of 10 is  
{..., -30, -20, -10, 10, 20, 30, ...}.

As you became bored with simply counting, you learned to add and subtract numbers. This led to a new set of numbers, the **whole numbers**.

The *whole numbers* are the counting numbers *and* zero  
{0, 1, 2, 3, ...}.

Remember getting negative answers? Those **negative numbers** made another set of numbers necessary. The *integers* are the counting numbers, their **opposites** (also called **additive inverses**), and zero.

The integers can be expressed (or written) as  
{..., -3, -2, -1, 0, 1, 2, 3, ...}.

**Even integers** are integers divisible by 2. The integers  
{..., -4, -2, 0, 2, 4, ...} form the set of *even integers*.



**Remember:** Every even integer ends with the **digit** 0, 2, 4, 6, or 8 in its ones (or units) place.



**Odd integers** are integers that are *not* divisible by 2. The integers  $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$  form the set of *odd integers*.



**Remember:** Every odd integer ends with the *digit* 1, 3, 5, 7, or 9 in its units place.

**Note:** There are *no fractions* or **decimals** listed in the set of integers above.

When you learned to divide and got answers that were *integers*, *decimals*, or *fractions*, your answers were all from the set of **rational numbers**.

*Rational numbers* can be expressed as fractions that can then be converted to **terminating decimals** (with a *finite* number of digits) or **repeating decimals** (with an *infinitely* repeating sequence of digits). For example,  $-\frac{3}{5} = -0.6$ ,  $\frac{6}{2} = 3$ ,  $-\frac{8}{4} = -2$  and  $\frac{1}{3} = 0.333\dots$  or  $0.\overline{3}$ .

As you learned more about mathematics, you found that some numbers are **irrational numbers**. *Irrational numbers* are numbers that cannot be written as a **ratio**, or a comparison of two quantities because their decimals never repeat a **pattern** and never end.

*Irrational numbers* like  $\pi$  (**pi**) and  $\sqrt{5}$  have non-terminating, non-repeating decimals.

If you put all of the rational numbers and all of the irrational numbers together in a set, you get the set of **real numbers**.

The set of *real numbers* is often symbolized with a capital R.

A diagram showing the *relationships* among all the sets mentioned is shown on the following page.



The diagram below is called a **Venn diagram**. A *Venn diagram* shows the relationships between different sets. In this case, the sets are types of numbers.

<b>rational numbers</b> (real numbers that can be expressed as a ratio $\frac{a}{b}$ , where $a$ and $b$ are integers and $b \neq 0$ )	<b>irrational numbers</b> (real numbers that <i>cannot</i> be expressed as a ratio of two integers)
<p><b>integers</b>                      (whole numbers and their opposites)  <math>\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}</math></p> <p><b>whole numbers</b>                      (zero and natural numbers)  <math>\{0, 1, 2, 3, 4, \dots\}</math></p> <p><b>natural numbers or counting numbers</b>  <math>\{1, 2, 3, 4, \dots\}</math></p> <p><math>\sqrt{0.16}</math>      <math>\frac{5}{6}</math></p> <p><math>-\frac{3}{7}</math>      <math>0</math></p> <p><math>0.83</math>      <math>0.09</math></p>	<p><math>\pi</math></p> <p><math>0.010010001\dots</math></p> <p><math>\sqrt{120}</math></p>



## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |   |  |
|---|--|
| _____ 1. the numbers in the set<br>$\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$  | A. even integer                          |
| _____ 2. the numbers in the set<br>$\{1, 2, 3, 4, 5, \dots\}$   | B. finite set                            |
| _____ 3. a number that can be expressed<br>as a ratio $\frac{a}{b}$ , where $a$ and $b$ are<br>integers and $b \neq 0$                | C. infinite set                          |
| _____ 4. a set in which a whole number can<br>be used to represent its number<br>of elements; a set that has bounds<br>and is limited | D. integers                              |
| _____ 5. the numbers in the set<br>$\{0, 1, 2, 3, 4, \dots\}$   | E. irrational number                     |
| _____ 6. a real number that cannot be<br>expressed as a ratio of two<br>integers  | F. multiples                             |
| _____ 7. any integer <i>not</i> divisible by 2  | G. natural numbers<br>(counting numbers) |
| _____ 8. a set that is <i>not</i> finite; a set that has<br><i>no</i> boundaries and <i>no</i> limits                                 | H. odd integer                           |
| _____ 9. any integer divisible by 2   | I. pi ( $\pi$ )                          |
| _____ 10. the numbers that result from<br>multiplying a given whole number<br>by the set of whole numbers                             | J. rational number                       |
| _____ 11. the set of all rational and irrational<br>numbers   | K. real numbers                          |
| _____ 12. the symbol designating the ratio of<br>the circumference of a circle to its<br>diameter                                     | L. whole numbers                         |





## Practice

Match each **description** with the correct **set**. Write the letter on the line provided.

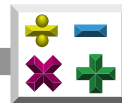
- |   |                                       |
|---|---------------------------------------|
| _____ 1. {2, 3, 4, 5, 6}                        | A. {counting numbers between 1 and 7} |
| _____ 2. {0, 1, 2, 3}                           | B. {even integers between -3 and 4}   |
| _____ 3. {3, 6, 9, 12, ...}                     | C. {first five counting numbers}      |
| _____ 4. {-2, 0, 2}                             | D. {first four whole numbers}         |
| _____ 5. {6, 12, 18, ...}                       | E. {integers that are multiples of 6} |
| _____ 6. {1, 2, 3, 4, 5}                        | F. {natural-number multiples of 3}    |
| _____ 7. {-3, -1, 1, 3}                         | G. {odd integers between -4 and 5}    |
| _____ 8. {..., -18, -12, -6, 0, 6, 12, 18, ...} | H. {whole number multiples of 6}      |



## Practice

Write **finite** if the **set** has bounds and is limited. Write **infinite** if the **set** has no boundaries and is not limited.

- \_\_\_\_\_ 1. {whole numbers less than 1,000,000}
- \_\_\_\_\_ 2. {natural numbers with four digits}
- \_\_\_\_\_ 3. {whole numbers with 0 as the last numeral}
- \_\_\_\_\_ 4. {real numbers between 6 and 8}
- \_\_\_\_\_ 5. {counting numbers between 2 and 10}
- \_\_\_\_\_ 6. {first five counting numbers}
- \_\_\_\_\_ 7. {natural-number multiples of 5}
- \_\_\_\_\_ 8. {integers less than 1,000,000}
- \_\_\_\_\_ 9. {counting numbers with three digits}
- \_\_\_\_\_ 10. {whole numbers with 5 as the last numeral}



## Practice

Write **True** if the statement is correct. Write **False** if the statement is not correct.

- \_\_\_\_\_ 1. 7 is a rational number.
- \_\_\_\_\_ 2.  $\frac{5}{3}$  is a real number.
- \_\_\_\_\_ 3. -9 is a whole number.
- \_\_\_\_\_ 4. 0 is a counting number.
- \_\_\_\_\_ 5.  $\sqrt{4}$  is irrational.
- \_\_\_\_\_ 6.  $\sqrt{7}$  is a rational number.
- \_\_\_\_\_ 7.  $\frac{10}{3}$  is a whole number.
- \_\_\_\_\_ 8. -9 is a natural number.
- \_\_\_\_\_ 9. 0 is an even integer.
- \_\_\_\_\_ 10.  $\pi$  is a real number.



## Practice

Use the list below to write the correct term for each definition on the line provided.

**additive inverses**  
**element or member**  
**negative numbers**

**null set ( $\emptyset$ ) or empty set**  
**positive numbers**

**repeating decimal**  
**terminating decimal**

- \_\_\_\_\_ 1. a set with no elements or members
- \_\_\_\_\_ 2. a decimal that contains a finite (limited) number of digits
- \_\_\_\_\_ 3. a decimal in which one digit or a series of digits repeat endlessly
- \_\_\_\_\_ 4. a number and its opposite whose sum is zero (0)
- \_\_\_\_\_ 5. numbers less than zero
- \_\_\_\_\_ 6. numbers greater than zero
- \_\_\_\_\_ 7. one of the objects in a set



## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Algebra Body of Knowledge**

#### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.
- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).



## The Order of Operations

Algebra can be thought of as a game. When you know the rules, you have a much better chance of winning! In addition to knowing how to add, subtract, multiply, and divide integers, fractions, and decimals, you must also use the **order of operations** correctly.

Although you have previously studied the rules for *order of operations*, here is a quick review.

### Rules for Order of Operations

Always start on the *left* and move to the *right*.

1. Do operations inside *grouping symbols* first. ( ), [ ], or  $\frac{x}{y}$
2. Then do all *powers* (exponents)  $x^2$  or  $\sqrt{x}$   
or *roots*.
3. Next do *multiplication or division*—  
as they occur from left to right. • or  $\div$
4. Finally, do *addition or subtraction*—  
as they occur from left to right. + or –

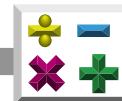


**Remember:** The fraction bar is considered a **grouping symbol**.

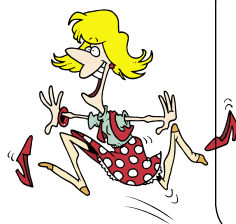
Example:  $\frac{3x^2 + 8}{2} = (3x^2 + 8) \div 2$

**Note:** In an **expression** where more than one set of *grouping symbols* occurs, work within the innermost set of symbols first, then work your way outward.

The order of operations makes sure everyone doing the problem correctly will get the same answer.



Some people remember these rules by using this mnemonic device to help their memory.



**Please Pardon My Dear Aunt Sally\***

**Please** ..... **Parentheses** (grouping symbols)

**Pardon**..... **Powers**

**My Dear**..... **Multiplication or Division**

**Aunt Sally**..... **Addition or Subtraction**

\*Also known as **Please Excuse My Dear Aunt Sally**—**Parentheses**, **Exponents**, **Multiplication or Division**, **Addition or Subtraction**.



**Remember:** You do multiplication **or** division—as they occur from *left to right*, and then addition **or** subtraction—as they occur from *left to right*.

*Study the following.*

$$25 - 3 \cdot 2 =$$

There are no grouping symbols. There are no **powers (exponents)** or **roots**. We look for multiplication or division and find multiplication. We multiply. We look for addition or subtraction and find subtraction. We subtract.

$$\begin{array}{r} 25 - 3 \cdot 2 = \\ 25 - 6 = \\ 19 \end{array}$$

*Study the following.*

$$12 \div 3 + 6 \div 2 =$$

There are no grouping symbols. There are no *powers* or *roots*. We look for multiplication or division and find division. We divide. We look for addition or subtraction and find addition. We add.

$$\begin{array}{r} 12 \div 3 + 6 \div 2 = \\ 4 + 3 = \\ 7 \end{array}$$



If the rules were ignored, one might divide 12 by 3 and get 4, then add 4 and 6 to get 10, then divide 10 by 2 to get 5—which is the wrong answer. Agreement is needed—using the agreed-upon *order of operations*.

*Study the following.*

$$30 - 3^3 =$$

There are no grouping symbols. We look for powers and roots and find powers,  $3^3$ . We calculate this. We look for multiplication or division and find none. We look for addition or subtraction and find subtraction. We subtract.

$$\begin{aligned} 30 - 3^3 &= \\ 30 - 27 &= \\ 3 \end{aligned}$$

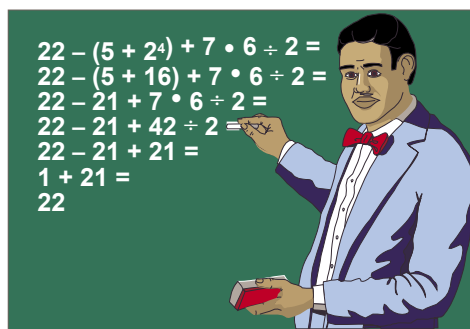
*Study the following.*

$$22 - (5 + 2^4) + 7 \cdot 6 \div 2 =$$

We look for grouping symbols and see them. We must do what is inside the parentheses first. We find addition and a power. We do the power first and then the addition. There are no roots. We look for multiplication or division and find both. We do them in the order they occur, left to right, so the multiplication occurs first. We look for addition or subtraction and find both. We do them in the order they occur, left to right, so the subtraction occurs first.

<b>Please</b>	<b>Parentheses</b>
<b>Pardon</b>	<b>Powers</b>
<b>My</b>	<b>Multiplication or</b>
<b>Dear</b>	<b>Division</b>
<b>Aunt</b>	<b>Addition or</b>
<b>Sally</b>	<b>Subtraction</b>

$$\begin{aligned} 22 - (5 + 2^4) + 7 \cdot 6 \div 2 &= \\ 22 - (5 + 16) + 7 \cdot 6 \div 2 &= \\ 22 - 21 + 7 \cdot 6 \div 2 &= \\ 22 - 21 + 42 \div 2 &= \\ 22 - 21 + 21 &= \\ 1 + 21 &= \\ 22 \end{aligned}$$

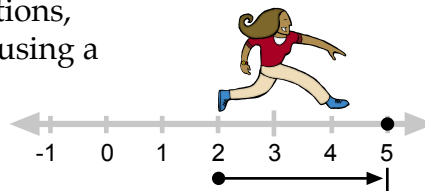






## Adding Numbers by Using a Number Line

After reviewing the rules for order of operations, let's get a visual feel for adding integers by using a **number line**.

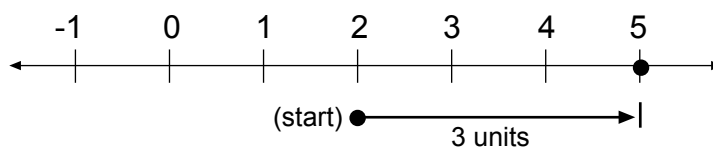


### Example 1

Add  $2 + 3$

1. Start at 2.
2. Move 3 units to the right in the *positive* direction.
3. Finish at 5.

So,  $2 + 3 = 5$ .

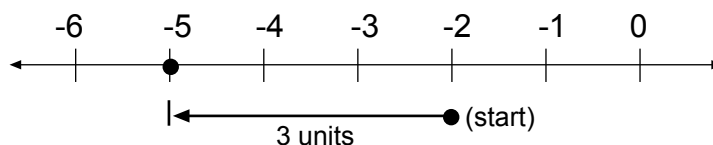


### Example 2

Add  $-2 + (-3)$

1. Start at -2.
2. Move 3 units to the left in the *negative* direction.
3. Finish at -5.

So,  $-2 + (-3) = -5$ .



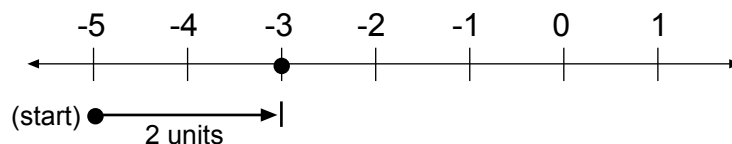


### Example 3

Add  $-5 + 2$

1. Start at -5.
2. Move 2 units to the right in a *positive* direction.
3. Finish at -3.

So,  $-5 + 2 = -3$ .

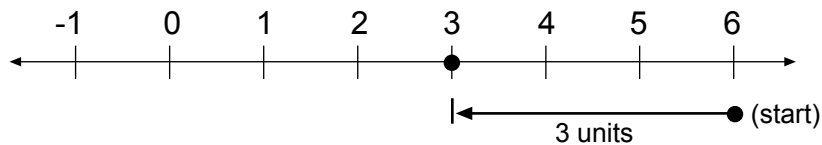


### Example 4

Add  $6 + (-3)$

1. Start at 6.
2. Move 3 units to the left in a *negative* direction.
3. Finish at 3.

So,  $6 + (-3) = 3$ .





## Addition Table

Look for *patterns* in the Addition Table below.

Addition Table									
+	4	3	2	1	0	-1	-2	-3	-4
4	8	7	6	5	4	3	2	1	0
3	7	6	5	4	3	2	1	0	-1
2	6	5	4	3	2	1	0	-1	-2
1	5	4	3	2	1	0	-1	-2	-3
0	4	3	2	1	0	-1	-2	-3	-4
-1	3	2	1	0	-1	-2	-3	-4	-5
-2	2	1	0	-1	-2	-3	-4	-5	-6
-3	1	0	-1	-2	-3	-4	-5	-6	-7
-4	0	-1	-2	-3	-4	-5	-6	-7	-8

← **addends**  
(any numbers being added)

**sums**  
(the result of adding numbers together)

**addends**      **sums**

- Look at the *positive sums* in the table. Note the *addends* that result in a positive sum.
- Look at the *negative sums* in the table. Note the *addends* that result in a negative sum.
- Look at the sums that are *zero*. Note the *addends* that result in a sum of zero.
- **Additive Identity Property**—when zero is added to any number, the sum is the number. Note that this property is true for addition of integers.
- **Commutative Property of Addition**—the order in which numbers are added does *not* change the sum. Note that this property is true for addition of integers.
- **Associative Property of Addition**—the way numbers are grouped when added does *not* change the sum. Note that this property is true for addition of integers.

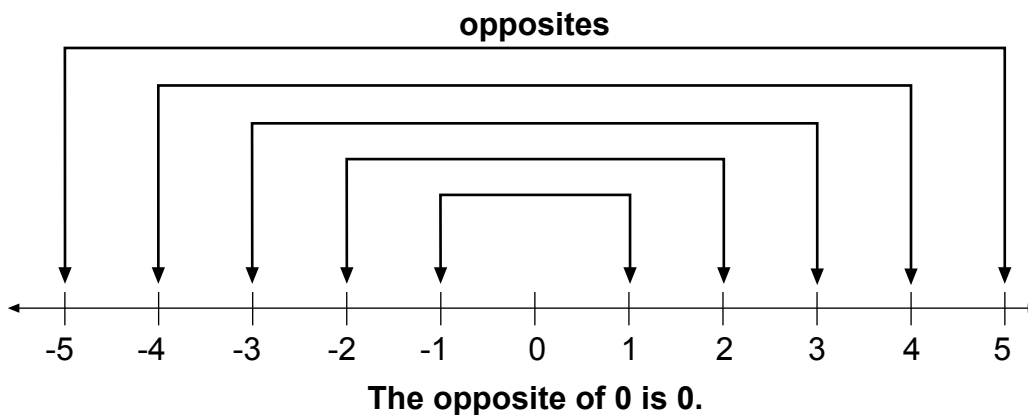


## Opposites and Absolute Value

Although we can visualize the process of adding by using a number line, there are faster ways to add. To accomplish this, we must know two things: *opposites* or *additive inverses* and **absolute value**.

### Opposites or Additives Inverses

5 and -5 are called *opposites*. Opposites are two numbers whose points on the number line are the same distance from 0 but in opposite directions.



Every **positive integer** can be paired with a **negative integer**. These pairs are called *opposites*. For example, the opposite of 4 is -4 and the opposite of -5 is 5.

The opposite of 4 can be written  $-(4)$ , so  $-(4)$  equals -4.

$$-(4) = -4$$

The opposite of -5 can be written  $-(-5)$ , so  $-(-5)$  equals 5.

$$-(-5) = 5$$

Two numbers are opposites or *additive inverses* of each other if their sum is zero.

For example:  $4 + -4 = 0$   
 $-5 + 5 = 0$

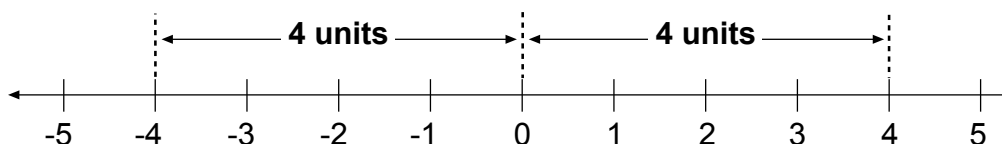


## Absolute Value

The *absolute value* of a number is the distance the number is from the *origin* or zero (0) on a number line. The symbol  $| |$  placed on either side of a number is used to show absolute value.

Look at the number line below. -4 and 4 are different numbers. However, they are the same distance in number of units from 0. Both have the same *absolute value* of 4. Absolute value is *always* positive because distance is always positive—you cannot go a negative distance. The absolute value of a number tells the number's *distance* from 0, not its *direction*.

$$|-4| = |4| = 4 \quad \text{The absolute value of a number is *always* positive.}$$



However, the number 0 is neither positive nor negative.  
The absolute value 0 is 0.

$|-4|$  denotes the  
absolute value of -4.

$$|-4| = 4$$

$|4|$  denotes the  
absolute value of 4.

$$|4| = 4$$

The absolute value of 10 is 10. We can use the following notation.

$$|10| = 10$$

The absolute value of -10 is also 10. We can use the following notation.

$$|-10| = 10$$

Both 10 and -10 are 10 units away from the origin. So, the absolute value of both numbers is 10.

The absolute value of 0 is 0.

$$|0| = 0$$

The *opposite* of the absolute value of a number is *negative*.

$$-|8| = -8$$



Now that we have this terminology under our belt, we can introduce two rules for adding numbers which will enable us to add quickly.

## Adding Positive and Negative Integers

There are specific rules for adding positive and negative numbers.

1. If the two integers have the *same sign*, *add their absolute values*, and *keep the sign*.

### Example

$$-5 + (-7)$$



**Think:** Both integers have the same signs and the signs are negative. Add their absolute values.

$$|-5| = 5$$

$$|-7| = 7$$

$$5 + 7 = 12$$

Keep the sign. The sign will be negative because both signs were negative. Therefore, the answer is -12.

$$-5 + -7 = -12$$



2. If the two integers have *opposite signs*, *subtract the absolute values*. The answer has the *sign* of the integer with the *greater absolute value*.

**Example**

$$-8 + 3$$



**Think:** Signs are opposite. Subtract the absolute values.

$$|-8| = 8$$

$$|3| = 3$$

$$8 - 3 = 5$$

The sign will be negative because -8 has the greater absolute value. Therefore, the answer is -5.

$$-8 + 3 = -5$$

**Example**

$$-6 + 8$$



**Think:** Signs are opposite. Subtract the absolute values.

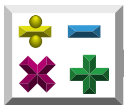
$$|-6| = 6$$

$$|8| = 8$$

$$8 - 6 = 2$$

The sign will be positive because 8 has a greater absolute value. Therefore, the answer is 2.

$$-6 + 8 = 2$$



### Example

$$5 + (-7)$$



**Think:** Signs are opposite. Subtract the absolute values.

$$|5| = 5$$

$$|-7| = 7$$

$$7 - 5 = 2$$

The sign will be negative because -7 has the greater absolute value. Therefore, the answer is -2.

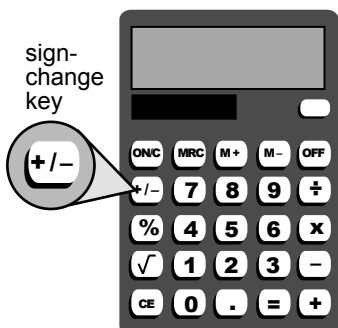
$$5 + -7 = -2$$

### Rules for Adding Integers

- The sum of two positive integers is  $(+) + (+) = +$  *positive*.
- The sum of two negative integers is  $(-) + (-) = -$  *negative*.
- The sum of a positive integer and a negative integer takes the *sign of the number with the greater absolute value*.  
 $(-) + (+) =$  } use sign of number with greater absolute value  
 $(+) + (-) =$  }
- The sum of a positive integer and a negative integer is zero if numbers have the *same absolute value*.  
 $(a) + (-a) = 0$   
 $(-a) + (a) = 0$



### Check Yourself Using a Calculator When Adding Positive and Negative Integers



Use a **calculator** with a  $\boxed{+/-}$  **sign-change** key.

For example, for  $-16 + 4$ , you would enter (for most calculators)  $16 \boxed{+/-} \boxed{+} 4 \boxed{=}$  and get the answer -12.





## Subtracting Integers

In the last section, we saw that 8 plus -3 equals 5.

$$8 + (-3) = 5$$

We know that 8 minus 3 equals 5.

$$8 - 3 = 5$$

Below are similar examples.

$$10 + (-7) = 3$$

$$12 + (-4) = 8$$

$$10 - 7 = 3$$

$$12 - 4 = 8$$

These three examples show that there is a connection between adding and subtracting. As a matter of fact, we can make any subtraction problem into an addition problem and any addition problem into a subtraction problem.

This idea leads us to the following definition.

<p><b>Definition of Subtraction</b> <math>a - b = a + (-b)</math></p>
---

### Examples

$$8 - 10 = 8 + (-10) = -2$$

$$12 - 20 = 12 + (-20) = -8$$

$$-2 - 3 = -2 + (-3) = -5$$

Even if we have

$$8 - (-8), \text{ this becomes}$$

8 plus the opposite of -8, which equals 8.

$$8 + [ -(-8) ] =$$

$$8 + 8 = 16$$



And  $-9 - (-3)$ , this becomes  
 $-9$  plus the opposite of  $-3$ , which equals  $3$ .  
 $-9 + -(-3)$   
 $-9 + 3 = -6$

**Shortcut** Two negatives become one positive!  
 $10 - (-3)$  becomes  $10$  plus  $3$ .  
 $10 + 3 = 13$

And  $-10 - (-3)$  becomes  $-10$  plus  $3$ .  
 $-10 + 3 = -7$

#### Generalization for Subtracting Integers

Subtracting an integer is the same as adding its opposite.

$$a - b = a + (-b)$$



#### Check Yourself Using a Calculator When Subtracting Negative Integers

Use a **calculator** with a  $\boxed{+/-}$  **sign-change** key.

For example, for  $18 - (-32)$ , you would enter  
 $18 \boxed{-} 32 \boxed{+/-} \boxed{=}$  and get the answer  $50$ .



## Practice

Answer the following.

- On May 22, 2004, in Ft. Worth, Texas, Annika Sorenstam became the first woman in 58 years to play on the PGA Tour. *Par* for the eighteen holes was 3 for four holes, 4 for twelve holes, and 5 for two holes, yielding a total par of 70 on the course. Sorenstam's scores on Day One in relation to par are provided in the table below. Determine her total for Day One.



**Note:** Par is the standard number of strokes a good golfer is expected to take for a certain hole on a given golf course. On this course, 70 is par. Therefore, add the total number of strokes in relation to par.

**Annika Sorenstam's Golf Scores for Day One**

Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Total Score Relative to Par	0	0	0	0	+1	0	0	0	+1	0	0	0	-1	0	0	0	0	0

Answer: \_\_\_\_\_

- Sorenstam's scores on Day One qualified her to continue to play on Day Two. However, her scores on Day Two did *not* qualify her to continue to play in the tournament. Sorenstam's scores in relation to par are provided in the table below. Determine her total for Day Two.

**Annika Sorenstam's Golf Scores for Day Two**

Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Total Score Relative to Par	0	-1	0	0	+1	+1	0	+1	0	+1	0	+1	0	0	0	0	0	0

Answer: \_\_\_\_\_



3. When an unknown integer is added to 12, the sum is *less than* -2. Give three examples of what the unknown number might be.

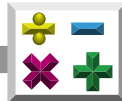
Answer: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

*Complete the following statements.*

4. a. The sum of *two positive numbers* is \_\_\_\_\_ (always, sometimes, never) positive.
- b. The sum of *two negative numbers* is \_\_\_\_\_ (always, sometimes, never) positive.
- c. The sum of *a number and its opposite* is \_\_\_\_\_ (always, sometimes, never) positive.
- d. The sum of *a positive number and a negative number* is \_\_\_\_\_ (always, sometimes, never) positive.



5. Complete the following statements.
- a. When a *positive* integer is subtracted from a *positive* integer,  
the result is \_\_\_\_\_ (always, sometimes, never) positive.
  - b. When a *negative* integer is subtracted from a *negative* integer,  
the result is \_\_\_\_\_ (always, sometimes, never) positive.
  - c. When a *negative* integer is subtracted from a *positive* integer,  
the result is \_\_\_\_\_ (always, sometimes, never) positive.
  - d. When a *positive* integer is subtracted from a *negative* integer,  
the result is \_\_\_\_\_ (always, sometimes, never) positive.



## Practice

Simplify *the following*.

1.  $6 - 4$

6.  $18 - 24$

2.  $5 - (-3)$

7.  $21 + (-3)$

3.  $-14 + 5$

8.  $26 - (-26)$

4.  $-12 - (-2)$

9.  $-37 + 17$

5.  $-57 + 3$

10.  $-37 - (-17)$



## Practice

**Simplify** the following **expressions**. Show **essential** steps.

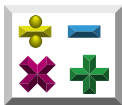
*Example:*  $5 - (8 + 3)$

$$\begin{array}{rcl} 5 - (8 + 3) & = & \\ 5 - 11 & = & -6 \end{array}$$

1.  $9 - (5 - 2 + 6)$
2.  $(7 - 3) + (-5 + 3)$
3.  $(5 + 32 - 36) + (12 + 5 - 10)$
4.  $(-26 + 15 - 13) - (4 - 16 + 43)$
5.  $(-15 + 3 - 7) - (26 - 14 + 10)$



**Check yourself:** The *sum* of the correct answers from numbers 1-5 above is -86.



## Multiplying Integers

What *patterns* do you notice?

$$3(4) = 12$$

$$3(-4) = -12$$

$$2 \bullet 4 = 8$$

$$2 \bullet -4 = -8$$

$$1(4) = 4$$

$$1(-4) = -4$$

$$0 \bullet 4 = 0$$

$$0 \bullet -4 = 0$$

$$-1(4) = -4$$

$$-1(-4) = 4$$

$$-2 \bullet 4 = -8$$

$$-2 \bullet -4 = 8$$

$$-3(4) = -12$$

$$-3(-4) = 12$$

Ask yourself:

- What is the sign of the **product** of two positive integers?

$$3(4) = 12 \quad 2 \bullet 4 = 8 \quad \text{positive}$$

- What is the sign of the *product* of two negative integers?

$$-1(-4) = 4 \quad -2 \bullet -4 = 8 \quad \text{positive}$$

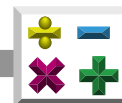
- What is the sign of the product of a positive integer and a negative integer or a negative integer and a positive integer?

$$3(-4) = -12 \quad -2 \bullet 4 = -8 \quad \text{negative}$$

- What is the sign of the product of any integer and 0?

$$0 \bullet 4 = 0 \quad 0 \bullet -4 = 0 \quad \text{neither; zero is neither positive nor negative}$$





You can see that the sign of a *product* depends on the signs of the numbers being multiplied. Therefore, you can use the following rules to multiply integers.

#### Rules for Multiplying Integers

- The product of two positive integers is *positive*.  $(+)(+) = +$
- The product of two negative integers is *positive*.  $(-)(-) = +$
- The product of two integers with different signs is *negative*.  $(+)(-) = -$   
 $(-)(+) = -$
- The product of any integer and 0 is 0.  $(a)(0) = 0$   
 $(-a)(0) = 0$



#### Check Yourself Using a Calculator When Multiplying Integers

Use a **calculator** with a  $\boxed{+/-}$  **sign-change** key.

For example, for  $-13 \cdot -7$ , you would enter

13  $\boxed{+/-}$   $\boxed{\times}$  7  $\boxed{+/-}$   $\boxed{=}$  and get the answer 91.



## Practice

**Simplify** the following. Do as many mentally as you can.

1.  $5 \times 6$

5.  $3 \times (-18)$

2.  $6 \times (-7)$

6.  $2 \times 4$

3.  $-4 \times 8$

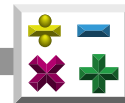
7.  $-20 \times (-20)$

4.  $-5 \times (-20)$

8.  $-6 \times (-6)$

9. The temperature was 83 degrees at 9:00 PM and dropped an average of 1.5 degrees per hour for the next 9 hours. What was the temperature at 6:00 AM?

*Answer:* \_\_\_\_\_ degrees



## Dividing Integers



**Think:**

1. What would you multiply 6 by to get 42?  
 $6 \cdot ? = 42$   
*Answer:* 7 because  $6 \cdot 7 = 42$
2. What would you multiply -6 by to get -54?  
 $-6 \cdot ? = -54$   
*Answer:* 9 because  $-6 \cdot 9 = -54$
3. What would you multiply -15 by to get 0?  
 $-15 \cdot ? = 0$   
*Answer:* 0 because  $-15 \cdot 0 = 0$



**Remember:** A **quotient** is the result of dividing two numbers.

### Example

42 divided by 7 results in a *quotient* of 6.

$$42 \div 7 = 6$$

↑  
quotient

To find the quotient of 12 and 4 we write:

$$4 \overline{)12} \quad \text{or} \quad 12 \div 4 \quad \text{or} \quad \frac{12}{4}$$

Each problem above is read “12 *divided by* 4.” In each form, the quotient is 3.

quotient ↓ 3 divisor → 4 $\overline{)12}$ ↑ dividend	or	divisor ↓ 12 ÷ 4 = 3 ↑     ↑ dividend     quotient	or	dividend ↓ $\frac{12}{4} = 3$ ← quotient ↑ divisor
---	----	--	----	--



In  $\frac{12}{4}$ , the bar separating 12 and 4 is called a *fraction bar*. Just as *subtraction* is the *inverse of addition*, *division* is the *inverse of multiplication*. This means that *division* can be *checked by multiplication*.

$$\begin{array}{r} 3 \\ 4 \overline{)12} \end{array} \quad \text{because} \quad 3 \cdot 4 = 12$$

Division of integers is *related to* multiplication of integers. The sign rules for division can be discovered by writing a related multiplication problem.

For example,

$$\frac{6}{2} = 3 \text{ because } 3 \cdot 2 = 6$$

$$\frac{-6}{2} = -3 \text{ because } -3 \cdot 2 = -6$$

$$\frac{6}{-2} = -3 \text{ because } 3 \cdot -2 = -6$$

$$\frac{-6}{-2} = 3 \text{ because } -3 \cdot -2 = 6$$

Below are the rules used to divide integers.

#### Rules for Dividing Integers

- The quotient of two positive integers is *positive*.  $(+) \div (+) = +$
- The quotient of two negative integers is *positive*.  $(-) \div (-) = +$
- The quotient of two integers with different signs is *negative*.  $(+) \div (-) = -$   
 $(-) \div (+) = -$
- The quotient of 0 divided by any nonzero integer is 0.  $0 \div a = 0$

Note the special division properties of 0.

$$0 \div 9 = 0$$

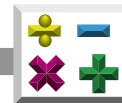
$$0 \div -9 = 0$$

$$\frac{0}{5} = 0$$

$$\frac{0}{-5} = 0$$

$$\begin{array}{r} 0 \\ 15 \overline{)0} \end{array}$$

$$\begin{array}{r} 0 \\ -15 \overline{)0} \end{array}$$



**Remember:** Division by 0 is *undefined*. The quotient of any number and 0 is not a number.

We say that  $\frac{9}{0}$ ,  $\frac{5}{0}$ ,  $\frac{15}{0}$ ,  $\frac{-9}{0}$ ,  $\frac{-5}{0}$ , and  $\frac{-15}{0}$  are *undefined*.

Likewise,  $\frac{0}{0}$  is undefined.

For example, try to divide 134 by 0. To divide, think of the related multiplication problem.

$$? \times 0 = 134$$

Any number times 0 is 0—so mathematicians say that division by 0 is undefined.

**Note:** On most calculators, if you divide by 0, you will get an *error* indicator.



### Check Yourself Using a Calculator When Dividing Integers

Use a **calculator** with a  $\boxed{+/-}$  **sign-change** key.

For example, for  $\frac{-54}{9}$ , you would enter

54  $\boxed{+/-}$   $\boxed{\div}$  9  $\boxed{=}$  and get the answer -6.



## Practice

**Simplify** the following. Do as many mentally as you can.

1.  $35 \div 5$

6.  $-400 \div 25$

2.  $49 \div (-7)$

7.  $-625 \div (-25)$

3.  $225 \div (-15)$

8.  $1,000 \div (-10)$

4.  $-121 \div 11$

9.  $-1,000 \div 100$

5.  $169 \div (-13)$

10.  $-10,000 \div (-100)$

11. The temperature of 69 degrees dropped to 44 degrees at an average rate of 6.25 degrees per hour. How many hours did the total drop of 25 degrees require?

Answer: \_\_\_\_\_ hours



## Practice

**Simplify** *the following. Show essential steps.*

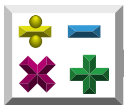
Example:  $\frac{2(-3 \cdot 6)}{-4}$

$$\frac{2(-3 \cdot 6)}{-4} =$$
$$\frac{2(-18)}{-4} =$$
$$\frac{-36}{-4} =$$
$$9$$

1.  $\frac{(6)(-5)(3)}{9}$

2.  $(-3)(5)(\frac{4}{3})(-2)$

3.  $(\frac{1}{2})(-4)(0)(5)$



4.  $\frac{-3(4)(-2)(5)}{(-16)}$

5.  $\frac{6(\frac{4}{7})(-\frac{3}{2})(-2)}{-(\frac{3}{7})}$

6.  $\left[ \frac{7 - (-3)}{5 - 3} \right] \left[ \frac{4 + (-8)}{3 - 5} \right]$

7.  $\left[ \frac{12 + (-2)}{3 + (-8)} \right] \left[ \frac{6 + (-15)}{8 - 5} \right]$

8.  $\frac{3(3 + 2) - 3 \cdot 3 + 2}{3 \cdot 2 + 2(2 - 1)}$





## Practice

Use the given **value** of each **variable** to **evaluate each expression**. Show **essential steps**.

Example: Evaluate  $5\left(\frac{F-32}{9}\right)$

$$F = 212$$

Replace  $F$  with 212 and simplify.

$$5\left(\frac{212-32}{9}\right) =$$

$$5\left(\frac{180}{9}\right) =$$

$$5(20) =$$

$$100$$

1.  $E = 18 \quad e = 2 \quad R = 6$

$$\frac{E-e}{R}$$

2.  $P = 1,000 \quad r = 0.04 \quad t = 5$

$$P + Prt$$

3.  $r = 8 \quad h = 6$

$$2r(r + h)$$



## Practice

**Simplify** the following. Show **essential steps**.

Example:  $(4 + 1)^2 - \frac{4 \cdot 3^2}{6}$

$$(4 + 1)^2 - \frac{4 \cdot 3^2}{6} =$$

$$5^2 - \frac{4 \cdot 9}{6} =$$

$$25 - \frac{36}{6} =$$

$$25 - 6 = 19$$

1.  $\frac{8 \cdot 2^2}{4^2} + (3 \cdot 1)^2$

2.  $\frac{5^2 \cdot 3^2}{4} - (2 + 1)^2$

3.  $\frac{3^2 \cdot 2^2}{7 - 2^2} + \frac{(-3)(2)^2}{6 - 3}$



$$4. \quad \frac{7^2 - 6^2}{10 + 3} + \frac{8^2 \cdot (-2)}{(-2)^4}$$

$$5. \quad \frac{(-5)^2 - 3^2}{4 - 6} + \frac{-(3)^2 \cdot 2}{5 + 1}$$

Use the given **value** of each **variable** to **evaluate** the following expressions.  
Show **essential steps**.

$x = 3$	$y = -2$
---------	----------

$$6. \quad \frac{-xy^2}{6} + 2x^2y$$

$$7. \quad (x + y)^2 + (x - y)^2$$



## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |       |  |                         |
|-------|--|-------------------------|
| _____ | 1. the order in which two numbers are added or multiplied does <i>not</i> change their sum or product, respectively  | A. absolute value       |
| _____ | 2. the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right) | B. additive identity    |
| _____ | 3. a number's distance from zero (0) on a number line; distance expressed as a positive value  | C. associative property |
| _____ | 4. the number of times the base occurs as a factor   | D. commutative property |
| _____ | 5. any symbol, usually a letter, which could represent a number  | E. exponent             |
| _____ | 6. the way in which three or more numbers are grouped for addition or multiplication does <i>not</i> change their sum or product, respectively   | F. order of operations  |
| _____ | 7. the number zero (0); when zero (0) is added to another number the sum is the number itself  | G. variable             |



## **Lesson Three Purpose**

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

#### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).



## Algebraic Expressions

A mathematical expression with a letter in it is called an **algebraic expression**. The letter represents an unknown or mystery number. The letter used can be any letter in the alphabet.

For example:  $7n$  means 7 times some number,  $n$ .

We use *algebraic expressions* to help us **solve equations**. Before we can use them, we must be able to translate them. Look at the following expressions translated into algebraic expressions.

- eight *more than* a number is expressed as

$$\underline{r + 8}$$

- sixteen *less than* a number is written as

$$\underline{y - 16}$$

- the *product* of a number and 12 looks like

$$\underline{12x}$$

- the **difference** between 19 and  $e$  is written as

$$\underline{19 - e}$$

- 4 *less than* 6 times a number means

$$\underline{6d - 4}$$

- the *quotient* of 18 and a number is

$$\underline{18 \div y \text{ or } \frac{18}{y}}$$

- four **cubed** is written as

$$\underline{4^3}$$

- three **squared** is written as

$$\underline{3^2}$$



## Practice

Translate *the following expressions into algebraic expressions.*

1. four *times* a number

---

2. a number *times* four

---

3. eleven *more than* a number

---

4. eleven *increased by* a number

---

5. the *quotient* of 15 and a number

---

6. the *quotient* of a number and 15

---

7. seven *squared*

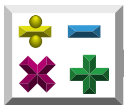
---

8. eight *cubed*

---

9. three *more than* twice a number

---



10. twice a number *less* three

\_\_\_\_\_

11. three *less than* twice a number

\_\_\_\_\_

12. twice the *sum* of a number and 21

\_\_\_\_\_

13. one-half the *square* of a number

\_\_\_\_\_

14. 22 *increased* by 4 *times* the square of a number

\_\_\_\_\_





## Practice

Translate *the following* algebraic expressions into words.

1.  $6y$

---

2.  $c - 5$

---

3.  $5 - c$

---

4.  $s + 21$

---

5.  $21 + s$

---

6.  $10r^2$

---

7.  $3d + 7$

---

8.  $8x - 11$

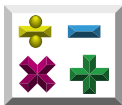
---

9.  $6(v + 9)$

---

10.  $\frac{1}{2}(5 + x^3)$

---



## **Lesson Four Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Algebra Body of Knowledge**

#### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.



## Working with Absolute Value

As discussed earlier in this unit, the *absolute value* of a number is actually the distance that number is from zero on a number line. Because distance is *always* positive, the result when taking the absolute value of a number is *always* positive.

The symbols for absolute value  $| \ |$  can also act as *grouping symbols*. Perform any operations within the grouping symbols first, just as you would within parentheses.

Look at these examples. Notice the digits are the same in each pair, but the answers are different due to the placement of the absolute value marks.

$$\begin{aligned}|-7| + |5| &= 7 + 5 = 12 \\|-7 + 5| &= |-2| = 2\end{aligned}$$

$$\begin{aligned}|6| - |-10| &= 6 - 10 = -4 \\|6 - -10| &= |6 + 10| = |16| = 16\end{aligned}$$



## Practice

Answer the following. Perform any operations *within* the grouping symbols first.

1.  $|-23 + 37|$

2.  $|21 - 44|$

3.  $|16 + 4| - |32|$

4.  $|16 + 4| - |-32|$

5.  $22 - |-10| + |56|$



**Check yourself:** The *sum* of the answers from numbers 1-5 is  $|-81|$ .



## Practice

Use the given **value** for each **variable** to **evaluate** the following expressions. Perform any operations *within* the grouping symbols first.

$a = -5$	$b = 7$	$c = -9$
----------	---------	----------

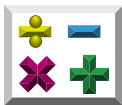
1.  $|a| + |b| - |c|$

2.  $|a + b| - |c|$

3.  $|c - a| - |b|$

4.  $|b + c| + |a|$

5.  $|c - b| + |a|$



Use the given **value** for each **variable** to **evaluate** the following expressions. Perform any operations *within* the grouping symbols first.

$a = -5$	$b = 7$	$c = -9$
----------	---------	----------

6.  $|a + c| - |-c|$

7.  $|a + b + c| - |c - b|$

8.  $|a| + |b| + |c|$

9.  $a - |b| - |c|$

10.  $a + |-b| - |c|$



## Practice

Use the given **value** for each **variable** to **evaluate** the following expressions. Perform any operations *within* the grouping symbols first.

$a = 6$	$b = -7$	$c = -8$
---------	----------	----------

1.  $|a| + |b| - |c|$

2.  $|a + b| - |c|$

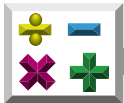
3.  $|c - a| - |b|$

4.  $|a + b + c| - |c - b|$

5.  $|a| + |b| + |c|$

6.  $a - |b| - |c|$

7.  $a + |-b| - |c|$



*Answer the following. Perform any operations within the grouping symbols first.*

8.  $|-33 + 57|$

9.  $|16 - 34|$

10.  $|26 + 4| - |36|$

11.  $|26 + 4| - |-36|$

12.  $22 - |20| + |32|$





## Practice

Use the list below to complete the following statements.

element or member	irrational	rational
even	finite	real numbers
grouping symbols	odd	variable

1. The color green is a(n) \_\_\_\_\_ of the set of colors in the rainbow.
2. A(n) \_\_\_\_\_ a real number that *cannot* be expressed as a ratio of two integers.
3. { } and [ ] are examples of \_\_\_\_\_ .
4. Rational numbers and irrational numbers together make up the set of \_\_\_\_\_
5. Any symbol, usually a letter, which could represent a number in a mathematical expression is a \_\_\_\_\_ .
6. A \_\_\_\_\_ is a number can be expressed as a ratio  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$ .
7. Any integer *not* divisible by 2 is called a(n) \_\_\_\_\_ integer.
8. Any integer divisible by 2 is called a(n) \_\_\_\_\_ integer.
9. A set that has bounds and is limited and a whole number can represent its number of elements is a \_\_\_\_\_ set.



## Unit Review

Specify the following sets by **listing the elements** of each.

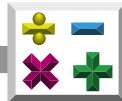
1. {whole numbers less than 8} \_\_\_\_\_
2. {odd counting numbers less than 12} \_\_\_\_\_
3. {even integers between -5 and 6} \_\_\_\_\_

Write **finite** if the set has bounds and is limited. Write **infinite** if the set has no boundaries and is not limited.

- \_\_\_\_\_ 4. {the colors in a crayon box}
- \_\_\_\_\_ 5. {rational numbers}
- \_\_\_\_\_ 6. {negative integers}

Write **True** if the statement is correct. Write **False** if the statement is not correct.

- \_\_\_\_\_ 7.  $\pi$  is rational.
- \_\_\_\_\_ 8. 0 is a whole number.
- \_\_\_\_\_ 9. -9 is a counting number.



Complete the following statements.

10. The sum of a *positive number* and a *negative number* is \_\_\_\_\_ (always, sometimes, never) positive.
11. The difference between a *negative number* and its *opposite* is \_\_\_\_\_ (always, sometimes, never) zero.

**Simplify the following. Show essential steps.**



**Remember:** Order of operations—Please Pardon My Dear Aunt Sally. (Also known as Please Excuse My Dear Aunt Sally.)

12.  $\frac{(5)(-2)(7)}{10}$

13.  $\frac{(-6)(4) - (8)(2)}{9 - 4}$

14.  $\left[ \frac{16 - (-4)}{10 - 6} \right] \left[ \frac{19 + (-8)}{(-2)(3)} \right]$



Use the given **value** of each **variable** to **evaluate** each expression. Show **essential steps**.

15.  $P = 100 \quad r = 0.02 \quad t = 6$

$$Prt$$

16.  $r = 6 \quad h = 8$

$$2r(r + h)$$

17.  $x = -2 \quad y = 3$

$$\frac{-xy^2}{6} + 2xy^2$$

**Simplify** the following. Show **essential steps**.

18.  $\frac{5^2 + (2^2 - 1)^3}{3^2 - 5}$

19.  $\frac{3^2 \cdot 2(4)}{3^2 - 2^2 + 1} + \frac{5^2 + 7}{2^3}$



**Translate the following expressions into algebraic expressions.**

20. eight *more than* a number

\_\_\_\_\_

21. 16 *less than* 2 times a number

\_\_\_\_\_

22. four *more than* the sum of 13 and the square of a number

\_\_\_\_\_

**Translate the following algebraic expressions into words.**

23.  $8c - 5$

\_\_\_\_\_

24.  $4(x^3 + 7)$

\_\_\_\_\_

25.  $13(x + 9)$

\_\_\_\_\_



Answer the following. **Perform any operations *within* the grouping symbols first.**

26.  $|13 - 24|$

27.  $|-19 + 17| - |41 + 8|$

28.  $36 - |14 - 10| + |3 - 15|$

Use the given **value** for each **variable** to evaluate the following expressions. **Perform any operations *within* the grouping symbols first. Show essential steps.**

$$a = -6$$

$$b = -2$$

$$c = 4$$

29.  $|a + b| - |c - a|$

30.  $|b + c| + |-a - b|$

## **Unit 2: Algebraic Thinking**

This unit emphasizes strategies used to solve equations and understand and solve inequalities.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Writing Process Strand**

##### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

##### **Standard 3: Linear Equations and Inequalities**

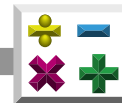
- MA.912.A.3.1  
Solve linear equations in one variable that include simplifying algebraic expressions.
- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.3  
Solve literal equations for a specified variable.
- MA.912.A.3.4  
Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

**Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.





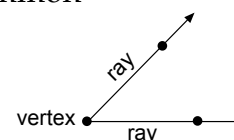
## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

**additive identity** .....the number zero (0); when zero (0) is added to another number the sum is the number itself  
*Example:*  $5 + 0 = 5$

**additive inverses** .....a number and its opposite whose sum is zero (0); also called *opposites*  
*Example:* In the equation  $3 + (-3) = 0$ , the additive inverses are 3 and -3.

**angle ( $\angle$ )** .....two rays extending from a common endpoint called the vertex; measures of angles are described in degrees ( $^\circ$ )



**area (A)** .....the measure, in square units, of the inside region of a closed two-dimensional figure; the number of square units needed to cover a surface  
*Example:* A rectangle with sides of 4 units by 6 units has an area of 24 square units.

**associative property** .....the way in which three or more numbers are grouped for addition or multiplication does *not* change their sum or product, respectively  
*Examples:*  $(5 + 6) + 9 = 5 + (6 + 9)$  or  
 $(2 \times 3) \times 8 = 2 \times (3 \times 8)$

**commutative property** .....the order in which any two numbers are added or multiplied does *not* change their sum or product, respectively  
*Examples:*  $2 + 3 = 3 + 2$  or  
 $4 \times 7 = 7 \times 4$



**consecutive** .....in order

*Example:* 6, 7, 8 are consecutive whole numbers  
and 4, 6, 8 are consecutive even numbers.

**cube (power)** .....the third power of a number

*Example:*  $4^3 = 4 \times 4 \times 4 = 64$ ;  
64 is the cube of 4

**cubic units** .....units for measuring volume

**decrease** .....to make less

**degree ( $^{\circ}$ )** .....common unit used in measuring angles

**difference** .....a number that is the result of subtraction

*Example:* In  $16 - 9 = 7$ , the difference is 7.

**distributive property** .....the product of a number and the sum or  
difference of two numbers is equal to the sum  
or difference of the two products

*Examples:*  $x(a + b) = ax + bx$   
 $5(10 + 8) = 5 \cdot 10 + 5 \cdot 8$

**equation** .....a mathematical sentence stating that the two  
expressions have the same value

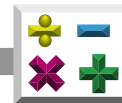
*Example:*  $2x = 10$

**equivalent**

**(forms of a number)** .....the same number expressed in different forms

*Example:*  $\frac{3}{4}$ , 0.75, and 75%

**even integer** .....any integer divisible by 2; any integer with  
the digit 0, 2, 4, 6, or 8 in the units place; any  
integer in the set  $\{\dots, -4, -2, 0, 2, 4, \dots\}$



- expression** ..... a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables  
*Examples:*  $4r^2$ ;  $3x + 2y$ ;  
An expression does not contain equal (=) or inequality (<, >,  $\leq$ ,  $\geq$ , or  $\neq$ ) signs.
- formula** ..... a way of expressing a relationship using variables or symbols that represent numbers
- graph of a number** ..... the point on a number line paired with the number
- increase** ..... to make greater
- inequality** ..... a sentence that states one expression is greater than (>), greater than or equal to ( $\geq$ ), less than (<), less than or equal to ( $\leq$ ), or not equal to ( $\neq$ ) another expression  
*Examples:*  $a \neq 5$  or  $x < 7$  or  $2y + 3 \geq 11$
- integers** ..... the numbers in the set  
 $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
- inverse operation** ..... an action that undoes a previously applied action  
*Example:* Subtraction is the inverse operation of addition.
- irrational number** ..... a real number that cannot be expressed as a ratio of two integers  
*Example:*  $\sqrt{2}$
- length ( $l$ )** ..... a one-dimensional measure that is the measurable property of line segments



**like terms** ..... terms that have the same variables and the same corresponding exponents  
*Example:* In  $5x^2 + 3x^2 + 6$ , the like terms are  $5x^2$  and  $3x^2$ .

**measure (m)**

**of an angle ( $\angle$ )** ..... the number of degrees ( $^\circ$ ) of an angle

**multiplicative identity** ..... the number one (1); the product of a number and the multiplicative identity is the number itself  
*Example:*  $5 \times 1 = 5$

**multiplicative inverse** ..... any two numbers with a product of 1; also called *reciprocals*  
*Example:* 4 and  $\frac{1}{4}$ ; zero (0) has no multiplicative inverse

**multiplicative**

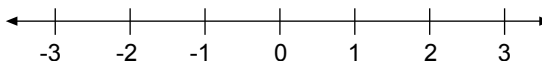
**property of -1** ..... the product of any number and -1 is the opposite or additive inverse of the number  
*Example:*  $-1(a) = -a$  and  $a(-1) = -a$

**multiplicative**

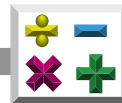
**property of zero** ..... for any number  $a$ ,  $a \cdot 0 = 0$  and  $0 \cdot a = 0$

**negative numbers** ..... numbers less than zero

**number line** ..... a line on which numbers can be written or visualized



**odd integer** ..... any integer not divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set  $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$



**order of operations** .....the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations*

*Example:*  $5 + (12 - 2) \div 2 - 3 \times 2 =$

$$5 + 10 \div 2 - 3 \times 2 =$$

$$5 + 5 - 6 =$$

$$10 - 6 =$$

$$4$$

**perimeter ( $P$ )** .....the distance around a figure

**positive numbers** .....numbers greater than zero

**power (of a number)** .....an exponent; the number that tells how many times a number is used as a factor

*Example:* In  $2^3$ , 3 is the power.

**product** .....the result of multiplying numbers together

*Example:* In  $6 \times 8 = 48$ , the product is 48.

**quotient** .....the result of dividing two numbers

*Example:* In  $42 \div 7 = 6$ , the quotient is 6.

**ratio** .....the comparison of two quantities

*Example:* The ratio of  $a$  and  $b$  is  $a:b$  or  $\frac{a}{b}$ , where  $b \neq 0$ .

**rational number** .....a number that can be expressed as a ratio  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$

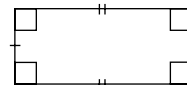
**real numbers** .....the set of all rational and irrational numbers



**reciprocals** .....any two numbers with a product of 1; also called *multiplicative inverse*

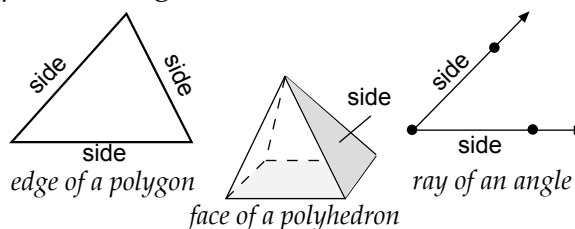
*Examples:* 4 and  $\frac{1}{4}$  are reciprocals because  $\frac{4}{1} \times \frac{1}{4} = 1$ ;  $\frac{3}{4}$  and  $\frac{4}{3}$  are reciprocals because  $\frac{3}{4} \times \frac{4}{3} = 1$ ; zero (0) has no multiplicative inverse

**rectangle** .....a parallelogram with four right angles



**side** .....the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle

*Example:* A triangle has three sides.



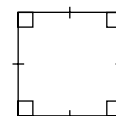
**simplify an expression** .....to perform as many of the indicated operations as possible

**solution** .....any value for a variable that makes an equation or inequality a true statement

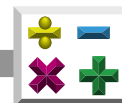
*Example:* In  $y = 8 + 9$   
 $y = 17$  17 is the solution.

**solve** .....to find all numbers that make an equation or inequality true

**square** .....a rectangle with four sides the same length



**square (of a number)** .....the result when a number is multiplied by itself or used as a factor twice  
*Example:* 25 is the square of 5.



**square units** .....units for measuring area; the measure of the amount of an area that covers a surface

**substitute** .....to replace a variable with a numeral  
*Example:*  $8(a) + 3$   
 $8(5) + 3$

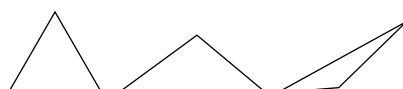
**substitution property of equality** .....for any numbers  $a$  and  $b$ , if  $a = b$ , then  $a$  may be replaced by  $b$

**sum** .....the result of adding numbers together  
*Example:* In  $6 + 8 = 14$ , the sum is 14.

**symmetric property of equality** .....for any numbers  $a$  and  $b$ , if  $a = b$ , then  $b = a$

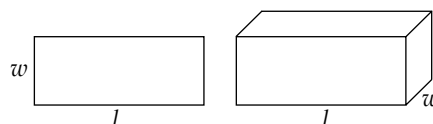
**table (or chart)** .....a data display that organizes information about a topic into categories

**triangle** .....a polygon with three sides



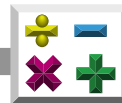
**variable** .....any symbol, usually a letter, which could represent a number

**width ( $w$ )** .....a one-dimensional measure of something side to side









## Unit 2: Algebraic Thinking

### Introduction

Algebraic thinking provides tools for looking at situations. You can state, simplify, and show relationships through algebraic thinking. When combining algebraic symbols with algebraic thinking, you can record information or ideas and gain insights into solving problems.

In this lesson you will use what you have learned to solve equations and inequalities on a more advanced level.

### Lesson One Purpose

#### Reading Process Strand

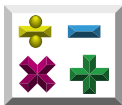
##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### Writing Process Strand

##### Standard 3: Prewriting

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.



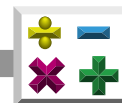
## **Algebra Body of Knowledge**

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.1  
Solve linear equations in one variable that include simplifying algebraic expressions.
- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.
- MA.912.A.3.4  
Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.



## Solving Equations

A mathematical sentence that contains an equal sign (=) is called an **equation**. An *equation* is a mathematical sentence stating that the two **expressions** have the same value. An *expression* is a mathematical phrase, or part of a number sentence that contains numbers, operation signs, and sometimes **variables**.

We also learned the rules to add and subtract and to multiply and divide **positive numbers** and **negative numbers**.

Rules for Adding and Subtracting Positive and Negative Integers				
(+)	+	(+)	=	+
(+)	-	(+)	=	positive if first number is greater, otherwise it is negative
(-)	+	(-)	=	-
(-)	-	(-)	=	positive if first number is greater, otherwise it is negative
(+)	+	(-)	=	use sign of <b>integer</b> with greater absolute value
(-)	+	(+)	=	
(+)	-	(-)	=	+
(-)	-	(+)	=	-

Rules for Multiplying and Dividing Positive and Negative Integers				
(+)	•	(+)	=	+
(-)	•	(-)	=	+
(+)	•	(-)	=	-
(-)	•	(+)	=	-
(+)	÷	(+)	=	+
(-)	÷	(-)	=	+
(+)	÷	(-)	=	-
(-)	÷	(+)	=	-



To **solve** the equation is to find the number that we can **substitute** for the *variable* to make the equation true.

Study these examples. Each equation has been *solved* and then checked by substituting the answer for the variable in the original equation. If the answer makes the equation a true sentence, it is called the **solution** of the equation.

Solve:

$$\begin{aligned}n + 14 &= -2 \\n + 14 - 14 &= -2 - 14 \\n &= -2 + -14 \\n &= -16\end{aligned}$$

Check:

$$\begin{aligned}n + 14 &= -2 \\-16 + 14 &= -2 \\-2 &= -2 \quad \text{It checks!}\end{aligned}$$

Solve:

$$\begin{aligned}-6x &= -66 \\\frac{-6x}{-6} &= \frac{-66}{-6} \\x &= 11\end{aligned}$$

Check:

$$\begin{aligned}-6x &= -66 \\-6(11) &= -66 \\-66 &= -66 \quad \text{It checks!}\end{aligned}$$

Solve:

$$\begin{aligned}y - (-6) &= 2 \\y + 6 - 6 &= 2 - 6 \\y &= 2 + -6 \\y &= -4\end{aligned}$$

Check:

$$\begin{aligned}y - (-6) &= 2 \\-4 - (-6) &= 2 \\-4 + 6 &= 2 \\2 &= 2 \quad \text{It checks!}\end{aligned}$$

Solve:

$$\begin{aligned}\frac{y}{-10} &= 5 \\(-10)\frac{y}{-10} &= 5(-10) \\y &= -50\end{aligned}$$

Check:

$$\begin{aligned}\frac{y}{-10} &= 5 \\\frac{-50}{-10} &= 5 \\5 &= 5 \quad \text{It checks!}\end{aligned}$$



## Practice

Solve each equation and **check**. Show **essential steps**.

1.  $y + 12 = 2$

2.  $a - (-2) = 2$

3.  $r + 15 = -25$

4.  $0 = y + -46$

5.  $15y = -30$



6.  $\frac{y}{15} = -2$

7.  $\frac{x}{5} = -9$

8.  $-9y = 270$

9.  $m - 9 = -8$

10.  $3 + x = -3$



11.  $\frac{n}{-5} = -2$

12.  $-55 = -5a$

13.  $12 = -6 + x$

14.  $t - 20 = -15$



## Interpreting Words and Phrases

Words and phrases can suggest relationships between numbers and mathematical operations. In Unit 1 we learned how words and phrases can be translated into mathematical expressions. Appendix B also contains a list of mathematical symbols and their meanings.

Relationships between numbers can be indicated by words such as **consecutive**, *preceding*, *before*, and *next*. Also, the same mathematical expression can be used to translate many different word expressions.

Below are some of the words and phrases we associate with the four mathematical operations and with powers of a number.

**Mathematical Symbols and Words**

<b>+</b>	<b>–</b>	<b>x</b>	<b>÷</b>	<b>power</b>
add	subtract	multiply	divide	power
sum	difference	product	quotient	square
plus	minus	times		cube
total	remainder	of		
more than	less than	twice		
increased by	decreased by	doubled		





## Practice

Write an **equation** and **solve** the problem.

*Example:* Sixteen less than a number  $n$  is 48. What is the number?



**Remember:** The word *is* means *is equal to* and translates to an = sign.

$$\begin{array}{rcl} 16 \text{ less than a number } n & = & 48 \\ \downarrow & & \downarrow \\ n - 16 & = & 48 \\ n - 16 + 16 & = & 48 + 16 \\ n & = & 64 \end{array}$$

*Note:* To write 16 less than  $n$ , you write  $n - 16$ .

So  $64 - 16 = 48$  or 16 less than 64 is 48.

1. A number increased by 9 equals -7. What is the number?  
(Let  $d$  = the number.)
2. A number times -12 equals -72. What is the number?  
(Let  $x$  = the number.)
3. A number decreased by 5 equals -9. What is the number?  
(Let  $y$  = the number.)



4. A number divided by 7 equals -25. What is the number?  
(Let  $n$  = the number.)

5. In a card game, Ann made 30 points on her first hand.  
After the second hand, her total score was 20 points.  
What was her score on the second hand?



6. A scuba diver is at the -30 foot level. How many feet  
will she have to rise to be at the -20 foot level?





## Solving Two-Step Equations

When solving an equation, you want to get the *variable* by itself on one side of the equal sign. You do this by *undoing* all the operations that were done on the variable. In general, undo the addition or subtraction first. Then undo the multiplication or division.

Study the following examples.

A. Solve:

$$\begin{aligned}2y + 2 &= 30 \\2y + 2 - 2 &= 30 - 2 && \leftarrow \text{subtract 2 from each side} \\ \frac{2y}{2} &= \frac{28}{2} && \leftarrow \text{divide each side by 2} \\ y &= 14\end{aligned}$$

Check:

$$\begin{aligned}2y + 2 &= 30 \\2(14) + 2 &= 30 && \leftarrow \text{replace } y \text{ with } 14 \\28 + 2 &= 30 \\30 &= 30 && \text{It checks!}\end{aligned}$$

B. Solve:

$$\begin{aligned}2x - 7 &= -29 \\2x - 7 + 7 &= -29 + 7 && \leftarrow \text{add 7 to each side} \\ \frac{2x}{2} &= \frac{-22}{2} && \leftarrow \text{divide each side by 2} \\ x &= -11\end{aligned}$$

Check:

$$\begin{aligned}2x - 7 &= -29 \\2(-11) - 7 &= -29 && \leftarrow \text{replace } x \text{ with } -11 \\-22 - 7 &= -29 \\-29 &= -29 && \text{It checks!}\end{aligned}$$



C. Solve:

$$\frac{n}{7} + 18 = 20$$

$$\frac{n}{7} + 18 - 18 = 20 - 18 \quad \leftarrow \text{subtract 18 from each side}$$

$$\cancel{(7)} \frac{n}{\cancel{7}} = 2(7) \quad \leftarrow \text{multiply each side by 7}$$

$$n = 14 \quad \leftarrow \text{simplify both sides}$$

Check:

$$\frac{n}{7} + 18 = 20$$

$$\frac{14}{7} + 18 = 20 \quad \leftarrow \text{replace } n \text{ with 14}$$

$$2 + 18 = 20$$

$$20 = 20 \quad \text{It checks!}$$

D. Solve:

$$\frac{t}{-2} + 4 = -10$$

$$\frac{t}{-2} + 4 - 4 = -10 - 4 \quad \leftarrow \text{subtract 4 from each side}$$

$$\cancel{(-2)} \frac{t}{\cancel{-2}} = -14(-2) \quad \leftarrow \text{multiply each side by -2}$$

$$t = 28 \quad \leftarrow \text{simplify both sides}$$

Check:

$$\frac{t}{-2} + 4 = -10$$

$$\frac{28}{-2} + 4 = -10 \quad \leftarrow \text{replace } t \text{ with 28}$$

$$-14 + 4 = -10$$

$$-10 = -10 \quad \text{It checks!}$$



## Practice

Solve *each equation and check. Show essential steps.*

1.  $4x + 8 = 16$

2.  $4y - 6 = 10$

3.  $5n + 3 = -17$

4.  $2y - 6 = -18$

5.  $-8y - 21 = 75$

6.  $\frac{a}{8} - 17 = 13$



7.  $13 + \frac{x}{-3} = -4$

8.  $\frac{n}{8} + 1 = 4$

9.  $-3b + 5 = 20$

10.  $6 = \frac{x}{4} - 14$

11.  $-7y + 9 = -47$

12.  $\frac{n}{-6} - 17 = -8$



Use the list below to decide which **equation** to use to solve each problem. Then **solve** the problem.

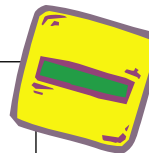


**Equation A:**  $\frac{n}{4} + 2 = 10$

**Equation B:**  $\frac{n}{4} - 2 = 10$

**Equation C:**  $4n + 2 = 10$

**Equation D:**  $4n - 2 = 10$



13. Two more than the product of 4 and Ann's age is 10.

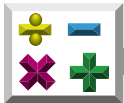
Equation: \_\_\_\_\_

How old is Ann?  $n =$  \_\_\_\_\_

14. If you multiply Sean's age by 4 and then subtract 2, you get 10.

Equation: \_\_\_\_\_

What is Sean's age?  $n =$  \_\_\_\_\_



15. If you divide Joe's age by 4 and then add 2, you get 10.

Equation: \_\_\_\_\_

What is Joe's age?  $n =$  \_\_\_\_\_

16. Divide Jenny's age by 4, then subtract 2, and get 10.

Equation: \_\_\_\_\_

What is Jenny's age?  $n =$  \_\_\_\_\_

*Circle the letter of the correct answer.*

17. The sentence that means the same as the equation

$\frac{1}{3}y + 8 = 45$  is \_\_\_\_\_ .

- a. Eight *more than* one-third of  $y$  is 45.
- b. One-third of  $y$  is eight *more than* 45.
- c.  $y$  is eight *less than* one-third of 45.
- d.  $y$  is eight *more than* one-third of 45.





## Special Cases

### Reciprocals: Two Numbers Whose Product is 1

**Note:**  $5 \cdot \frac{1}{5} = 1$  and  $\frac{5}{5} = 1$

When you multiply 5 by  $\frac{1}{5}$  and divide 5 by 5, both equations yield 1.

We see that 5 is the **reciprocal** of  $\frac{1}{5}$  and  $\frac{1}{5}$  is the *reciprocal* of 5. Every number but zero has a reciprocal. (Division by zero is undefined.) Two numbers are reciprocals if their product is 1.

Below are some examples of numbers and their reciprocals.

Number	Reciprocal
$-\frac{1}{4}$	-4
1	1
$-\frac{2}{3}$	$-\frac{3}{2}$
$\frac{7}{8}$	$\frac{8}{7}$
-2	$-\frac{1}{2}$
$\frac{1}{7}$	7
$x$	$\frac{1}{x}$

#### Multiplication Property of Reciprocals

any nonzero number times its reciprocal is 1

$$x \cdot \frac{1}{x} = 1$$

If  $x \neq 0$



**Remember:** When two numbers are reciprocals of each other, they are also called **multiplicative inverses** of each other.



Study the following two examples.

**Method 1: Division Method**

$$\begin{aligned}
 5x - 6 &= 9 \\
 5x - 6 + 6 &= 9 + 6 \\
 5x &= 15 \\
 \frac{5x}{5} &= \frac{15}{5} \\
 x &= 3
 \end{aligned}$$

**Method 2: Reciprocal Method**

$$\begin{aligned}
 5x - 6 &= 9 \\
 5x - 6 + 6 &= 9 + 6 \\
 5x &= 15 \\
 \frac{1}{5} \bullet 5x &= \frac{1}{5} \bullet 15 \\
 x &= 3
 \end{aligned}$$

Both methods work well. However, the *reciprocal method* is probably easier in the next two examples, which have fractions.

$$\begin{aligned}
 -\frac{1}{5}x - 1 &= 9 \\
 -\frac{1}{5}x - 1 + 1 &= 9 + 1 \\
 -\frac{1}{5}x &= 10 \\
 -5 \bullet -\frac{1}{5}x &= -5 \bullet 10 \quad \leftarrow \text{multiply by reciprocal of } -\frac{1}{5} \text{ which is } -5 \\
 x &= -50
 \end{aligned}$$

Here is another equation with fractions.

$$\begin{aligned}
 -\frac{3}{4}x + 12 &= 36 \\
 -\frac{3}{4}x + 12 - 12 &= 36 - 12 \\
 -\frac{3}{4}x &= 24 \\
 -\frac{4}{3} \bullet -\frac{3}{4}x &= -\frac{4}{3} \bullet 24 \quad \leftarrow \text{multiply by reciprocal of } -\frac{3}{4} \text{ which is } -\frac{4}{3} \\
 1 \bullet x &= -32 \\
 x &= -32
 \end{aligned}$$



## Multiplying by -1

Here is another equation which sometimes gives people trouble.

$$5 - x = -10$$



**Remember:**  $5 - x$  is not the same thing as  $x - 5$ . To solve this equation we need to make the following observation.

### Property of Multiplying by -1

**-1 times a number equals the opposite of that number**

$$-1 \cdot x = -x$$

This property is also called the **multiplicative property of -1**, which says the *product* of any number and -1 is the opposite or **additive inverse** of the number.

See the following examples.

$$-1 \cdot 5 = -5$$

$$-1 \cdot (-6) = 6$$



Now let's go back to  $5 - x = -10$  using the property of multiplying by  $-1$ . We can rewrite the equation as follows.

$$\begin{aligned}
 5 - 1x &= -10 \\
 5 - 1x - 5 &= -10 - 5 && \leftarrow \text{subtract 5 from both sides to} \\
 -1x &= -15 && \text{isolate the variable} \\
 \frac{-1x}{-1} &= \frac{-15}{-1} \\
 x &= 15
 \end{aligned}$$

This example requires great care with the positive numbers and negative signs.

$$\begin{aligned}
 11 - \frac{1}{9}x &= -45 \\
 11 - \frac{1}{9}x - 11 &= -45 - 11 && \leftarrow \text{subtract 11 from both sides} \\
 -\frac{1}{9}x &= -56 && \text{to isolate the variable} \\
 -9 \cdot -\frac{1}{9}x &= -9 \cdot -56 && \leftarrow \text{multiply by reciprocal of } -\frac{1}{9} \text{ which is } -9 \\
 x &= 504
 \end{aligned}$$

Consider the following example.



**Remember:** *Decreased by* means *subtract*, *product* means *multiply*, and *is* translates to the  $=$  sign.

Five decreased by the product of 7 and  $x$  is  $-6$ . Solve for  $x$ .



Five decreased by the product of 7 and  $x$  is  $-6$ .

$$\begin{aligned}
 &\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 5 &- \quad 7x = -6 \\
 5 - 7x - 5 &= -6 - 5 && \leftarrow \text{subtract 5 from both sides to} \\
 -7x &= -11 && \text{isolate the variable} \\
 \frac{-7x}{-7} &= \frac{-11}{-7} \\
 x &= \frac{11}{7} \text{ or } 1\frac{4}{7}
 \end{aligned}$$



## Practice

Write the **reciprocals** of the following. If none exist, write **none**.

1. 10

2. -6

3.  $\frac{5}{6}$

4.  $-\frac{9}{10}$

5. 0

Solve the following. Show **essential steps**.

6.  $\frac{1}{5}x + 3 = 9$

9.  $10 - 6x = 11$

7.  $\frac{1}{4}x - 7 = 2$

10.  $15 - x = 10$

8.  $-\frac{1}{2}x - 7 = 23$

11.  $\frac{1}{8}x + 4 = -6$



12.  $-6 - x = 10$

14.  $4 - \frac{3}{7}x = 10$

13.  $2 + \frac{5}{6}x = -8$



**Check yourself:** Use the list of **scrambled answers** below and check your answers to problems 6-14.

-80 -60 -16 -14 -12  $-\frac{1}{6}$  5 30 36

*Answer the following.*

15. The difference between 12 and  $2x$  is -8. Solve for  $x$ .



## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |   |                     |
|---|---------------------|
| _____ 1. to find all numbers that make an equation or inequality true   | A. equation         |
| _____ 2. numbers less than zero   | B. expression       |
| _____ 3. a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables | C. negative numbers |
| _____ 4. any value for a variable that makes an equation or inequality a true statement                                     | D. positive numbers |
| _____ 5. any symbol, usually a letter, which could represent a number   | E. reciprocals      |
| _____ 6. to replace a variable with a numeral   | F. solution         |
| _____ 7. a mathematical sentence stating that the two expressions have the same value                                       | G. solve            |
| _____ 8. numbers greater than zero  | H. substitute       |
| _____ 9. any two numbers with a product of 1; also called <i>multiplicative inverse</i>                                     | I. variable         |



## Practice

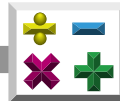
Use the list below to write the correct term for each definition on the line provided.

**additive inverses**  
**decrease**  
**difference**  
**increase**

**multiplicative inverses**  
**multiplicative property of -1**  
**product**

- \_\_\_\_\_ 1. any two numbers with a product of 1; also called *reciprocals*
- \_\_\_\_\_ 2. the result of multiplying numbers together
- \_\_\_\_\_ 3. to make greater
- \_\_\_\_\_ 4. to make less
- \_\_\_\_\_ 5. the product of any number and -1 is the opposite or additive inverse of the number
- \_\_\_\_\_ 6. a number that is the result of subtraction
- \_\_\_\_\_ 7. a number and its opposite whose sum is zero (0); also called *opposites*





## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

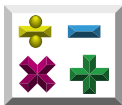
#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.1  
Solve linear equations in one variable that include simplifying algebraic expressions.
- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.



- MA.912.A.3.4  
Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

### Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

### The Distributive Property

Consider  $4(2 + 6)$ . The rules for **order of operations** would have us add inside the parentheses first.

$$\begin{array}{rcl} 4(2 + 6) & = & \\ 4(8) & = & \\ 32 & & \end{array}$$



**Remember:** Rules for the order of operations

Always start on the *left* and move *to the right*.

1. Do operations inside *grouping symbols* first.
2. Then do all *powers* (exponents) **or** *roots*.
3. Next do *multiplication* **or** *division*—as they occur from left to right.
4. Finally, do *addition* **or** *subtraction*—as they occur from left to right.



However, there is a second way to do the problem.

$$\begin{aligned}4(2 + 6) &= \\4(2) + 4(6) &= \\8 + 24 &= \\32\end{aligned}$$

In the second way, the 4 is *distributed* over the addition. This second way of doing the problem illustrates the **distributive property**.

### The Distributive Property

For any numbers  $a$ ,  $b$ , and  $c$ ,  
 $a(b + c) = ab + ac$

Also, it works for subtraction:  
 $a(b - c) = ab - ac$

This property is most useful in simplifying expressions that contain variables, such as  $2(x + 4)$ .

To **simplify an expression** we must perform as many of the indicated operations as possible. However, in the expression  $2(x + 4)$ , we can't add first, unless we know what number  $x$  represents. The *distributive property* allows us to rewrite the equation:

$$\begin{aligned}\overset{\curvearrowright}{2(x + 4)} &= \\2x + 2(4) &= \\2x + 8\end{aligned}$$

The distributive property allows you to multiply each term *inside* a set of parentheses by a factor *outside* the parentheses. We say multiplication is *distributive over* addition and subtraction.

$$\begin{aligned}\overset{\curvearrowright}{5(3 + 1)} &= (5 \cdot 3) + (5 \cdot 1) \\5(4) &= 15 + 5 \\20 &= 20\end{aligned}$$

$$\begin{aligned}\overset{\curvearrowright}{5(3 - 1)} &= (5 \cdot 3) - (5 \cdot 1) \\5(2) &= 15 - 5 \\10 &= 10\end{aligned}$$



Not all operations are distributive. You cannot distribute division over addition.

$$\begin{aligned} 14 \div (5 + 2) &\neq 14 \div 5 + 14 \div 2 \\ 14 \div 7 &\neq 2.8 + 7 \\ 2 &\neq 9.8 \end{aligned}$$

Study the chart below.

Properties	
Addition	Multiplication
Commutative: $a + b = b + a$	Commutative: $ab = ba$
Associative: $(a + b) + c = a + (b + c)$	Associative: $(ab)c = a(bc)$
Identity: 0 is the identity. $a + 0 = a$ and $0 + a = a$	Identity: 1 is the identity. $a \cdot 1 = a$ and $1 \cdot a = a$
Addition	Subtraction
Distributive: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	Distributive: $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$

These properties deal with the following:

order—**commutative property** of addition and commutative property of multiplication

grouping—**associative property** of addition and associative property of multiplication

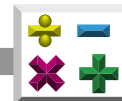
identity—**additive identity** property and **multiplicative identity** property

zero—**multiplicative property of zero**

distributive—**distributive property** of multiplication over addition and over subtraction

Notice in the distributive property that it does *not* matter whether  $a$  is placed on the *right* or the *left* of the expression in parentheses.

$$a(b + c) = (b + c)a \text{ or } a(b - c) = (b - c)a$$



The **symmetric property of equality** (if  $a = b$ , then  $b = a$ ) says that if one quantity equals a second quantity, then the second quantity also equals the first quantity. We use the **substitution property of equality** when replacing a variable with a number or when two quantities are equal and one quantity can be replaced by the other. Study the chart and examples below that describe properties of equality.

### Properties of Equality

Reflexive:	$a = a$
Symmetric:	If $a = b$ , then $b = a$ .
Transitive:	If $a = b$ and $b = c$ , then $a = c$ .
Substitution:	If $a = b$ , then $a$ may be replaced by $b$ .

### Examples of Properties of Equality

Reflexive:	$8 - e = 8 - e$
Symmetric:	If $5 + 2 = 7$ , then $7 = 5 + 2$ .
Transitive:	If $9 - 2 = 4 + 3$ and $4 + 3 = 7$ , then $9 - 2 = 7$ .
Substitution:	If $x = 8$ , then $x \div 4 = 8 \div 4$ . $x$ is replaced by 8. or If $9 + 3 = 12$ , then $9 + 3$ may be replaced by 12.

Study the following examples of how to simplify expressions. Refer to the charts above and on the previous page as needed.

$$\begin{array}{lcl}
 5(6x + 3) + 8 & & \\
 5(6x + 3) + 8 = & \longleftarrow & \text{use the distributive property to} \\
 5(6x) + 5(3) + 8 = & & \text{distribute 5 over } 6x \text{ and } 3 \\
 30x + 15 + 8 = & \longleftarrow & \text{use the associative property to} \\
 30x + 23 & & \text{associate 15 and 8}
 \end{array}$$

and

$$\begin{array}{lcl}
 6 + 2(4x - 3) & & \\
 6 + 2(4x - 3) = & \longleftarrow & \text{use order of operations to multiply} \\
 & & \text{before adding, then} \\
 6 + 2(4x) + 2(-3) = & \longleftarrow & \text{distribute 2 over } 4x \text{ and } -3 \\
 6 + 8x + -6 = & \longleftarrow & \text{use the associative property to} \\
 & & \text{associate 6 and } -6 \\
 8x + 0 = & \longleftarrow & \text{use the identity property of addition} \\
 8x & &
 \end{array}$$



## Practice

Simplify *by using the distributive property*. Show essential steps.

1.  $10(x + 9)$

7.  $4(3x + 7) - 2$

2.  $16(z - 3)$

8.  $-6(x + 3) + 18$

3.  $a(b + 5)$

9.  $30 + 2(x + 8)$

4.  $5(x + 3) + 9$

10.  $x(x + 3)$

5.  $4(x - 5) + 20$

11.  $a(b + 10)$

6.  $5(3 + x) - 9$



*Circle the letter of the correct answer.*

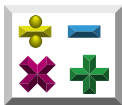
12. Mrs. Smith has 5 children. Every fall she buys each child a new book bag for \$20, a new notebook for \$3.50, and other school supplies for \$15. Which expression is a correct representation for the amount she spends?



- a.  $5(20 + 3.50 + 15)$
- b.  $5 + (20 + 3.50 + 15)$
- c.  $5(20 \cdot 3.50 \cdot 15)$
- d.  $5 \cdot 20 + 3.50 + 15$

*Number the **order of operations** in the correct **order**. Write the numbers 1-4 on the line provided.*

- \_\_\_\_\_ 13. addition *or* subtraction
- \_\_\_\_\_ 14. powers (exponents)
- \_\_\_\_\_ 15. parentheses
- \_\_\_\_\_ 16. multiplication *or* division



## Simplifying Expressions

Here's how to use the distributive property and the definition of subtraction to simplify the following expressions.

### Example 1:

Simplify

$$-7a - 3a$$

$$-7a - 3a = -7a + -3a$$

$$= (-7 + -3)a \quad \leftarrow \text{use the distributive property}$$

$$= -10a$$

### Example 2:

Simplify

$$10c - c$$

$$10c - c = 10c - 1c$$

$$= 10c + -1c$$

$$= (10 + -1)c \quad \leftarrow \text{use the distributive property}$$

$$= 9c$$

The expressions  $-7a - 3a$  and  $-10a$  are called **equivalent expressions**. The expressions  $10c - c$  and  $9c$  are also called *equivalent* expressions. Equivalent expressions express the same number. An expression is in simplest form when it is replaced by an equivalent expression having no **like terms** and no parentheses.

*Study these examples.*

$$-5x + 4x = (-5 + 4)x$$

$$= -x$$

$$5y - 5y = 5y + -5y$$

$$= (5 + -5)y$$

$$= 0y \quad \leftarrow \text{multiplicative property of zero}$$

$$= 0$$

The multiplicative property of 0 says for any number  $a$ ,  
 $a \cdot 0 = 0 \cdot a = 0$ .





The following shortcut is frequently used to simplify expressions.

First

- rewrite each subtraction as adding the opposite
- then combine *like terms* (terms that have the same variable) by adding.

Simplify

$$\begin{array}{l} 2a + 3 - 6a \\ 2a + 3 - 6a = 2a + 3 + -6a \quad \leftarrow \text{rewrite } -6a \text{ as } + -6a \\ = -4a + 3 \quad \leftarrow \text{combine like terms by adding} \end{array}$$

**like terms**

Simplify

$$\begin{array}{l} 8b + 7 - b - 6 \\ 8b + 7 - b - 6 = 8b + 7 + -1b + -6 \quad \leftarrow \text{rewrite } -b \text{ as } + -1b \text{ and } -6 \text{ as } + -6 \\ = 7b + 1 \quad \leftarrow \text{combine like terms by adding} \end{array}$$

**like terms**

Simplify

$$7x + 5 + 3x = 10x + 5 \quad \leftarrow \text{combine like terms}$$

**like terms**



## Practice

**Simplify** *by combining like terms.* **Show essential steps.**

1.  $5n + 3n$

7.  $4x + 11x$

2.  $6n - n$

8.  $4x - 11x$

3.  $8y - 8y$

9.  $-4x - 11x$

4.  $7n + 3n - 6$

10.  $10y - 4y + 7$

5.  $-7n - 3n - 6$

11.  $10y + 4y - 7$

6.  $6n - 3 + 7$

12.  $10y - 4 - 7$



13.  $8c - 12 - 6c$

20.  $20n - 6n - 1 + 8$

14.  $8c - 12c - 6$

21.  $12c - 15 - 12c - 17$

15.  $-10y - y - 15$

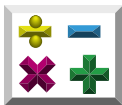
22.  $12c - 15c - 12 - 18$

16.  $-10y + y - 15$

17.  $15x - 15x + 6$

18.  $15x - 15 + 8x$

19.  $20n - 6 - n + 8$



## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |   |                                      |
|---|--------------------------------------|
| _____ 1. $a(b + c) = ab + ac$   | A. associative property              |
| _____ 2. $a + b = b + a$  | B. commutative property              |
| _____ 3. $(a + b) + c = a + (b + c)$  | C. distributive property             |
| <hr/>   |                                      |
| _____ 4. $a \cdot 1 = a$  | A. additive identity                 |
| _____ 5. $a + 0 = a$  | B. multiplicative identity           |
| _____ 6. $a \cdot 0 = 0$  | C. multiplicative property of zero   |
| <hr/>   |                                      |
| _____ 7. if $a = b$ , then $b = a$  | A. substitution property of equality |
| _____ 8. if $a = b$ , then $a$ may be replaced by $b$   | B. symmetric property of equality    |
| <hr/>   |                                      |
| _____ 9. terms that have the same variables and the same corresponding exponents  | A. like terms                        |
| _____ 10. to perform as many of the indicated expressions as possible   | B. order of operations               |
| _____ 11. the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right) | C. simplify an expression            |



## Equations with Like Terms

Consider the following equation.

$$2x + 3x + 4 = 19$$

Look at both sides of the equation and see if either side can be simplified.

Always simplify first  
by combining like terms.

$$2x + 3x + 4 = 19$$

$$5x + 4 = 19$$

← add like terms

$$5x + 4 - 4 = 19 - 4$$

← subtract 4 from each side

$$5x = 15$$

$$\frac{5x}{5} = \frac{15}{5}$$

← divide each side by 5

$$x = 3$$

Always mentally check your answer by *substituting* the solution for the variable in the original equation.

Substitute 3 for  $x$  in the equation.

$$2x + 3x + 4 = 19$$

$$2(3) + 3(3) + 4 = 19$$

$$6 + 9 + 4 = 19$$

$$19 = 19$$

*It checks!*



Consider this example.

The product of  $x$  and 7 plus the product of  $x$  and 3 is 45.



**Remember:** To work a problem like this one, we need to remember two things. The word *product* means *multiply* and the word *is* always translates to  $=$ .

The product of  $x$  and 7 plus the product of  $x$  and 3 is 45.

$$\begin{array}{lcl} 7x + 3x & = & 45 \\ 10x & = & 45 \quad \leftarrow \text{add like terms} \\ \frac{10x}{10} & = & \frac{45}{10} \quad \leftarrow \text{divide both sides by 10} \\ x & = & 4.5 \end{array}$$

Check by substituting 4.5 for  $x$  in the original equation.

$$\begin{array}{lcl} 7x + 3x & = & 45 \\ 7(4.5) + 3(4.5) & = & 45 \\ 31.5 + 13.5 & = & 45 \\ 45 & = & 45 \end{array} \quad \text{It checks!}$$

Here is another example which appears to be more challenging.

$$\begin{array}{lcl} 3x - 2 - x + 10 & = & -12 \\ 3x - 2 - 1x + 10 & = & -12 \quad \leftarrow \text{remember: } 1 \cdot x = x \\ 3x - 1x - 2 + 10 & = & -12 \quad \leftarrow \text{add like terms} \\ 2x + 8 & = & -12 \\ 2x + 8 - 8 & = & -12 - 8 \quad \leftarrow \text{subtract 8 from both sides} \\ 2x & = & -20 \\ \frac{2x}{2} & = & \frac{-20}{2} \quad \leftarrow \text{divide both sides by 2} \\ x & = & -10 \end{array}$$

Check by substituting -10 into the original equation.

$$\begin{array}{lcl} 3x - 2 - x + 10 & = & -12 \\ 3(-10) - 2 - (-10) + 10 & = & -12 \\ -30 - 2 + 10 + 10 & = & -12 \\ -32 + 20 & = & -12 \\ -12 & = & -12 \end{array} \quad \text{It checks!}$$



## Practice

Solve these equations by first **simplifying** each side. Show **essential steps**.

1.  $4x + 6x = -30$

5.  $3y - y - 8 = 30$

2.  $-2x + 10x - 6x = -12$

6.  $x + 10 - 2x = -2$

3.  $12m - 6m + 4 = -32$

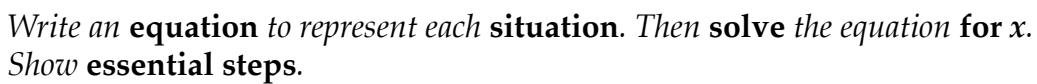
7.  $13x + 105 - 8x = 0$

4.  $3 = 4x + x - 2$

8.  $2x + 10 + 3x - 8 = -13$



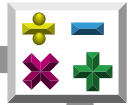
**Check yourself:** Add all your answers for problems 1-8. Did you get a *sum of -7*? If yes, complete the practice. If no, correct your work before continuing.



- |        |                   |                    |                      |                     |                    |                                    |
|--------|-------------------|--------------------|----------------------|---------------------|--------------------|------------------------------------|
|        |                   |                    |                      |                     |                    | 1                                  |
| Sunday | Monday<br>6 hours | Tuesday<br>8 hours | Wednesday<br>5 hours | Thursday<br>4 hours | Friday<br>10 hours | Saturday<br>No baby sitting today! |
| 2      | 3                 | 4                  | 5                    | 6                   | 7                  | 8                                  |
| Sunday | Monday            | Tuesday            | Wednesday            | Thursday            | Friday             | Saturday                           |

Your total salary for the week was \$198.00. How much do you earn per hour?

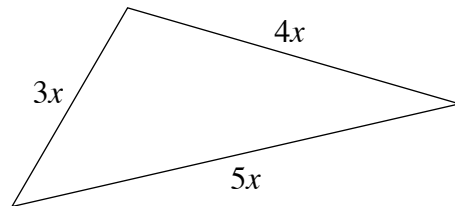




13. The **perimeter** ( $P$ ) of the **triangle** is 48 inches. What is  $x$ ?

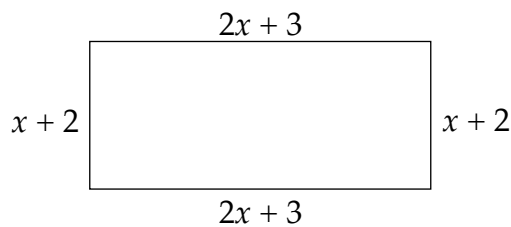


**Remember:** The *perimeter* of a figure is the distance around a figure, or the *sum* of the **lengths** ( $l$ ) of the **sides**.



14. Use the answer from problem 13 to find the *length* ( $l$ ) of each side of the triangle. Do the sides add up to 48 inches?

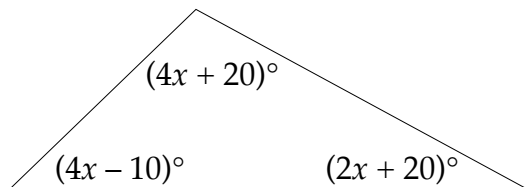
15. The perimeter of the **rectangle** is 58 inches. What is  $x$ ?



16. Use the answer from problem 15 to find the *length* and **width** ( $w$ ) of the rectangle. Do the sides add up to 58 inches?



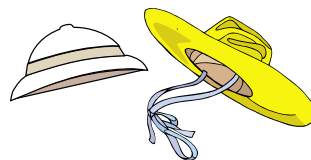
17. In any triangle, the sum of the **measures (m) of the angles ( $\angle$ )** is always 180 **degrees ( $^\circ$ )**. What is  $x$ ?



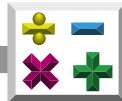
18. Using the answer from problem 17, find the measure of each **angle ( $\angle$ )**. Do the angles add up to 180 degrees?

*Circle the letter of the correct answer.*

19. You and your friend go to a popular theme park in central Florida. Admission for two comes to a total of \$70. Both of you immediately buy 2 hats to wear during the day. Later, as you are about to leave you decide to buy 4 more hats for your younger brothers and sisters who didn't get to come. The total bill for the day is \$115.00. Which equation could you use to find the cost of a single hat?



- a.  $6x = 115$
- b.  $70 + 2x + 4x = 115$
- c.  $70 - 6x = 115$
- d.  $70 + 2x = 115 + 6x$



Complete the following.

20. A common mistake in algebra is to say that

$$3x + 4x = 7x^2,$$

instead of

$$3x + 4x = 7x.$$

Let  $x = 2$  and substitute into both expressions below.



**Remember:** When you are doing  $7x^2$  the rules for the order of operation require that you square *before* you multiply!

$x = 2$
---------

$$3x + 4x = 7x$$

$$3x + 4x = 7x^2$$

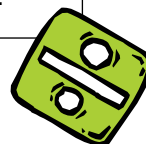
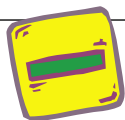
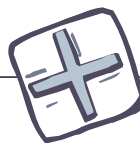
Are you convinced that  $3x + 4x$  is not equal to ( $\neq$ )  $7x^2$ ?



## Putting It All Together

### Guidelines for Solving Equations

1. Use the distributive property to clear parentheses.
2. Combine like terms. We want to isolate the variable.
3. Undo addition or subtraction using **inverse operations**.
4. Undo multiplication or division using *inverse operations*.
5. Check by substituting the solution in the original equation.



SAM = Simplify (steps 1 and 2) then  
Add (or subtract)  
Multiply (or divide)

### Example 1

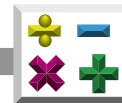
Solve:

$$\begin{aligned}6y + 4(y + 2) &= 88 \\6y + 4y + 8 &= 88 && \leftarrow \text{use distributive property} \\10y + 8 - 8 &= 88 - 8 && \leftarrow \text{combine like terms and undo addition} \\&&& \text{by subtracting 8 from each side} \\\frac{10y}{10} &= \frac{80}{10} && \leftarrow \text{undo multiplication by dividing by 10} \\y &= 8\end{aligned}$$

**Check** solution in the original equation:

$$\begin{aligned}6y + 4(y + 2) &= 88 \\6(8) + 4(8 + 2) &= 88 \\48 + 4(10) &= 88 \\48 + 40 &= 88 \\88 &= 88\end{aligned}$$

*It checks!*



### Example 2

**Solve:**

$$\begin{aligned}-\frac{1}{2}(x + 8) &= 10 \\ -\frac{1}{2}x - 4 &= 10 && \leftarrow \text{use distributive property} \\ -\frac{1}{2}x - 4 + 4 &= 10 + 4 && \leftarrow \text{undo subtraction by adding 4 to both sides} \\ -\frac{1}{2}x &= 14 \\ (-2)-\frac{1}{2}x &= 14(-2) && \leftarrow \text{isolate the variable by multiplying} \\ x &= -28 && \leftarrow \text{each side by the reciprocal of } -\frac{1}{2}\end{aligned}$$

**Check** solution in the original equation:

$$\begin{aligned}-\frac{1}{2}(x + 8) &= 10 \\ -\frac{1}{2}(-28 + 8) &= 10 \\ -\frac{1}{2}(-20) &= 10 \\ 10 &= 10 && \text{It checks!}\end{aligned}$$

### Example 3

**Solve:**

$$\begin{aligned}26 &= \frac{2}{3}(9x - 6) \\ 26 &= \frac{2}{3}(9x) - \frac{2}{3}(6) && \leftarrow \text{use distributive property} \\ 26 &= 6x - 4 \\ 26 + 4 &= 6x - 4 + 4 && \leftarrow \text{undo subtraction by adding 4 to each side} \\ \frac{30}{6} &= \frac{6x}{6} && \leftarrow \text{undo multiplication by dividing each side by 6} \\ 5 &= x\end{aligned}$$

**Check** solution in the original equation:

$$\begin{aligned}26 &= \frac{2}{3}(9x - 6) \\ 26 &= \frac{2}{3}(9 \cdot 5 - 6) \\ 26 &= \frac{2}{3}(39) \\ 26 &= 26 && \text{It checks!}\end{aligned}$$



#### Example 4

Solve:

$$\begin{array}{rcll} x - (2x + 3) & = & 4 & \\ x - 1(2x + 3) & = & 4 & \leftarrow \text{use the multiplicative property of } -1 \\ x - 2x - 3 & = & 4 & \leftarrow \begin{array}{l} \text{use the multiplicative identity of } 1 \\ \text{and use the distributive property} \end{array} \\ -1x - 3 & = & 4 & \leftarrow \text{combine like terms} \\ -1x - 3 + 3 & = & 4 + 3 & \leftarrow \text{undo subtraction} \\ \frac{-1x}{-1} & = & \frac{7}{-1} & \leftarrow \text{undo multiplication} \\ x & = & -7 & \end{array}$$

Examine the solution steps above. See the use of the *multiplicative property of -1* in front of the parentheses on line two.

line 1:  $x - (2x + 3) = 4$

line 2:  $x - 1(2x + 3) = 4$

Also notice the use of *multiplicative identity* on line three.

line 3:  $1x - 2x - 3 = 4$

The simple variable  $x$  was multiplied by 1 ( $1 \cdot x$ ) to equal  $1x$ . The  $1x$  helped to clarify the number of variables when combining like terms on line four.

**Check** solution in the original equation:

$$\begin{array}{rcl} x - (2x + 3) & = & 4 \\ -7 - (2 \cdot -7 + 3) & = & 4 \\ -7 - (-11) & = & 4 \\ 4 & = & 4 \end{array} \quad \text{It checks!}$$



## Practice

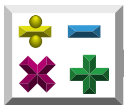
**Solve and check.** *Show essential steps.*

1.  $10(2n + 3) = 130$

2.  $4(y - 3) = -20$

3.  $\frac{x}{-2} + 4 = -10$

4.  $6x + 6(x - 4) = 24$



5.  $6 = \frac{2}{3}(3n - 6)$

6.  $10p - 4(p - 7) = 42$

7.  $28 = \frac{1}{2}(x - 8)$

8.  $x - (3x - 7) = 11$



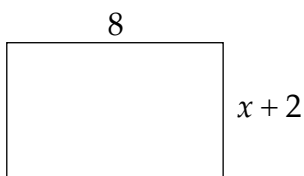


**Solve and Check. Show essential steps.**

9. Write an equation for the area of the rectangle. Then solve for  $x$ .

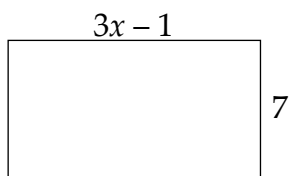


**Remember:** To find the **area ( $A$ )** of a rectangle, multiply the length ( $l$ ) times the width ( $w$ ).  $A = (lw)$



The area is 64 **square units**.

10. Write an equation for the area of the rectangle. Then solve for  $x$ .

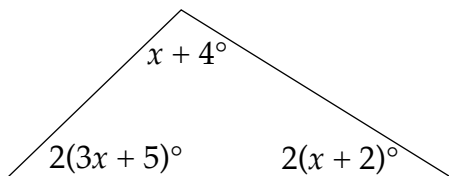


The area is 56 square units.

11. Write an equation for the measure of degrees ( $^\circ$ ) in the triangle. Then solve for  $x$ .



**Remember:** For any triangle, the sum of the measures of the angles is 180 degrees.





## Practice

Use the list below to write the correct term for each definition on the line provided.

area ( $A$ )  
length ( $l$ )  
perimeter ( $P$ )

rectangle  
sum

triangle  
width ( $w$ )

- |       |    |   |
|-------|----|---|
| _____ | 1. | a one-dimensional measure that is the measurable property of line segments  |
| _____ | 2. | a parallelogram with four right angles  |
| _____ | 3. | a one-dimensional measure of something side to side   |
| _____ | 4. | the result of adding numbers together   |
| _____ | 5. | the distance around a figure  |
| _____ | 6. | the measure, in square units, of the inside region of a closed two-dimensional figure; the number of square units needed to cover a surface |
| _____ | 7. | a polygon with three sides  |

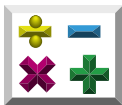


## Practice

Use the list below to write the correct term for each definition on the line provided.

<b>angle (<math>\angle</math>)</b>	<b>side</b>
<b>degree (<math>^\circ</math>)</b>	<b>square (of a number)</b>
<b>inverse operation</b>	<b>square units</b>
<b>measure (m) of an angle (<math>\angle</math>)</b>	

- \_\_\_\_\_ 1. two rays extending from a common endpoint called the vertex
- \_\_\_\_\_ 2. the number of degrees ( $^\circ$ ) of an angle
- \_\_\_\_\_ 3. units for measuring area; the measure of the amount of an area that covers a surface
- \_\_\_\_\_ 4. common unit used in measuring angles
- \_\_\_\_\_ 5. the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle
- \_\_\_\_\_ 6. an action that cancels a previously applied action
- \_\_\_\_\_ 7. the result when a number is multiplied by itself or used as a factor twice



## **Lesson Three Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.1  
Solve linear equations in one variable that include simplifying algebraic expressions.



- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.
- MA.912.A.3.4  
Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

### Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

### Solving Equations with Variables on Both Sides

I am thinking of a number. If you multiply my number by 3 and then subtract 2, you get the same answer that you do when you add 4 to my number. What is my number?



To solve this riddle, begin by translating these words into an algebraic sentence. Let  $x$  represent my number.

If you multiply my number by 3 and then subtract 2  
 $3x - 2$

you get the same answer  
 $=$

that you do when you add 4 to my number  
 $4 + x$

Putting it all together, we get the equation  $3x - 2 = 4 + x$ . Note that this equation is different from equations in previous units. There is a *variable* on both sides. To solve such an equation, we do what we've done in the past: make sure both sides are simplified, and that there are no parentheses.



**Strategy:** Collect the variables on one side. Collect the numbers on the other side.

Now let's go back to the equation which goes with our riddle.

**Solve:**

$$\begin{array}{ll} 3x - 2 = 4 + x & \leftarrow \text{both sides are simplified} \\ 3x - 2 = 4 + 1x & \leftarrow \text{multiplicative identity of 1} \\ 3x - 2 - 1x = 4 + 1x - 1x & \leftarrow \text{collect variables on the left} \\ 2x - 2 = 4 & \leftarrow \text{combine like terms; simplify} \\ 2x - 2 + 2 = 4 + 2 & \leftarrow \text{collect numbers on the right} \\ \frac{2x}{2} = \frac{6}{2} & \leftarrow \text{divide both sides by 2} \\ x = 3 & \end{array}$$

**Check** solution in the original equation and the original riddle:

$$\begin{array}{ll} 3x - 2 = 4 + x & \\ 3 \bullet 3 - 2 = 4 + 3 & \\ 9 - 2 = 7 & \\ 7 = 7 & \text{It checks!} \end{array}$$

Study the equation below.

**Solve:**

$$\begin{array}{ll} 2(3x + 4) = 5(x - 2) & \\ 6x + 8 = 5x - 10 & \leftarrow \text{distributive property} \\ 6x + 8 - 5x = 5x - 10 - 5x & \leftarrow \text{variables on the left} \\ x + 8 = -10 & \leftarrow \text{simplify} \\ x + 8 - 8 = -10 - 8 & \leftarrow \text{numbers on the right} \\ x = -18 & \end{array}$$

**Check** solution in the original equation:

$$\begin{array}{ll} 2(3x + 4) = 5(x - 2) & \\ 2(3 \bullet -18 + 4) = 5(-18 - 2) & \\ 2(-50) = 5(-20) & \\ -100 = -100 & \text{It checks!} \end{array}$$



Let's work this next example in two different ways.

1. Collect the *variables* on the *left* and the *numbers* on the *right*.

**Solve:**

$$\begin{aligned}6y &= 4(5y - 7) \\6y &= 20y - 28 && \leftarrow \text{distributive property} \\6y - 20y &= 20y - 28 - 20y && \leftarrow \text{variables on the left} \\\frac{-14y}{-14} &= \frac{-28}{-14} && \leftarrow \text{divide both sides by -14} \\y &= 2\end{aligned}$$

**Check** solution in the original equation:

$$\begin{aligned}6y &= 4(5y - 7) \\6 \cdot 2 &= 4(5 \cdot 2 - 7) \\12 &= 4 \cdot 3 \\12 &= 12 && \text{It checks!}\end{aligned}$$

2. Collect the *variables* on the *right* and *numbers* on the *left*.

**Solve:**

$$\begin{aligned}6y &= 4(5y - 7) \\6y &= 20y - 28 && \leftarrow \text{distributive property} \\6y - 6y &= 20y - 28 - 6y && \leftarrow \text{variables on the right} \\0 &= 14y - 28 && \leftarrow \text{simplify} \\0 + 28 &= 14y - 28 + 28 && \leftarrow \text{numbers on the left} \\\frac{28}{14} &= \frac{14y}{14} && \leftarrow \text{divide both sides by 14} \\2 &= y\end{aligned}$$

We get the *same* answer, so the choice of which side you put the variable on is up to you!



## Practice

Solve each **equation** below. Then **find** your **solution** at the bottom of the page. Write the **letter** next to the number of that equation on the line provided above the solution. Then you will have the answer to this question:



Which great explorer's last words were,

"I have not told half of what I saw!"

**r** 1.  $2x - 4 = 3x + 6$

**p** 5.  $-2x + 6 = -x$

**l** 2.  $2(-12 - 6x) = -6x$

**m** 6.  $7x = 3(5x - 8)$

**a** 3.  $x - 3 = 2(-11 + x)$

**o** 7.  $2(12 - 8x) = 1x - 11x$

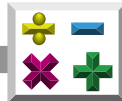
**c** 4.  $-7(1 - 4m) = 13(2m - 3)$

8.  $\frac{\quad}{3} \quad \frac{\quad}{19} \quad \frac{\quad}{-10} \quad \frac{\quad}{-16} \quad \frac{\quad}{4} \quad \frac{\quad}{6} \quad \frac{\quad}{4} \quad \frac{\quad}{-4} \quad \frac{\quad}{4}$



**Check yourself:** Use the answer above to check your solutions to problems 1-7. Did your solutions spell out the great explorer's name? If not, correct your work before continuing.





**Solve and check. Show essential steps.**

9. Six more than 5 times a number is the same as 9 less than twice the number. Find the number.
  
  
  
  
  
  
  
  
  
  
10. Twelve less than a number is the same as 6 decreased by 8 times the number. Find the number.
  
  
  
  
  
  
  
  
  
  
11. The product of 5 and a number, plus 17, is equal to twice the sum of the same number and -5. Find the number.



**Complete to solve the following.**

12.  $-12 + 7(x + 3) = 4(2x - 1) + 3$



**Remember:** Always multiply before you add.

$$-12 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} + 3$$

$$7x + \underline{\hspace{1cm}} = 8x - \underline{\hspace{1cm}}$$

Now finish the problem.

13.  $-56 + 10(x - 1) = 4(x + 3)$

14.  $5(2x + 4) + 3(-2x - 3) = 2x + 3(x + 4)$

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 2x + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

distributive property                      distributive property

$$\underline{\hspace{1cm}} x + 11 = \underline{\hspace{1cm}} x + \underline{\hspace{1cm}}$$

add like terms                      add like terms

Now finish the problem.



15.  $-16x + 10(3x - 2) = -3(2x + 20)$

$-16x + \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$   
distributive property    distributive property

Add like terms and finish the problem.

16.  $6(-2x - 4) + 2(3x + 12) = 37 + 5(x - 3)$



## Problems That Lead to Equations

Joshua presently weighs 100 pounds, but is steadily growing at a rate of 2 pounds per week. When will he weigh 140 pounds?



The answer is 20 weeks. Let's use this simple problem to help us think algebraically.

**Step 1: Read the problem and label the variable. Underline all clues.**

Joshua presently weighs 100 pounds, but is steadily growing at a rate of 2 pounds per week. When will he weigh 140 pounds?

Let  $x$  represent the number of weeks.

**Step 2: Plan.**

Let  $2x$  represent the weight Joshua will gain.

**Step 3: Write the equation.**

$$\begin{array}{rclcl} \text{present weight} & + & \text{gain} & = & \text{desired weight} \\ 100 & + & 2x & = & 140 \end{array}$$

**Step 4: Solve the equation.**

$$\begin{array}{rcl} 100 + 2x & = & 140 \\ 100 + 2x - 100 & = & 140 - 100 \end{array} \quad \leftarrow \text{subtract 100 from both sides}$$

$$\begin{array}{rcl} 2x & = & 40 \\ \frac{2x}{2} & = & \frac{40}{2} \\ x & = & 20 \end{array} \quad \leftarrow \text{divide both sides by 2}$$

**Step 5: Check your solution. Does your answer make sense?**

$$\begin{array}{l} \text{now} \quad \text{gain} \\ 100 + 2(20) = 140 \end{array}$$



We will use this 5-step approach on the following problems. You will find that many times a picture or chart will also help you arrive at an answer. Remember, we are learning to think algebraically, and to do that the procedure is as important as the final answer!

### 5-Step Plan for Thinking Algebraically

**Step 1:** **Read** the problem and **label** the variable. Underline all clues.

Decide what  $x$  represents.

**Step 2:** **Plan.**

**Step 3:** **Write** an equation.

**Step 4:** **Solve** the equation.

**Step 5:** **Check** your solution. Does your answer make sense?



## Practice

Use the **5-step plan** to solve and check the following. Show **essential steps**.

1. Leon's television breaks down. Unfortunately he has only \$100.00 in savings for emergencies. The repairman charges \$35.00 for *coming to Leon's house* and then another \$20.00 *per hour* for fixing the television. *How many hours* can Leon pay for the repairman to work?



- a. Step 1: Read the problem and label the variable. Underline all clues. (*Note: The clues have been italicized for you.*)

Let  $x$  represent \_\_\_\_\_ .

- b. Step 2: Plan. Let  $20x$  represent \_\_\_\_\_  
\_\_\_\_\_ .

- c. Step 3: Write an equation. \_\_\_\_\_

- d. Step 4: Solve the equation.

- e. Step 5: Check your solution. Does it make sense? \_\_\_\_\_



2. Samantha charges \$16.00 to deliver sand to your house, plus \$3.50 per **cubic** foot for the sand that you buy. How much sand can you buy for \$121.00?

- a. Step 1: Read the problem and label the variable. Underline all clues.

Let  $x$  represent \_\_\_\_\_ .

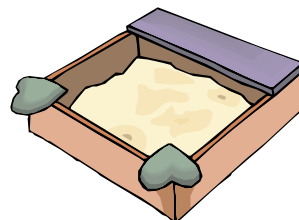
- b. Step 2: Plan. Let \_\_\_\_\_  $x$  represent \_\_\_\_\_ .

- c. Step 3: Write an equation. \_\_\_\_\_

- d. Step 4: Solve the equation.

- e. Step 5: Check your solution. Does it make sense? \_\_\_\_\_

- f. If the installation of a child's sand box requires 29 cubic feet of sand, will you be able to complete this project for \$121.00?



Explain. \_\_\_\_\_  
\_\_\_\_\_



3. Suppose that the gas tank of a car holds 20 gallons, and that the car uses  $\frac{1}{10}$  of a gallon per mile. How far has the car gone when 5 gallons remain?



- a. Step 1: Read the problem and label the variable. Underline all clues.

Let  $x$  represent \_\_\_\_\_ .

- b. Step 2: Plan. Let  $\frac{1}{10}x$  represent \_\_\_\_\_

\_\_\_\_\_ .

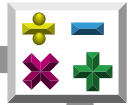
- c. Step 3: Explain why the appropriate equation is  $20 - \frac{1}{10}x = 5$ .

\_\_\_\_\_  
\_\_\_\_\_

- d. Step 4: Solve the equation.

- e. Step 5: Check your solution. Does it make sense? \_\_\_\_\_





4. Jared weighs 250 pounds and is steadily losing 3 pounds per week. How long will it take him to weigh 150 pounds?

- a. Step 1: Read the problem and label the variable. Underline all clues.

Let  $x$  represent \_\_\_\_\_ .

- b. Step 2: Let  $3x$  represent \_\_\_\_\_ .

- c. Step 3: Write an equation. \_\_\_\_\_

\_\_\_\_\_

- d. Step 4: Solve the equation.

- e. Step 5: Check your solution. Does it make sense? \_\_\_\_\_



5. Batman has \$100.00 and spends \$3.00 per day. Robin has \$20.00 but is adding to it at the rate of \$5.00 per day. When will they have the same amount of money?

- a. Step 1: Read the problem and label the variable. Underline all clues.

Let  $x$  represent \_\_\_\_\_ .

- b. Step 2: Plan.  $100 - 3x$  is what Batman will have after  $x$  days.

How much will Robin have after  $x$  days? \_\_\_\_\_

\_\_\_\_\_

- c. Step 3: Write an equation stating that Batman's money is the same amount as Robin's money after  $x$  days.

\_\_\_\_\_

- d. Step 4: Solve the equation.

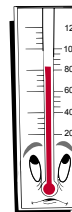
- e. What does your solution mean? \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- f. Step 5: Check your solution. Does it make sense? \_\_\_\_\_



6. Suppose you live in Tallahassee, Florida, where the temperature is 84 degrees and going down 3 degrees per hour. A friend lives in Sydney, Australia, where the temperature is 69 degrees and going up at a rate of 2 degrees per hour. How long would you and your friend have to wait before the temperatures in both places are equal?

- a. Step 1: Read the problem and label the variable. Underline all clues.

Let  $x$  represent \_\_\_\_\_ .

- b. Step 2: Plan. Let \_\_\_\_\_ represent

Tallahassee's temperature and \_\_\_\_\_

represent Sydney's temperature.

- c. Step 3: Write an equation. Let the Tallahassee temperature *equal* Sydney's temperature. \_\_\_\_\_

- d. Step 4: Solve the equation.

- e. Step 5: Check your solution. Does it make sense? \_\_\_\_\_



## Practice

Use the **5-step plan** to **solve** and **check** the following. Show **essential steps**.

Sometimes a *chart* helps *organize* the information in a problem.

1. I am thinking of 3 *numbers*. The *second number* is 4 more than the *first number*. The *third number* is twice the *first number*. The *sum* of all 3 numbers is 28. Find the numbers.

*Algebraic Thinking:*

- Step 1: Read the problem and label the variable.  
Underline all clues. (*Note: The clues have been italicized for you.*)

What does  $x$  represent? Since the second and third numbers are described in terms of the first number, let  $x$  represent the *first* number.

- Step 2: Plan. See the **table** below.

Description		Value
first number	$x$	=
second number	$x + 4$	=
third number	$2x$	=
sum	$4x + 4$	= 28

- Step 3: Write an equation.

$$4x + 4 = 28$$



- Step 4: Solve the equation.
  - a. The equation  $4x + 4 = 28$  will give you the *value* of only the *first* number. Substitute your answer back into the *expressions* in the *table* on the previous page to find the *second* and *third* numbers.
  - b. Solve the equation to find the value of the *first* number.

$$4x + 4 = 28$$

- c. Substitute the first number's value in the expression from the *table* on the previous page to get the value of the *second* number.

$$x + 4$$

- d. Substitute the first number's value in the expression from the *table* on the previous page to get the value of the *third* number.

$$2x$$

- Step 5: Check your solution. Does it make sense?

- e. Check solution in original equation.

- f. Do your numbers add up to 28? \_\_\_\_\_

Write an equation and solve to prove that the sum of the 3 numbers equal 28.



Sometimes a *picture* helps *organize* the information in a problem.

2. A triangle has a perimeter ( $P$ ) of 30 inches. The longest side is 8 inches longer than the shortest side. The third side is 1 inch shorter than the longest side. Find the sides.



**Remember:** The *perimeter* is the sum of all the lengths of all sides.

- Step 1: Read the problem and label the variable. Underline all clues.

**Hint:** Let the shortest side be  $x$  inches long.

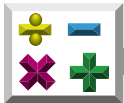
- Step 2: Plan.
  - a. Draw a triangle. Label the shortest side  $x$ . Label the other two sides in terms of  $x$ .
  - b. Let \_\_\_\_\_ represent adding up the sides of the triangle.
- Step 3: Write an equation.
  - c. Use the fact that the perimeter is 30 inches to write an equation.

\_\_\_\_\_



- Step 4: Solve the equation.
  - d. Find  $x$  by solving the equation.
  
- Step 5: Check your solution. Does it make sense?
  - e. Check solution in original equation.
  
  - f. Use the value of  $x$  to find the other 2 sides.
  
  - g. Do the sides add up to 30? \_\_\_\_\_

Write an equation and solve to prove that the sum of the 3 sides of the triangle equals 30.



3. A rectangle has a perimeter of 38 inches and a width of  $x$  inches. The length of the rectangle is *4 more than twice the width*. Label all 4 sides.

Draw and label a rectangle and use the 5-step plan to find the dimensions of the rectangle.

4. The measures of the angles in any triangle add up to 180 degrees. Let the smallest angle be  $x$  degrees. The second angle is *twice the smallest*. The third angle is *30 degrees more than the second angle*. Find the measures of all the angles.

Draw and label a triangle. Use the 5-step plan to find all angles.

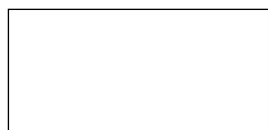




5. Write an equation for the area of the rectangle. \_\_\_\_\_



**Remember:** To find the *area* of a rectangle we multiply the length times the width.  $A = l \cdot w$



$2x - 1$

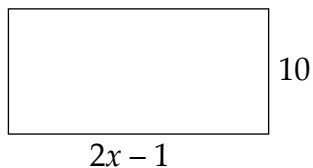
5 The area is 35 square inches.

Solve the equation for  $x$ , then substitute it in  $2x - 1$  to find the length.

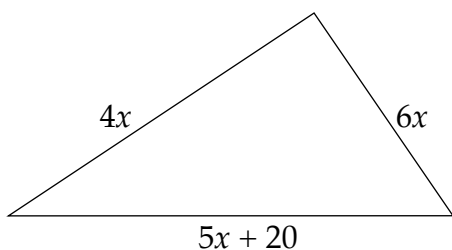
Is the product of the length and width 35 square inches? \_\_\_\_\_



6. Consider the rectangle and the triangle below. What is the value of  $x$  if the area of the rectangle equals the perimeter of the triangle?



Let \_\_\_\_\_ equal  
the area of the rectangle.

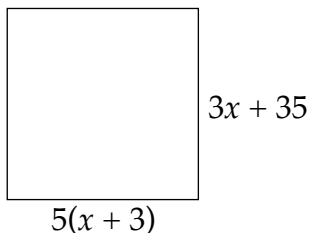


Let \_\_\_\_\_ equal  
the perimeter of the triangle.

Now set up an equation. Let the area of the rectangle equal the perimeter of the triangle. Solve for  $x$ .



7. A **square** is a four-sided figure with *all sides the same length*. Find the value of  $x$  so that the figure is a square.



*Circle the letter of the correct answer.*

8. Mrs. Jones brings \$142.50 to pay for her family's expenses to see Florida A&M University play football. She has to pay \$10.00 to park. An adult ticket costs \$45.00. She has 4 children who qualify for student tickets. She has \$27.50 left at the end of the day. Which equation can you use to find the cost of a student ticket?
- a.  $4x + 45 = \$142.50$
  - b.  $\$27.50 + 4x + 45 = \$142.50$
  - c.  $\$142.50 - 10 - 45 - 4x = \$27.50$
  - d.  $\$142.50 - 10 - 45 + 4x = \$27.50$



Answer the following. Show **essential steps**.

Consecutive **even integers** are numbers like 6, 8, and 10 or 14, 16, and 18. Note that you add 2 to the smallest to get the second number and 4 to the smallest to get the third number. Use this information to solve the following problem.

9. The sum of three consecutive even integers is 198. Find the numbers.

Description		Value
first number	$x$	=
second number	$x + 2$	=
third number	$x + 4$	=
sum	_____	= 198

Set up an equation and solve for  $x$ . Substitute your answer back into the table above to find all answers. Do the numbers add up to 198?

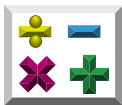


## Practice

Use the list below to complete the following statements.

consecutive even integers	square table
---------------------------------	-----------------

1. A data display that organizes information about a topic into categories is called a(n) \_\_\_\_\_ or chart.
2. A rectangle with four sides the same length is called a \_\_\_\_\_ .
3. Consecutive even \_\_\_\_\_ are numbers like 6, 8, and 10 or 14, 16, and 18.
4. When numbers are in order they are \_\_\_\_\_ .
5. Any integer divisible by 2 is a(n) \_\_\_\_\_ integer.



## **Lesson Four Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.1  
Solve linear equations in one variable that include simplifying algebraic expressions.



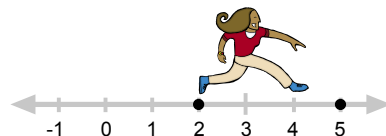
- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.
- MA.912.A.3.4  
Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

### Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

## Graphing Inequalities on a Number Line

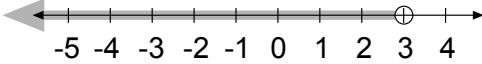
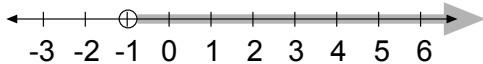
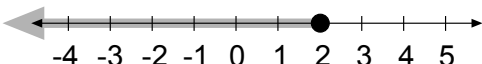
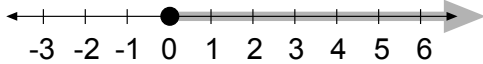
In this unit we will graph **inequalities** on a **number line**. A **graph of a number** is the point on a *number line* paired with the number. Graphing solutions on a number line will help you visualize solutions.





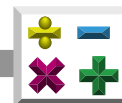
Here are some examples of *inequalities*, their verbal meanings, and their graphs.

### Inequalities

Inequality	Meaning	Graph
a. $x < 3$	All real numbers less than 3.	 <p>The open circle means that 3 is <i>not</i> a solution. Shade to left.</p>
b. $x > -1$	All real numbers greater than -1.	 <p>The open circle means that -1 is <i>not</i> a solution. Shade to right.</p>
c. $x \leq 2$	All real numbers less than or equal to 2.	 <p>The solid circle means that 2 is a solution. Shade to left.</p>
d. $x \geq 0$	All real numbers greater than or equal to 0.	 <p>The solid circle means that 0 is a solution. Shade to right.</p>

For each example, the inequality is written with the variable on the left. Inequalities can also be written with the variable on the right. However, graphing is easier if the variable is on the left.

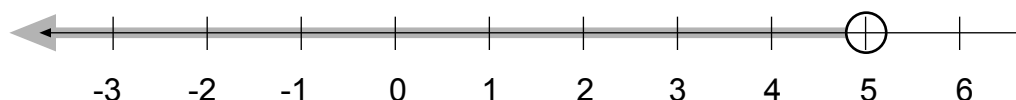




Consider  $x < 5$ , which means the same as  $5 > x$ . Note that the graph of  $x < 5$  is all real numbers less than 5.



The graph of  $5 > x$  is all real number that 5 is greater than.



To write an inequality that is *equivalent* to (or the same as)  $x < 5$ , move the number and variable to the opposite side of the inequality, and then reverse the inequality.

$$\begin{array}{c} x < 5 \\ \swarrow \searrow \\ 5 > x \end{array}$$

$x < 5$  means the same as

$$5 > x$$

The inequality  $y \geq -2$  is equivalent to  $-2 \leq y$ . Both inequalities can be written as the *set of all* **real numbers** that are *greater than or equal to* -2.

The inequality  $0 \leq x$  is equivalent to  $x \geq 0$ . Each can be written as the *set of all real numbers* that are *greater than or equal to* zero.

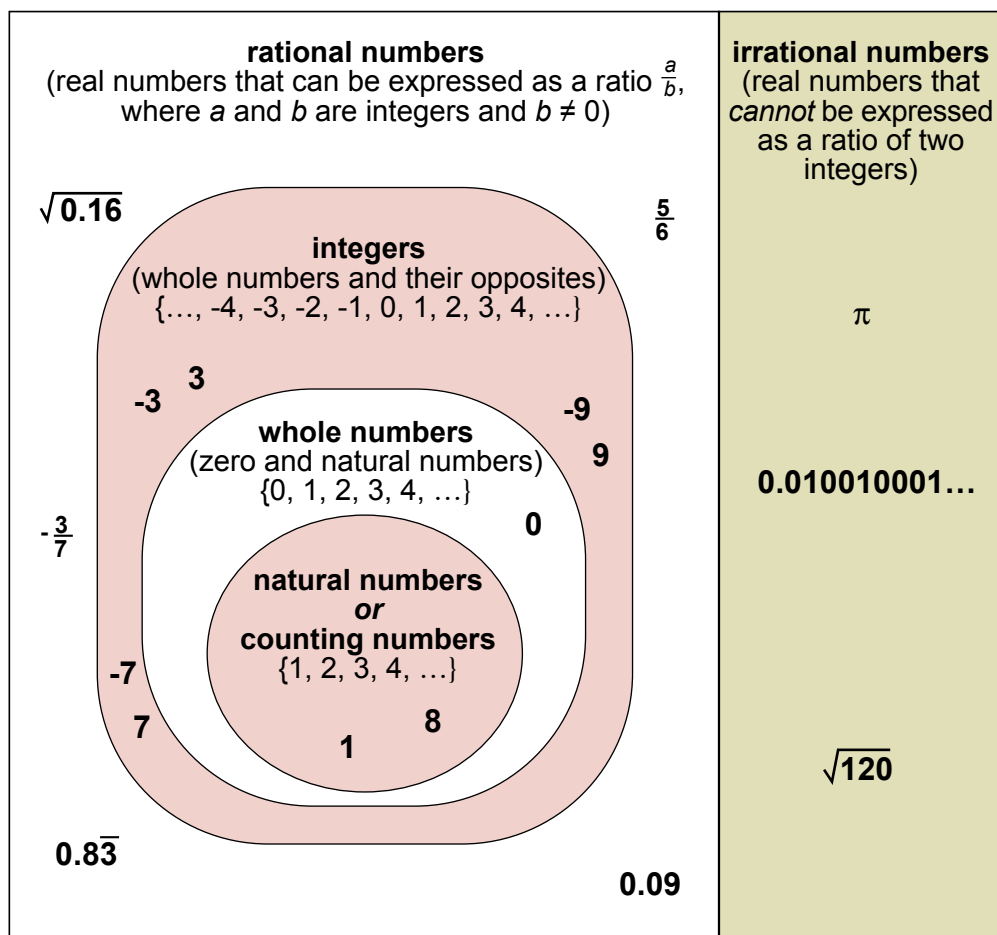


**Remember:** *Real numbers* are all **rational numbers** and all **irrational numbers**.



The Venn diagram below is a graphic organizer that aids in visualizing what real numbers are.

### The Set of Real Numbers

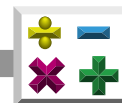


Rational numbers can be expressed as a **ratio**  $\frac{a}{b}$ , where  $a$  and  $b$  are *integers* and  $b \neq 0$ .

<b>rational numbers</b>	4	$-3\frac{3}{4}$	0.250	0	$0.\bar{3}$
<b>expressed as ratio of two integers</b>	$\frac{4}{1}$	$-\frac{15}{4}$	$\frac{1}{4}$	$\frac{0}{1}$	$\frac{1}{3}$

**Note:** All integers are rational numbers.

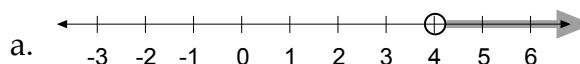
A *ratio* is the comparison of two quantities. For example, a ratio of 8 and 11 is 8:11 or  $\frac{8}{11}$ .



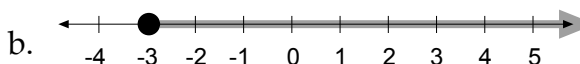
## Practice

Match each **inequality** with the correct **graph**.

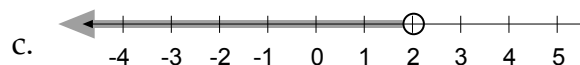
\_\_\_\_\_ 1.  $x \geq -3$



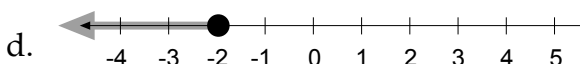
\_\_\_\_\_ 2.  $x \leq 0$



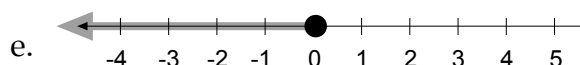
\_\_\_\_\_ 3.  $x > 4$



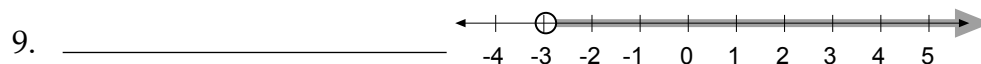
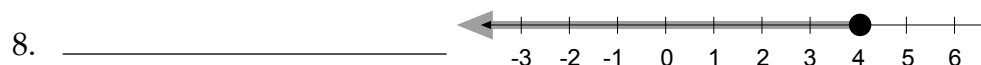
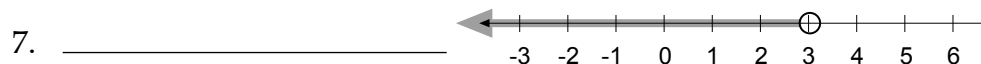
\_\_\_\_\_ 4.  $2 > x$



\_\_\_\_\_ 5.  $x \leq -2$



Write an **inequality** for each **graph**.





Graph each **inequality**.

10.  $x \geq -1$

11.  $x < 0$

12.  $x > 5$

13.  $x \leq -3$



## Solving Inequalities

We have been solving *equations* since Unit 1. When we solve inequalities, the procedures are the same except for one important difference.

**When we multiply or divide both sides of an inequality by the same *negative number*, we reverse the direction of the inequality symbol.**

**Example:** Solve by *dividing* by a *negative number* and *reversing* the inequality sign.

$$\begin{array}{l} -3x < 6 \\ \frac{-3x}{-3} > \frac{6}{-3} \quad \leftarrow \text{divide each side by } -3 \text{ and} \\ x > -2 \quad \quad \quad \text{reverse the inequality symbol} \end{array}$$

To check this solution, pick any number *greater than* -2 and substitute your choice into the original inequality. For instance, -1, 0, or 3, or 3,000 could be substituted into the original problem.

Check with different solutions of numbers *greater than* -2:

substitute -1

$$\begin{array}{l} -3x < 6 \\ -3(-1) < 6 \\ 3 < 6 \end{array} \quad \text{It checks!}$$

substitute 3

$$\begin{array}{l} -3x < 6 \\ -3(3) < 6 \\ -9 < 6 \end{array} \quad \text{It checks!}$$

substitute 0

$$\begin{array}{l} -3x < 6 \\ -3(0) < 6 \\ 0 < 6 \end{array} \quad \text{It checks!}$$

substitute 3,000

$$\begin{array}{l} -3x < 6 \\ -3(3,000) < 6 \\ -9,000 < 6 \end{array} \quad \text{It checks!}$$

Notice that -1, 0, 3, and 3,000 are all *greater than* -2 and each one *checks* as a solution.



Study the following examples.

**Example:** Solve by *multiplying* by a *negative number* and *reversing* the inequality sign.

$$\begin{aligned} -\frac{1}{3}y &\geq 4 \\ (-3) -\frac{1}{3}y &\leq 4(-3) && \leftarrow \text{multiply each side by } -3 \text{ and} \\ & && \text{reverse the inequality symbol} \\ y &\leq -12 \end{aligned}$$

**Example:** Solve by first adding, then *dividing* by a *negative number*, and *reversing* the inequality sign.

$$\begin{aligned} -3a - 4 &> 2 \\ -3a - 4 + 4 &> 2 + 4 && \leftarrow \text{add 4 to each side} \\ -3a &> 6 \\ \frac{-3a}{-3} &< \frac{6}{-3} && \leftarrow \text{divide each side by } -3 \text{ and} \\ & && \text{reverse the inequality symbol} \\ a &< -2 \end{aligned}$$

**Example:** Solve by first subtracting, then *multiplying* by a *negative number*, and *reversing* the inequality sign.

$$\begin{aligned} \frac{y}{-2} + 5 &\leq 0 \\ \frac{y}{-2} + 5 - 5 &\leq 0 - 5 && \leftarrow \text{subtract 5 from each side} \\ \frac{y}{-2} &\leq -5 \\ \frac{(-2)y}{-2} &\geq (-5)(-2) && \leftarrow \text{multiply each side by } -2 \text{ and} \\ & && \text{reverse the inequality symbol} \\ y &\geq 10 \end{aligned}$$



**Example:** Solve by first subtracting, then *multiplying* by a *positive number* and **not reversing** the inequality sign.

$$\begin{aligned}\frac{n}{2} + 5 &\leq 2 \\ \frac{n}{2} + 5 - 5 &\leq 2 - 5 && \leftarrow \text{subtract 5 from each side} \\ \frac{n}{2} &\leq -3 \\ \frac{(2)n}{2} &\leq -3(2) && \leftarrow \text{multiply each side by 2, but do not reverse} \\ n &\leq -6 && \text{the inequality symbol because we} \\ &&& \text{multiplied by a positive number}\end{aligned}$$

When multiplying or dividing both sides of an inequality by the same *positive number*, do *not* reverse the inequality symbol—leave it alone.

**Example:** Solve by first adding, then *dividing* by a positive number, and **not reversing** the inequality sign.

$$\begin{aligned}7x - 3 &> -24 \\ 7x - 3 + 3 &> -24 + 3 && \leftarrow \text{add 3 to each side} \\ 7x &> -21 \\ \frac{7x}{7} &> \frac{-21}{7} && \leftarrow \text{divide each side by 7 do not reverse} \\ x &> -3 && \text{the inequality symbol because we} \\ &&& \text{divided by a positive number}\end{aligned}$$



## Practice

Solve each **inequality**. Show **essential steps**. Then **graph the solutions**.

1.  $x + 5 \geq 2$

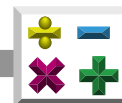
2.  $y - 1 \leq 5$

3.  $4 < n - 1$

4.  $2 \geq y - 4$

5.  $5a - 2 \leq 3$





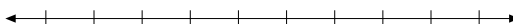
6.  $-\frac{1}{4}y > 0$



7.  $-2a \geq -12$



8.  $\frac{a}{3} - 3 < 1$



9.  $\frac{y}{2} - 6 < -5$



10.  $\frac{a}{-3} + 9 < 8$





## Practice

Solve the following. Show **essential steps**.

1.  $2y + 1 \leq 4$

4.  $\frac{1}{5}y + 9 \leq 8$

2.  $-\frac{1a}{3} - 4 > 2$

5.  $-10 < 2b - 14$

3.  $-11a + 3 < -30$

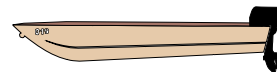
6.  $10y + 3 \leq 8$



Study the following.

Many problems in everyday life involve inequalities.

**Example:** A summer camp needs a boat with a motor. A local civic club will donate the money on the condition that the camp will spend *less than* \$1,500 for both. The camp decides to buy a boat for \$1,050. How much can be spent on the motor?



1. Choose a variable. Let  $x$  = cost of the motor,  
then let  $x + 1,050$  = cost of motor and boat,  
and cost of motor + cost of boat < total money.
2. Write as an inequality.  $x + 1,050 < 1,500$
3. Solve.  $x + 1,050 - 1,050 < 1,500 - 1,050$   
 $x < \$450$
4. Interpretation of solution: The camp can spend *any* amount *less than* \$450 for the motor. (**Note:** The motor *cannot* cost \$450.)

Use the **steps** below for the **word problems** on the following pages.

1. **Choose a variable**
2. **Write as an inequality**
3. **Solve**
4. **Interpret your solution**



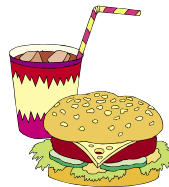
7. If \$50 is added to 2 times the amount of money in a wallet, the result is less than \$150. What is the greatest amount of money that could be in the wallet?

Interpretation of solution: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

8. Sandwiches cost \$2.50 and a drink is \$1.50. If you want to buy one drink, what is the greatest number of sandwiches you could also buy and spend less than \$10.00?



Interpretation of solution: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

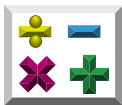


9. Annie babysits on Friday nights and Saturdays for \$3.00 an hour. Find the fewest number of hours she can babysit and earn more than \$20.00 a week.

Interpretation of solution: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



## Practice

Write **True** if the statement is correct. Write **False** if the statement is not correct.

- \_\_\_\_\_ 1. Graphing solutions on a number line will help you visualize solutions.
- \_\_\_\_\_ 2. An inequality can only be written with the variable on the right.
- \_\_\_\_\_ 3. The graph below of  $x < 5$  shows all real numbers greater than 5.



- \_\_\_\_\_ 4. *Real numbers* are all rational and irrational numbers.
- \_\_\_\_\_ 5. A *ratio* is the comparison of two quantities.
- \_\_\_\_\_ 6. To write an inequality that is equivalent to  $x < 5$ , move the number and variable to the opposite side of the inequality, and then reverse the inequality.
- \_\_\_\_\_ 7. When we multiply or divide each side of an inequality by the same negative number, we reverse the direction of the inequality symbol.
- \_\_\_\_\_ 8. There are no problems in everyday life that involve inequalities.
- \_\_\_\_\_ 9. An *inequality* is a sentence that states one expression is greater than, greater than or equal to, less than, less than or equal to, or not equal to another expression.



## Practice

Use the list below to write the correct term for each definition on the line provided.

decrease difference equation	increase reciprocals simplify an expression	solve sum
------------------------------------	---	--------------

- \_\_\_\_\_ 1. a number that is the result of subtraction
- \_\_\_\_\_ 2. to find all numbers that make an equation or inequality true
- \_\_\_\_\_ 3. to make less
- \_\_\_\_\_ 4. a mathematical sentence stating that the two expressions have the same value
- \_\_\_\_\_ 5. to make greater
- \_\_\_\_\_ 6. any two numbers with a product of 1; also called *multiplicative inverse*
- \_\_\_\_\_ 7. the result of adding numbers together
- \_\_\_\_\_ 8. to perform as many of the indicated operations as possible

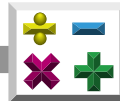


## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |   |   |
|---|---|
| _____ 1. a sentence that states one expression is greater than, greater than or equal to, less than, less than or equal to, or not equal to another expression          | A. angle ( $\angle$ )                   |
| _____ 2. the distance around a figure   | B. graph (of a number)                  |
| _____ 3. the point on a number line paired with the number  | C. inequality                           |
| _____ 4. a one-dimensional measure that is the measurable property of line segments   | D. length ( $l$ )                       |
| _____ 5. a data display that organizes information about a topic into categories  | E. measure (m) of an angle ( $\angle$ ) |
| _____ 6. the number of degrees ( $^\circ$ ) of an angle   | F. odd integer                          |
| _____ 7. a polygon with three sides   | G. perimeter ( $P$ )                    |
| _____ 8. two rays extending from a common endpoint called the vertex  | H. rectangle                            |
| _____ 9. any integer not divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$ | I. table (or chart)                     |
| _____ 10. a parallelogram with four right angles  | J. triangle                             |





## **Lesson Five Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.

### **Algebra Body of Knowledge**

#### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.3  
Solve literal equations for a specified variable.

#### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.



## Formulas Using Variables

There may be times when you need to solve an equation, such as

$$d = r \cdot t \quad \text{distance} = \text{rate} \cdot \text{time}$$

for one of its variables. When you know both rate and time, it is easy to calculate the distance using the **formula** above.

If you drive 60 mph for 5 hours, how far will you go? Use the following *formula*.

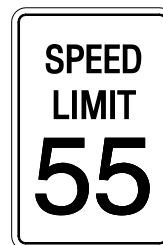
$$d = r \cdot t$$

Substitute 60 for the rate and 5 for the time to get the following.

$$d = 60 \cdot 5$$

$$d = 300 \text{ miles}$$

But what if you know your destination is 385 miles away and that the speed limit is 55 mph? It would be helpful to have a formula that gives you the amount of time you will need to get there. Rather than trying to remember a new formula for each situation, you could transform the one you already have using the rules you know.



Use the same algebraic rules that we used before, and solve the following.



**Remember:**  $t$  is for time.

$$d = r \cdot t$$

We want to get  $t$  alone on one side of the equation.

$$d = r \cdot t$$

$$\frac{d}{r} = \frac{r^1 \cdot t}{r_1}$$

$$\frac{d}{r} = t \quad \leftarrow \text{divide both sides by } r$$



Now see below how dividing distance (385) by rate (55), you get time, 7 hours.

$$\begin{aligned}
 d &= r \cdot t \\
 385 &= 55 \cdot t \\
 \frac{385}{55} &= \frac{55 \cdot t}{55} \\
 7 &= t \\
 7 \text{ hours}
 \end{aligned}$$

Let's try the examples below.

### Example 1

**Solve**

$$\begin{aligned}
 A &= \frac{1}{2}bh \text{ for } b. \\
 A &= \frac{1}{2}bh \\
 2A &= bh && \longleftarrow \text{multiply both sides by 2} \\
 \frac{2A}{h} &= b && \longleftarrow \text{divide both sides by } h
 \end{aligned}$$

Now let's try a more challenging example.

### Example 2

**Solve**

$$\begin{aligned}
 A &= \frac{1}{2}(b_1 + b_2)h \text{ for } h \\
 A &= \frac{1}{2}(b_1 + b_2)h \\
 2A &= (b_1 + b_2)h && \longleftarrow \text{multiply both sides by 2} \\
 \frac{2A}{(b_1 + b_2)} &= h && \longleftarrow \text{divide both sides by } (b_1 + b_2)
 \end{aligned}$$

Your turn.



## Practice

Solve *each* **formula** or **equation** for the **variable** given.

1.  $ax + by = c$  Solve for  $x$ .

2.  $(n - 2)180 = x$  Solve for  $n$ .

3.  $2a + b = c$  Solve for  $b$ .

4.  $a(1 + b) = c$  Solve for  $a$ .



5.  $2a + 2b = 4c$  Solve for  $b$ .

6.  $4(x + 5) = y$  Solve for  $x$ .

7.  $t = a + (n - 1)$  Solve for  $a$ .

8.  $c = \frac{5}{9}(F - 32)$  Solve for  $F$ .



## Unit Review

Solve these equations. *Show essential steps.*

1.  $4y + 2 = 30$

6.  $\frac{-3x}{4} - 8 = -2$

2.  $-5x - 6 = 34$

7.  $5 - x = 12$

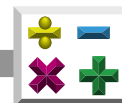
3.  $\frac{x}{3} + 7 = -3$

8.  $12 = -7 - x$

4.  $\frac{x}{4} - 2 = 10$

9.  $8 - \frac{2x}{3} = 12$

5.  $\frac{1x}{6} + 2 = 8$



10. What is the reciprocal of  $-\frac{3}{4}$ ? \_\_\_\_\_

Number 11 is a **gridded-response item**.

Write answer along the top of the grid and correctly mark it below.

11. What is the reciprocal of 8?

**Mark your answer on the grid to the right.**

	$\diagup$	$\diagup$	$\diagup$	
<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

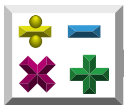
**Simplify** the following.

12.  $-5(x + 2) + 16$

14.  $5x - 7x$

13.  $15 + 2(x + 8)$

15.  $-8x - 14 + 10x - 20$



**Solve these equations. *Show essential steps.***

16.  $7x + 3 - 8x + 12 = -6$

17.  $7x + 3(x + 2) = 36$

18.  $-\frac{1}{2}(x + 10) = -15$

19.  $5x - 8 = 4x + 10$

20.  $-8(1 - 2x) = 5(2x - 6)$





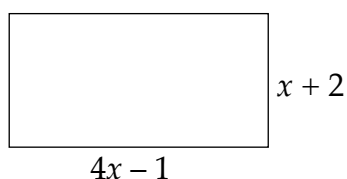
Write an **equation** and **solve for  $x$** .

21. The *sum* of  $2x$  and 7 equals 19. What is the number?

22.  $\frac{1}{2}$  of  $x$  *decreased* by 7 is -10. What is the number?

23. The *difference* between 14 and  $2x$  is -10. What is the number?

24. The *perimeter* ( $P$ ) is the distance around a figure, or sum of the lengths of the sides of a figure. The perimeter of the rectangle below is 52. Write an equation and solve for  $x$ .





Answer the following. Show **essential steps**.

Consecutive **odd integers** are numbers like 3, 5, and 7 or 15, 17, and 19. Note that you add 2 to the smallest to get the second number and 4 to the smallest to get to the third number. Use this information to solve the following problem.

25. The sum of three consecutive *odd* integers is 159.

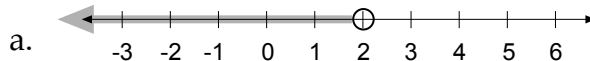
Description		Value
first number	$x$	=
second number	$x + 2$	=
third number	$x + 4$	=
sum	_____	= 159

Set up an equation and solve for  $x$ . Substitute your answer back into the table to find all answers. Do the numbers add up to 159?

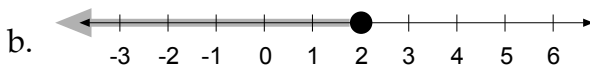


Match each **inequality** with its **graph**.

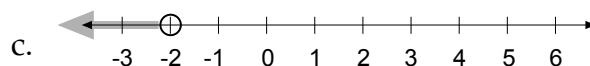
\_\_\_\_\_ 26.  $x \geq 2$



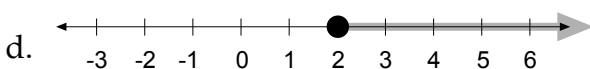
\_\_\_\_\_ 27.  $x < 2$



\_\_\_\_\_ 28.  $2 \geq x$

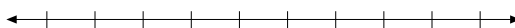


\_\_\_\_\_ 29.  $x < -2$



**Solve and graph.**

30.  $-5x + 6 > -34$



31.  $\frac{x}{-2} + 6 \leq 0$



**Solve the equation for the variable given.**

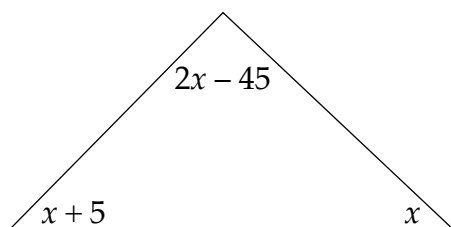
32.  $x(1 + b) = y$  Solve for  $x$ .



### Bonus Problems

*Answer the following.*

33. The sum of the measures of the angles in any triangle is 180 degrees. Find  $x$ , and then find the *measure* of each angle for the triangle below.



34. Solve and graph this inequality.

$$-20 < -2x - 14$$

## **Unit 4: Making Sense of Rational Expressions**

This unit emphasizes performing mathematical operations on rational expressions and using these operations to solve equations and inequalities.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Writing Process Strand**

##### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

#### **Algebra Body of Knowledge**

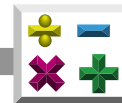
##### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.4  
Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

**Standard 4: Polynomials**

- MA.912.A.4.1  
Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2  
Add, subtract, and multiply polynomials.
- MA.912.A.4.3  
Factor polynomial expressions.
- MA.912.A.4.4  
Divide polynomials by monomials and polynomials with various techniques, including synthetic division.



## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

**canceling** .....dividing a numerator and a denominator by a common factor to write a fraction in lowest terms or before multiplying fractions

Example:  $\frac{15}{24} = \frac{\cancel{3} \cdot 5}{2 \cdot \cancel{2} \cdot 2 \cdot \cancel{2}} = \frac{5}{8}$

**common denominator** .....a common multiple of two or more denominators

Example: A common denominator for  $\frac{1}{4}$  and  $\frac{5}{6}$  is 12.

**common factor** .....a number that is a factor of two or more numbers

Example: 2 is a common factor of 6 and 12.

**common multiple** .....a number that is a multiple of two or more numbers

Example: 18 is a common multiple of 3, 6, and 9.



**cross multiplication** ..... a method for solving and checking proportions; a method for finding a missing numerator or denominator in equivalent fractions or ratios by making the cross products equal

*Example:* Solve this proportion by doing the following.

$$\frac{n}{9} = \frac{8}{12}$$

$$12 \times n = 9 \times 8$$

$$12n = 72$$

$$n = \frac{72}{12}$$

$$n = 6$$

Solution:

$$\frac{⑥}{9} = \frac{8}{12}$$

**decimal number** ..... any number written with a decimal point in the number

*Examples:* A decimal number falls between two whole numbers, such as 1.5, which falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called *decimal fractions*, such as five-tenths, or  $\frac{5}{10}$ , which is written 0.5.

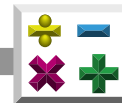
**denominator** ..... the bottom number of a fraction, indicating the number of equal parts a whole was divided into

*Example:* In the fraction  $\frac{2}{3}$  the denominator is 3, meaning the whole was divided into 3 equal parts.

**difference** ..... a number that is the result of subtraction

*Example:* In  $16 - 9 = 7$ , the difference is 7.





**distributive property** ..... the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products

*Examples:*  $x(a + b) = ax + bx$

$$5(10 + 8) = 5 \cdot 10 + 5 \cdot 8$$

**equation** ..... a mathematical sentence stating that the two expressions have the same value

*Example:*  $2x = 10$

**equivalent**

**(forms of a number)** ..... the same number expressed in different forms

*Example:*  $\frac{3}{4}$ , 0.75, and 75%

**expression** ..... a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables

*Examples:*  $4r^2$ ;  $3x + 2y$ ;  $\sqrt{25}$

An expression does *not* contain equal (=) or inequality (<, >, ≤, ≥, or ≠) signs.

**factor** ..... a number or expression that divides evenly into another number; one of the numbers multiplied to get a product

*Example:* 1, 2, 4, 5, 10, and 20 are factors of 20 and  $(x + 1)$  is one of the factors of  $(x^2 - 1)$ .

**factoring** ..... expressing a polynomial expression as the product of monomials and polynomials

*Example:*  $x^2 - 5x + 4 = 0$

$$(x - 4)(x - 1) = 0$$



**fraction** .....any part of a whole  
*Example:* One-half written in fractional form is  $\frac{1}{2}$  .

**inequality** .....a sentence that states one expression is greater than ( $>$ ), greater than or equal to ( $\geq$ ), less than ( $<$ ), less than or equal to ( $\leq$ ), or not equal to ( $\neq$ ) another expression  
*Examples:*  $a \neq 5$  or  $x < 7$  or  $2y + 3 \geq 11$

**integers** .....the numbers in the set  
 $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

**inverse operation** .....an action that undoes a previously applied action  
*Example:* Subtraction is the inverse operation of addition.

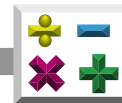
**irrational number** .....a real number that cannot be expressed as a ratio of two integers  
*Example:*  $\sqrt{2}$

**least common**

**denominator (LCD)** .....the smallest common multiple of the denominators of two or more fractions  
*Example:* For  $\frac{3}{4}$  and  $\frac{1}{6}$ , 12 is the least common denominator.

**least common**

**multiple (LCM)** .....the smallest of the common multiples of two or more numbers  
*Example:* For 4 and 6, 12 is the least common multiple.



- like terms** ..... terms that have the same variables and the same corresponding exponents  
*Example:* In  $5x^2 + 3x^2 + 6$ , the like terms are  $5x^2$  and  $3x^2$ .
- minimum** ..... the smallest amount or number allowed or possible
- multiplicative identity** ..... the number one (1); the product of a number and the multiplicative identity is the number itself  
*Example:*  $5 \times 1 = 5$
- multiplicative property of -1** ..... the product of any number and -1 is the opposite or additive inverse of the number  
*Example:*  $-1(a) = -a$  and  $a(-1) = -a$
- negative numbers** ..... numbers less than zero
- numerator** ..... the top number of a fraction, indicating the number of equal parts being considered  
*Example:* In the fraction  $\frac{2}{3}$ , the numerator is 2.
- order of operations** ..... the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations*  
*Example:*  $5 + (12 - 2) \div 2 - 3 \times 2 =$   
 $5 + 10 \div 2 - 3 \times 2 =$   
 $5 + 5 - 6 =$   
 $10 - 6 =$   
 $4$



**polynomial** ..... a monomial or sum of monomials; any rational expression with no variable in the denominator

*Examples:*  $x^3 + 4x^2 - x + 8$      $5mp^2$   
 $-7x^2y^2 + 2x^2 + 3$

**positive numbers** ..... numbers greater than zero

**product** ..... the result of multiplying numbers together  
*Example:* In  $6 \times 8 = 48$ , the product is 48.

**quotient** ..... the result of dividing two numbers  
*Example:* In  $42 \div 7 = 6$ , the quotient is 6.

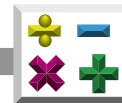
**ratio** ..... the comparison of two quantities  
*Example* The ratio of  $a$  and  $b$  is  $a:b$  or  $\frac{a}{b}$ , where  $b \neq 0$ .

**rational expression** ..... a fraction whose numerator and/or denominator are polynomials  
*Examples:*  $\frac{x}{8}$      $\frac{5}{x+2}$      $\frac{4x^2+1}{x^2+1}$

**rational number** ..... a number that can be expressed as a ratio  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$

**real numbers** ..... the set of all rational and irrational numbers

**reciprocals** ..... two numbers whose product is 1; also called *multiplicative inverses*  
*Examples:* 4 and  $\frac{1}{4}$  are reciprocals because  $\frac{4}{1} \times \frac{1}{4} = 1$ ;  $\frac{3}{4}$  and  $\frac{4}{3}$  are reciprocals because  $\frac{3}{4} \times \frac{4}{3} = 1$ ; zero (0) has no multiplicative inverse



**simplest form**

**(of a fraction)** ..... a fraction whose numerator and denominator have no common factor greater than 1

*Example:* The simplest form of  $\frac{3}{6}$  is  $\frac{1}{2}$ .

**simplify an expression** ..... to perform as many of the indicated operations as possible

**solution** ..... any value for a variable that makes an equation or inequality a true statement

*Example:* In  $y = 8 + 9$   
 $y = 17$       17 is the solution.

**substitute** ..... to replace a variable with a numeral

*Example:*  $8(a) + 3$   
 $8(5) + 3$

**sum** ..... the result of adding numbers together

*Example:* In  $6 + 8 = 14$ , the sum is 14.

**term** ..... a number, variable, product, or quotient in an expression

*Example:* In the expression  $4x^2 + 3x + x$ , the terms are  $4x^2$ ,  $3x$ , and  $x$ .

**variable** ..... any symbol, usually a letter, which could represent a number





## Unit 4: Making Sense of Rational Numbers

### Introduction

Algebra students must be able to add, subtract, multiply, divide, and simplify rational expressions efficiently. These skills become more important as you progress in using mathematics. As an algebra student, you will have the opportunity to work with methods you will need for future mathematical success.

### Lesson One Purpose

#### Reading Process Strand

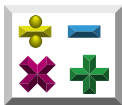
##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### Writing Process Strand

##### Standard 3: Prewriting

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

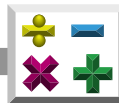


## Algebra Body of Knowledge

### Standard 4: Polynomials

- MA.912.A.4.1  
Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2  
Add, subtract, and multiply polynomials.
- MA.912.A.4.3  
Factor polynomial expressions.
- MA.912.A.4.4  
Divide polynomials by monomials and polynomials with various techniques, including synthetic division.





## Simplifying Rational Expressions

An **expression** is a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes **variables**. A **fraction**, or any part of a whole, is an *expression* that represents a **quotient**—the result of dividing two numbers. The same *fraction* may be expressed in many different ways.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10}$$

If the **numerator** (top number) and the **denominator** (bottom number) are both **polynomials**, then we call the fraction a **rational expression**. A *rational expression* is a fraction whose *numerator* and/or *denominator* are *polynomials*. The fractions below are all rational expressions.

$$\frac{x}{x+y} \quad \frac{a^2 - 2a + 1}{a} \quad \frac{1}{y^2 + 4} \quad \frac{a}{b-3}$$

When the *variables* or any symbols which could represent numbers (usually letters) are replaced, the result is a numerator and a denominator that are **real numbers**. In this case, we say the entire *expression* is a *real number*. Real numbers are all **rational numbers** and **irrational numbers**.

*Rational numbers* are numbers that can be expressed as a **ratio**  $\frac{a}{b}$ , where  $a$  and  $b$  are **integers** and  $b \neq 0$ . *Irrational numbers* are real numbers that *cannot* be expressed as a *ratio* of two *integers*. Of course, there is an exception: when a denominator is equal to 0, we say the fraction is *undefined*.

**Note:** In this unit, we will agree that *no* denominator equals 0.



Fractions have some interesting properties. Let's examine them.

- If  $\frac{a}{b} = \frac{c}{d}$ , then  $ad = bc$ .  $\frac{4}{8} = \frac{6}{12}$  therefore  $4 \cdot 12 = 8 \cdot 6$

$$\begin{array}{c} \text{a} \\ \text{b} \end{array} = \begin{array}{c} \text{c} \\ \text{d} \end{array}$$

$$a \cdot d = b \cdot c$$

$$ad = bc$$

In other words, if two fractions are equal, then the **products** are equal when you **cross multiply**.

$$\begin{array}{c} 4 \\ 8 \end{array} = \begin{array}{c} 6 \\ 12 \end{array}$$

$$4 \cdot 12 = 8 \cdot 6$$

$$48 = 48$$

- $\frac{a}{b} = \frac{ac}{bc}$   $\frac{4}{7} = \frac{4 \cdot 3}{7 \cdot 3}$  therefore  $\frac{4}{7} = \frac{12}{21}$

Simply stated, if you *multiply* both the numerator and the denominator by the *same* number, the new fraction will be **equivalent** to the original fraction.

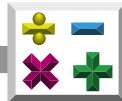
- $\frac{ac}{bc} = \frac{a}{b}$   $\frac{9}{21} = \frac{9 \div 3}{21 \div 3}$  therefore  $\frac{9}{21} = \frac{3}{7}$

In other words, if you *divide* both the numerator and the denominator by the *same* number, the new fraction will be *equivalent* to the original fraction. The same rules are true for **simplifying** rational expressions by performing as many indicated operations as possible. Many times, however, it is necessary to **factor** and find numbers or expressions that divide the numerator or the denominator, or both, so that the **common factors** become easier to see. Look at the following example:

$$\frac{3x + 3y}{3} = \frac{3(x + y)}{3} = x + y$$

Notice that, by **factoring** a 3 out of the numerator, we can divide (or **cancel**) the 3s, leaving  $x + y$  as the final result.

Before we move on, do the practice on the following pages.



## Practice

**Simplify each expression.** Refer to **properties** and **examples** on the previous pages as needed. Show **essential steps**.

1.  $\frac{4x-4}{x-1}$

2.  $\frac{4m-2}{2m-1}$

3.  $\frac{6x-3y}{3}$



*Example:*

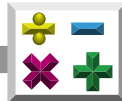
$$\frac{4x - 6}{6} = \frac{\overset{1}{2}(2x - 3)}{\underset{3}{\cancel{6}}} = \frac{1(2x - 3)}{3} = \frac{2x - 3}{3}$$

4.  $\frac{5a - 10}{15}$

5.  $\frac{2y - 8}{4}$

6.  $\frac{3m + 6n}{3}$

7.  $\frac{14r^3s^4 + 28rs^2 - 7rs}{7r^2s^2}$



## Practice

**Simplify each expression.** Refer to **properties** and **examples** on the previous pages as needed. Show **essential steps**.

*Example:*

$$\frac{2x^2 - 8}{x + 2} = \frac{2(x^2 - 4)}{x + 2} = \frac{2(x - 2)(\cancel{x + 2})^1}{\cancel{x + 2}_1} = 2(x - 2)$$

**Note:** In the above example, notice the following:

- After we factored 2 from the numerator,
- we were left with  $x^2 - 4$ ,
- which can be factored into  $(x + 2)(x - 2)$ .
- Then the  $(x + 2)$  is cancelled,
- leaving  $2(x - 2)$  as the final answer.

1.  $\frac{3y^2 - 27}{y - 3}$

2.  $\frac{a - b}{a^2 - b^2}$



3.  $\frac{b-a}{a^2-b^2}$

4.  $\frac{9x+3}{6x+2}$

5.  $\frac{9x^2+3}{6x+3}$



## Additional Factoring

Look carefully at numbers 2-5 in the previous practice. What do you notice about them?



**Alert!** You cannot cancel individual **terms** (numbers, variables, products, or quotients in an expression)—you can only cancel *factors* (numbers or expressions that exactly divide another number)!

$$\frac{2x+4}{4} \neq \frac{2x}{4} \quad \frac{3x+6}{3} \neq \frac{x+6}{3} \quad \frac{9x^2+3}{6x+3} \neq \frac{9x^2x}{6x}$$

Look at how simplifying these expressions was taken a step further. Notice that additional factoring was necessary.

### Example

$$\frac{x^2 + 5x + 6}{x + 3} = \frac{\overset{1}{(x+3)}(x+2)}{\underset{1}{x+3}} = (x+2) = x+2$$

Look at the denominator above. It is one of the factors of the numerator. Often, you can use the problem for hints as you begin to factor.



## Practice

**Factor** each of these and then **simplify**. Look for **hints** within the problem. Refer to the previous page as necessary. Show **essential steps**.

1.  $\frac{a^2 - 3a + 2}{a - 2}$

2.  $\frac{b^2 - 2b - 3}{b - 3}$

Sometimes, it is necessary to **factor both** the numerator and denominator. Examine the example below, then **simplify** each of the following **expressions**.

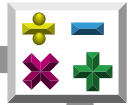
Example:

$$\frac{x^2 - 4}{x^2 + x - 6} = \frac{(x + 2)(\cancel{x - 2})^1}{(x + 3)(\cancel{x - 2})_1} = \frac{(x + 2)}{(x + 3)} = \frac{x + 2}{x + 3} \quad \textbf{Note: The } x\text{'s do not cancel.}$$

3.  $\frac{2r^2 + r - 6}{r^2 + r - 2}$

4.  $\frac{x^2 + x - 2}{x^2 - 1}$





## Practice

Simplify *each* expression. Show essential steps.

1.  $\frac{5b - 10}{b - 2}$

2.  $\frac{6a - 9}{10a - 15}$

3.  $\frac{9x + 3}{9}$

4.  $\frac{6b + 9}{12}$



5.  $\frac{3a^2b + 6ab - 9b^2}{3b}$

6.  $\frac{x^2 - 16}{x + 4}$

7.  $\frac{2a - b}{b^2 - 4a^2}$

8.  $\frac{6x^2 + 2}{9x^2 + 3}$



## Practice

**Factor** each of these **expressions** and then **simplify**. Show **essential steps**.

1.  $\frac{y^2 + 5y - 14}{y - 2}$

2.  $\frac{a^2 - 5a + 4}{a - 4}$

3.  $\frac{6m^2 - m - 1}{2m^2 + 9m - 5}$

4.  $\frac{4x^2 - 9}{2x^2 + x - 6}$



## Practice

Use the list below to write the correct term for each definition on the line provided.

**denominator**  
**expression**  
**fraction**

**numerator**  
**polynomial**  
**quotient**

**rational expression**  
**real numbers**  
**variable**

- \_\_\_\_\_ 1. a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables
- \_\_\_\_\_ 2. the top number of a fraction, indicating the number of equal parts being considered
- \_\_\_\_\_ 3. the bottom number of a fraction, indicating the number of equal parts a whole was divided into
- \_\_\_\_\_ 4. the set of all rational and irrational numbers
- \_\_\_\_\_ 5. any part of a whole
- \_\_\_\_\_ 6. a fraction whose numerator and/or denominator are polynomials
- \_\_\_\_\_ 7. any symbol, usually a letter, which could represent a number
- \_\_\_\_\_ 8. a monomial or sum of monomials; any rational expression with no variable in the denominator
- \_\_\_\_\_ 9. the result of dividing two numbers



## Practice

Use the list below to complete the following statements.

canceling  
cross multiplication  
equivalent  
factor

integers  
product  
simplify an expression  
terms

1. If you *multiply* both the numerator and the denominator by the *same* number, the new fraction will be \_\_\_\_\_ because it is the same number expressed in a different form.
2. The numbers in the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$  are \_\_\_\_\_.
3. If you divide a numerator and a denominator by a common factor to write a fraction in lowest terms, or before multiplying fractions, you are \_\_\_\_\_.
4. To \_\_\_\_\_, you need to perform as many of the indicated operations as possible.
5. Numbers, variables, products, or quotients in an expression are called \_\_\_\_\_.



6. A \_\_\_\_\_ is a number or expression that divides evenly into another number.
7. When you multiply numbers together, the result is called the \_\_\_\_\_ .
8. To find a missing numerator or denominator in equivalent fractions or ratios, you can use a method called \_\_\_\_\_ and make the cross products equal.



## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

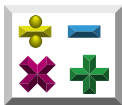
#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 4: Polynomials**

- MA.912.A.4.1  
Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2  
Add, subtract, and multiply polynomials.
- MA.912.A.4.3  
Factor polynomial expressions.



- MA.912.A.4.4  
Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

## Addition and Subtraction of Rational Expressions

In order to add and subtract rational expressions in fraction form, it is necessary for the fractions to have a **common denominator** (the same bottom number). We find those *common denominators* in the same way we did with simple fractions. The process requires careful attention.

- When we add  $\frac{3}{7} + \frac{5}{8}$ , we find a common denominator by multiplying 7 and 8.
- Then we change each fraction to an equivalent fraction whose denominator is 56.

$$\frac{3 \cdot 8}{7 \cdot 8} = \frac{24}{56} \quad \text{and} \quad \frac{5 \cdot 7}{8 \cdot 7} = \frac{35}{56}$$

- Next we add  $\frac{24}{56} + \frac{35}{56} = \frac{59}{56}$ .

## Finding the Least Common Multiple (LCM)

By multiplying the denominators of the terms we intend to add or subtract, we can always find a common denominator. However, it is often to our advantage to find the **least common denominator (LCD)**, which is also the **least common multiple (LCM)**. The *LCD* or *LCM* is the smallest of the **common multiples** of two or more numbers. This makes simplifying the result easier. Look at the example on the following page.





Let's look at finding the LCM of 36, 27, and 15.

1. Factor each of the denominators and examine the results.

$36 = 2 \cdot 2 \cdot 3 \cdot 3$  ← The new denominator must contain at least **two 2s** and **two 3s**.

$27 = 3 \cdot 3 \cdot 3$  ← The new denominator must contain at least **three 3s**.

$45 = 3 \cdot 3 \cdot 5$  ← The new denominator must contain at least **two 3s** and **one 5**.

2. Find the **minimum** combination of factors that is described by the combination of all the statements above—two 2s, three 3s, and one 5.

$$\text{LCM} = \underbrace{2 \cdot 2}_{\text{two 2s}} \cdot \underbrace{3 \cdot 3 \cdot 3}_{\text{three 3s}} \cdot \underbrace{5}_{\text{one 5}} = 540$$

3. Convert the *terms* to equivalent fractions using the new common denominator and then proceed to add or subtract.

$$\frac{5}{36} = \frac{75}{540}, \frac{8}{27} = \frac{160}{540}, \frac{4}{15} = \frac{144}{540} \rightarrow \frac{75}{540} + \frac{160}{540} - \frac{144}{540} = \frac{91}{540}$$



Now, let's look at an algebraic example.

$$\frac{y}{y^2 - 9} - \frac{1}{y^2 - 4y - 21} =$$

1. Factor each denominator and examine the results.

$$y^2 - 9 = (y + 3)(y - 3) \quad \leftarrow \text{The new denominator must contain } (y + 3) \text{ and } (y - 3).$$

$$y^2 - 4y - 21 = (y - 7)(y + 3) \quad \leftarrow \text{The new denominator must contain } (y - 7) \text{ and } (y + 3).$$

2. Find the *minimum* combination of factors.

$$\text{LCM} = (y + 3)(y - 3)(y - 7)$$

3. Convert each fraction to an equivalent fraction using the new common denominator and proceed to subtract.

$$\begin{aligned} \frac{y(y - 7)}{(y + 3)(y - 3)(y - 7)} - \frac{1(y - 3)}{(y + 3)(y - 3)(y - 7)} &= \text{notice how the minus sign between the fractions} \\ &\text{distributes to make} \\ &\text{-}y + 3 \text{ in the numerator} \\ &\text{(distributive property)} \\ \frac{y^2 - 7y - y + 3}{(y + 3)(y - 3)(y - 7)} &= \\ \frac{y^2 - 8y + 3}{(y + 3)(y - 3)(y - 7)} \end{aligned}$$

**Hint:** Always check to see if the numerator can be factored and then reduce, if possible. Do this to be sure the answer is in the lowest terms.



## Practice

Write each **sum** or **difference** as a single fraction in **lowest terms**. Show **essential steps**.

1.  $\frac{a}{7} + \frac{2a}{7} - \frac{5}{7}$

4.  $\frac{x+1}{5} - \frac{x+1}{5}$

2.  $\frac{x-2}{2y} + \frac{x}{2y}$

5.  $\frac{5}{6} + \frac{y}{4}$

3.  $\frac{x+1}{5} + \frac{x-1}{5}$

6.  $\frac{2}{x+2} - \frac{3}{x+3}$



## Practice

Write each **sum** or **difference** as a single fraction in **lowest terms**. Show **essential steps**.

Example:  $\frac{5}{b^2-9} - \frac{1}{b-3} = \frac{5}{(b+3)(b-3)} - \frac{1}{b-3} =$

$$\frac{5}{(b+3)(b-3)} - \frac{1(b+3)}{(b+3)(b-3)} =$$
$$\frac{5-1(b+3)}{(b+3)(b-3)} =$$
$$\frac{5-b-3}{(b+3)(b-3)} =$$
$$\frac{2-b}{(b+3)(b-3)} =$$

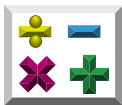
1.  $\frac{1}{2z+1} + \frac{3}{z-2}$

2.  $\frac{r}{r^2-16} + \frac{r+1}{r^2-5r+4}$



3.  $\frac{8}{a^2 - 4} - \frac{2}{a^2 - 5a + 6}$

4.  $m + \frac{1}{m - 1} - \frac{1}{(m - 1)^2}$



## Practice

Write each **sum** or **difference** as a single fraction in **lowest terms**. Show **essential steps**.

1.  $\frac{x}{3} - \frac{3y}{3} + \frac{4z}{3}$

2.  $\frac{x-2}{2y} - \frac{x}{2y}$

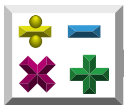
3.  $\frac{x+1}{5} - \frac{x-1}{5}$



4.  $\frac{2}{2a-4b} - \frac{b-2}{2a-4b} + \frac{7b}{2a-4b}$

5.  $\frac{x+3}{4} + \frac{5-x}{10}$

6.  $\frac{5}{2m-6} - \frac{3}{m-3}$



## Practice

Write each **sum** or **difference** as a single fraction in **lowest terms**. Show **essential steps**.

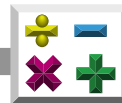
1.  $\frac{2}{x^2 - x - 2} - \frac{2}{x^2 + 2x + 1}$

2.  $\frac{1}{b^2 - 1} - \frac{1}{b^2 + 2b + 1}$

3.  $\frac{3x}{x^2 + 3x - 10} - \frac{2x}{x^2 + x - 6}$

4.  $\frac{1}{x^2 - 7x + 12} + \frac{2}{x^2 - 5x + 6} - \frac{3}{x^2 - 6x + 8}$





## **Lesson Three Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

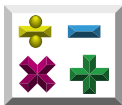
#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 4: Polynomials**

- MA.912.A.4.1  
Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2  
Add, subtract, and multiply polynomials.
- MA.912.A.4.3  
Factor polynomial expressions.



## Multiplication and Division of Rational Expressions

To multiply fractions, you learned to multiply the numerators together, then multiply the denominators together, and then reduce, if possible.

$$\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$$

We use this same process with rational expressions.

$$\frac{4}{5x} \times \frac{11x}{13} = \frac{44x}{65x} = \frac{44}{65}$$

Sometimes it is simpler to reduce or cancel *common factors* before multiplying.

$$\frac{4}{5x} \times \frac{11x}{13} = \frac{4}{\cancel{5x}} \times \frac{\cancel{11x}}{13} = \frac{44}{65}$$

When we need to divide fractions, we invert (flip over) the fraction to the right of the division symbol and then multiply.

$$\frac{2x^2}{3y} \div \frac{4x}{5y^3} = \frac{2x^2}{3y} \cdot \overset{\text{invert}}{\frac{5y^3}{4x}} = \frac{10x^2y^3}{12xy} = \frac{5xy^2}{6}$$

Pay careful attention to *negative* signs in the factors.

Decide before you multiply whether the answer will be positive *or* negative.

- If the number of negative factors is *even*, the result will be *positive*.
- If the number of negative factors is *odd*, the answer will be *negative*.



**Remember:** In this unit, we agreed that *no* denominator equals 0.



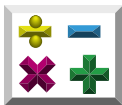
## Practice

Write each **product** as a single fraction in **simplest terms**. Show **essential steps**.

1.  $\frac{6x^3}{3} \cdot \frac{4b}{2x}$

2.  $\frac{14a^3b}{3b} \cdot \frac{-6}{7ab}$

3.  $\frac{-12ab^2}{5bc} \cdot \frac{10b^2c}{6ab}$



## Practice

Write each **product** as a single fraction in **simplest terms**. Show **essential steps**.

Example:

$$\frac{4a^2 - 1}{a^2 - 4} \cdot \frac{a + 2}{4a + 2} =$$
$$\frac{\cancel{(2a+1)}(2a-1)}{\cancel{(a+2)}(a-2)} \cdot \frac{\cancel{a+2}}{2\cancel{(2a+1)}} =$$
$$\frac{(2a-1)}{2(a-2)} =$$
$$\frac{2a-1}{2(a-2)}$$

1.  $\frac{5x + 25}{4x} \cdot \frac{2x}{3x + 15}$

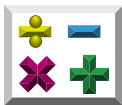
2.  $\frac{y^2 - y - 2}{y^2 + 4y + 3} \cdot \frac{y^2 - 4y - 5}{y^2 - 3y - 10}$

**Hint:** If you have trouble factoring, review the examples and explanation of processes on pages 304-306.



$$3. \quad \frac{2a^2 - a - 6}{3a^2 - 4a + 1} \cdot \frac{3a^2 + 7a + 2}{2a^2 + 7a + 6}$$

$$4. \quad \frac{3x^2 - 3x}{5} \cdot \frac{x^2 - 9x - 10}{6x - 60} \cdot \frac{4}{1 - x^2}$$



## Practice

Write each **quotient** as a single fraction in **simplest terms**. Show **essential steps**.



**Remember:** Invert and then multiply!

1.  $\frac{9ab}{x} \div \frac{3a}{2x^2}$

2.  $\frac{x^2 - x - 6}{x^2 - 2x - 15} \div \frac{x^2 - 4}{x^2 - 6x + 5}$

3.  $\frac{10a^2 - 13a - 3}{2a^2 - a - 3} \div \frac{5a^2 - 9a + -2}{3a^2 + 2a - 1}$

4.  $\frac{9r^2 + 3r - 2}{12r^2 + 5r - 2} \div \frac{9r^2 - 6r + 1}{8r^2 + 10r - 3}$



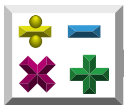
## Practice

Write each **product** as a single fraction in **simplest terms**. Show **essential steps**.

1.  $\frac{4a^3}{3} \cdot \frac{6b}{2a}$

2.  $\frac{-18ab^2}{5bc} \cdot \frac{15b^3c}{6ab}$

3.  $\frac{24a^3b}{3b} \cdot \frac{-9}{6ab}$



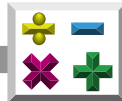
4.  $\frac{9b^2 - 25}{2b - 2} \cdot \frac{b^2 - 1}{6b - 10}$

5.  $\frac{x^2 - x - 20}{x^2 + 7x + 12} \cdot \frac{x + 3}{x - 5}$

6.  $\frac{7x + 14}{14x - 28} \cdot \frac{4 - 2x}{x + 2} \cdot \frac{x + 3}{x + 1}$

**Hint:**  $a - b = -(b - a)$





## Practice

Write each **quotient** as a single fraction in **simplest terms**. Show **essential steps**.

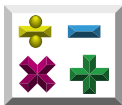
1.  $\frac{28x^2y^3}{10a^2} \div \frac{21x^3y}{5a}$

2.  $\frac{4x-8}{3} \div \frac{-(6x-12)}{9}$

3.  $\frac{6a^3b}{4x} \div \frac{3a}{2x^3}$

4.  $\frac{r^2+2r-15}{r^2+3r-10} \div \frac{r^2-9}{r^2-9r+14}$

5.  $\frac{y^2+y-2}{y^2+2y-3} \div \frac{y^2+7y+10}{y^2-2y-15}$



## **Lesson Four Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

### **Standard 4: Polynomials**

- MA.912.A.4.1  
Simplify monomials and monomial expressions using the laws of integral exponents.



- MA.912.A.4.2  
Add, subtract, and multiply polynomials.
- MA.912.A.4.3  
Factor polynomial expressions.
- MA.912.A.4.4  
Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

## Solving Equations

Recall that an **equation** is a mathematical sentence stating the two expressions have the same value. The equality symbol or equal sign (=) shows that two quantities are equal. An *equation* equates one expression to another.

$3x - 7 = 8$  is an example of an equation.

You may be able to solve this problem mentally, without using paper and pencil.

$$3x - 7 = 8$$

The problem reads—3 times what number minus 7 equals 8?

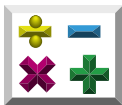


**Think:**  $3 \cdot 4 = 12$   
 $12 - 7 = 5$  ← too small



**Think:**  $3 \cdot 5 = 15$   
 $15 - 7 = 8$  ← That's it!

$$\begin{aligned} 3x - 7 &= 8 \\ 3(5) - 7 &= 8 \end{aligned}$$



## Practice

Solve each of the following **mentally**, writing only the answer.

1.  $4y + 6 = 22$

$y =$

2.  $2a - 4 = 10$

$a =$

3.  $5x - 15 = -20$

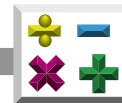
$x =$

4.  $-7b + 6 = -22$

$b =$



**Check yourself:** Add all your answers for problems 1-4. Did you get a *sum* of 14? If not, correct your work before continuing.



## Step-by-Step Process for Solving Equations

A problem like  $\frac{x+12}{5} = -2(x-10)$  is a bit more challenging. You could use a guess and check process, but that would take more time, especially when answers involve **decimals** or fractions.

So, as problems become more difficult, you can see that it is important to have a process in mind and to write down the steps as you go.

Unfortunately, there is *no* exact process for solving equations. Every rule has an exception. That is why creative thinking, reasoning, and practice are necessary and keeping a written record of the steps you have used is extremely helpful.

### Example 1

Let's look at a step-by-step process for solving the problem above.

$$\frac{x+12}{5} = -2(x-10) \quad \leftarrow \text{Step 1: Copy the problem **carefully!**}$$

$$\frac{x+12}{5} = -2x + 20 \quad \leftarrow \text{Step 2: Simplify each side of the equation as needed by *distributing* the 2.}$$

$$\left(\frac{x+12}{5}\right) \cdot 5 = (-2x + 20) \cdot 5 \quad \leftarrow \text{Step 3: Multiply both sides of the equation by 5 to "undo" the division by 5, which eliminates the fraction.}$$

$$x + 12 = -10x + 100 \quad \leftarrow \text{Step 4: Simplify by distributing the 5.}$$

$$\begin{aligned} (+10x) + 1x + 12 &= -10x + (+10x) + 100 \quad \leftarrow \text{Step 5: Add } 10x \text{ to both sides.} \\ 11x + 12 &= 100 \end{aligned}$$

$$\begin{aligned} 11x + 12 (-12) &= 100 (-12) \quad \leftarrow \text{Step 6: Subtract 12 from both sides.} \\ 11x &= 88 \end{aligned}$$

$$\begin{aligned} 11x \div 11 &= 88 \div 11 \quad \leftarrow \text{Step 7: Divide both sides by 11.} \\ x &= 8 \end{aligned}$$

$$\begin{aligned} \frac{x+12}{5} &= -2(x-10) \quad \leftarrow \text{Step 8: Check by replacing the variable in the original problem.} \\ \frac{8+12}{5} &= -2(8) + 20 \\ 4 &= -16 + 20 \\ 4 &= 4 \end{aligned}$$

*It checks!*



### Example 2

What if the original problem had been  $5x + 12 = -2(x - 10)$ ? The process would have been different. Watch for differences.

$$5x + 12 = -2(x - 10) \quad \leftarrow \text{Step 1: Copy the problem **carefully!**}$$

$$5x + 12 = -2x + 20 \quad \leftarrow \text{Step 2: Simplify each side of the equation as needed by *distributing* the 2.}$$

$$5x + 12 \textcircled{-12} = -2x + 20 \textcircled{-12} \quad \leftarrow \text{Step 3: Subtract 12 from both sides of the equation.}$$
$$5x = -2x + 8$$

$$5x \textcircled{+2x} = -2x \textcircled{+2x} + 8 \quad \leftarrow \text{Step 4: Add } 2x \text{ to both sides of the equation.}$$
$$7x = 8$$

$$7x \textcircled{\div 7} = 8 \textcircled{\div 7} \quad \leftarrow \text{Step 5: Divide both sides by 7.}$$
$$x = \frac{8}{7}$$

$$5x + 12 = -2(x - 10) \quad \leftarrow \text{Step 6: Check by replacing the variable in the original problem.}$$
$$5\left(\frac{8}{7}\right) + 12 = -2\left(\frac{8}{7}\right) + 20$$

$$\frac{40}{7} + 12 = \frac{-16}{7} + 20$$

$$5\frac{5}{7} + 12 = -2\frac{2}{7} + 20$$

$$17\frac{5}{7} = 17\frac{5}{7}$$

*It checks!*

Did you notice that the steps were *not* always the same? The rules for solving equations change to fit the individual needs of each problem. You can see why it is a good idea to check your answers each time. You may need to do some steps in a different order than you originally thought.



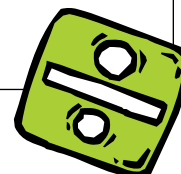
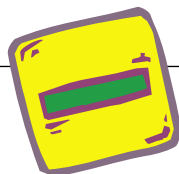
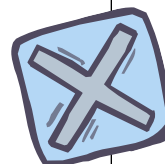
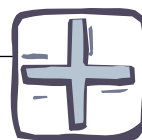
Generally speaking the processes for solving equations are as follows.

- Simplify both sides of the equation as needed.
- “Undo” additions and subtractions.
- “Undo” multiplications and divisions.

You might notice that this seems to be the *opposite* of the **order of operations**. Typically, we “undo” in the *reverse* order from the original process.

### Guidelines for Solving Equations

1. Use the **distributive property** to clear parentheses.
2. Combine **like terms**. We want to isolate the variable.
3. Undo addition or subtraction using **inverse operations**.
4. Undo multiplication or division using *inverse operations*.
5. Check by **substituting** the **solution** in the original equation.



SAM = Simplify (steps 1 and 2) then  
Add (or subtract)  
Multiply (or divide)



Here are some additional examples.

### Example 3

**Solve:**

$$\begin{aligned}6y + 4(y + 2) &= 88 \\6y + 4y + 8 &= 88 &< \text{ use } \textit{distributive property} \\10y + 8 - 8 &= 88 - 8 &< \text{ combine } \textit{like terms} \text{ and undo addition} \\&&\text{ by subtracting 8 from each side} \\ \frac{10y}{10} &= \frac{80}{10} &< \text{ undo multiplication by dividing} \\y &= 8 &\text{ by 10}\end{aligned}$$

**Check** *solution* in the original equation:

$$\begin{aligned}6y + 4(y + 2) &= 88 \\6(8) + 4(8 + 2) &= 88 \\48 + 4(10) &= 88 \\48 + 40 &= 88 \\88 &= 88 &< \text{ It checks!}\end{aligned}$$

### Example 4

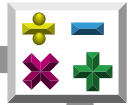
**Solve:**

$$\begin{aligned}-\frac{1}{2}(x + 8) &= 10 \\-\frac{1}{2}x - 4 &= 10 &< \text{ use distributive property} \\-\frac{1}{2}x - 4 + 4 &= 10 + 4 &< \text{ undo subtraction by adding 4 to} \\&&\text{ both sides} \\-\frac{1}{2}x &= 14 \\(-2)-\frac{1}{2}x &= 14(-2) &< \text{ isolate the variable by multiplying} \\x &= -28 &\text{ each side by the } \textbf{reciprocal} \text{ of } -\frac{1}{2}\end{aligned}$$

**Check** *solution* in the original equation:

$$\begin{aligned}-\frac{1}{2}(x + 8) &= 10 \\-\frac{1}{2}(-28 + 8) &= 10 \\-\frac{1}{2}(-20) &= 10 \\10 &= 10 &< \text{ It checks!}\end{aligned}$$





### Example 5

**Solve:**

$$\begin{aligned} 26 &= \frac{2}{3}(9x - 6) \\ 26 &= \frac{2}{3}(9x) - \frac{2}{3}(6) \quad \leftarrow \text{use distributive property} \\ 26 &= 6x - 4 \\ 26 + 4 &= 6x - 4 + 4 \quad \leftarrow \text{undo subtraction by adding 4 to each side} \\ \frac{30}{6} &= \frac{6x}{6} \quad \leftarrow \text{undo multiplication by dividing each side by 6} \\ 5 &= x \end{aligned}$$

**Check** solution in the original equation:

$$\begin{aligned} 26 &= \frac{2}{3}(9x - 6) \\ 26 &= \frac{2}{3}(9 \cdot 5 - 6) \\ 26 &= \frac{2}{3}(39) \\ 26 &= 26 \quad \leftarrow \text{It checks!} \end{aligned}$$



### Example 6

Solve:

$$\begin{aligned}x - (2x + 3) &= 4 \\x - 1(2x + 3) &= 4 &< \text{ use the } \mathbf{multiplicative \text{ property of } -1} \\x - 2x - 3 &= 4 &< \text{ use the } \mathbf{multiplicative \text{ identity of } 1} \\&&\text{ and use the distributive property} \\-1x - 3 &= 4 &< \text{ combine like terms} \\-1x - 3 + 3 &= 4 + 3 &< \text{ undo subtraction} \\\frac{-1x}{-1} &= \frac{7}{-1} &< \text{ undo multiplication} \\x &= -7\end{aligned}$$

Examine the solution steps above. See the use of the *multiplicative property of -1* in front of the parentheses on line two.

$$\begin{aligned}\text{line 1: } x - (2x + 3) &= 4 \\ \text{line 2: } x - 1(2x + 3) &= 4\end{aligned}$$

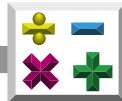
Also notice the use of *multiplicative identity* on line three.

$$\text{line 3: } 1x - 2x - 3 = 4$$

The simple variable  $x$  was multiplied by 1 ( $1 \cdot x$ ) to equal  $1x$ . The  $1x$  helped to clarify the number of variables when combining like terms on line four.

**Check** solution in the original equation:

$$\begin{aligned}x - (2x + 3) &= 4 \\-7 - (2 \cdot -7 + 3) &= 4 \\-7 - (-11) &= 4 \\4 &= 4 &< \text{ It checks!}\end{aligned}$$



## Practice

**Solve and check each equation.** Use the examples on pages 325-330 for reference. Show **essential steps**.

**Hint:** Find a step that looks similar to the problem you need help with and follow from that point.



**Remember:** To check your work, replace the variable in the original problem with the answer you found.

1.  $3x - 7 = 17$

2.  $4x + 20 = x - 4$

3.  $\frac{x}{6} = 1.5$

4.  $\frac{2x}{5} = 3.2$



5.  $5(x - 4) = 20$

6.  $5(4x - 7) = 0$

7.  $8x - 2x = 42$

8.  $5x - 3 = 2x + 18$

9.  $-2x + 4 = -4x - 10$



## Practice

**Solve and check each equation.** Use the examples on pages 325-330 for reference. Show **essential steps**.

1.  $2(3x - 4) + 6 = 10$

2.  $3(x - 7) - x = -9$

3.  $\frac{2}{3}x = 1$

**Hint:**  $\frac{2}{3}x = \frac{2x}{3}$ . Rewrite 1 as  $\frac{1}{1}$  and cross multiply.

4.  $\frac{-1}{2}x - \frac{3}{4} = 4$



5.  $-3x = \frac{-33}{8}$

6.  $\frac{-2}{x} = 8$

7.  $-3x - \frac{3}{2} = \frac{11}{2}$



## Practice

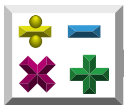
Solve *and* check *each* equation.

1.  $-87 = 9 - 8x$

2.  $4k + 3 = 3k + 1$

3.  $5a + 9 = 64$

4.  $\frac{b}{3} + 5 = -2$

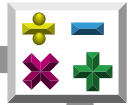


5.  $4x = -(9 - x)$

6.  $\frac{5}{x} = -10$

7.  $3x - 1 = -x + 19$





## Practice

Solve *and* check *each* equation. Reduce *fractions* to simplest form.

1.  $5x - 3 = 2x + 18$

2.  $6x - (4x - 12) = 3x + 5$

3.  $\frac{x}{6} = \frac{-24}{5}$

4.  $4(x - 2) = -3(x + 5)$

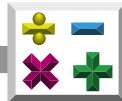


5.  $5\left(\frac{1}{3}x - 2\right) = 4$

6.  $\frac{4}{x} + \frac{3}{2} = \frac{5}{8}$

7.  $\frac{2}{9}x = \frac{1}{5}$

8.  $-\frac{1}{2} + \frac{8x}{5} = -\frac{7}{8}$



## **Lesson Five Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

#### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.
- MA.912.A.3.4  
Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.



## Standard 4: Polynomials

- MA.912.A.4.1  
Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2  
Add, subtract, and multiply polynomials.
- MA.912.A.4.3  
Factor polynomial expressions.
- MA.912.A.4.4  
Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

## Solving Inequalities

**Inequalities** are mathematical sentences that state two expressions are not equal. Instead of using the equal symbol ( $=$ ), we use the following with *inequalities*.

- greater than  $>$
- less than  $<$
- greater than or equal to  $\geq$
- less than or equal to  $\leq$
- not equal to  $\neq$



**Remember:** The “is greater than” ( $>$ ) or “is less than” ( $<$ ) symbols always *point to the lesser number*.

For example:

$$\begin{array}{ccc} 5 & > & 3 \\ 3 & < & 5 \end{array}$$



We have been solving *equations* in this unit. When we solve inequalities, the procedures are the same except for one important difference.

**When we multiply or divide both sides of an inequality by the same *negative number*, we reverse the direction of the inequality symbol.**

### Example

Solve by *dividing* by a **negative number** and *reversing* the inequality sign.

$$\begin{array}{l} -3x < 6 \\ \frac{-3x}{-3} > \frac{6}{-3} \quad \leftarrow \text{divide each side by } -3 \text{ and} \\ x > -2 \quad \quad \quad \text{reverse the inequality symbol} \end{array}$$

To check this solution, pick any number *greater than* -2 and substitute your choice into the original inequality. For instance, -1, 0, or 3, or 3,000 could be substituted into the original problem.

Check with different solutions of numbers *greater than* -2:

substitute -1

$$\begin{array}{l} -3x < 6 \\ -3(-1) < 6 \\ 3 < 6 \quad \leftarrow \text{It checks!} \end{array}$$

substitute 3

$$\begin{array}{l} -3x < 6 \\ -3(3) < 6 \\ -9 < 6 \quad \leftarrow \text{It checks!} \end{array}$$

substitute 0

$$\begin{array}{l} -3x < 6 \\ -3(0) < 6 \\ 0 < 6 \quad \leftarrow \text{It checks!} \end{array}$$

substitute 3,000

$$\begin{array}{l} -3x < 6 \\ -3(3,000) < 6 \\ -9,000 < 6 \quad \leftarrow \text{It checks!} \end{array}$$

Notice that -1, 0, 3, and 3,000 are all *greater than* -2 and each one *checks* as a solution.



Study the following examples.

### Example 1

Solve by *multiplying* by a *negative number* and *reversing* the inequality sign.

$$\begin{aligned} -\frac{1}{3}y &\geq 4 \\ (-3) \cdot -\frac{1}{3}y &\leq 4(-3) \quad \leftarrow \text{multiply each side by } -3 \text{ and} \\ &\quad \text{reverse the inequality symbol} \\ y &\leq -12 \end{aligned}$$

### Example 2

Solve by first *adding*, then *dividing* by a *negative number*, and *reversing* the inequality sign.

$$\begin{aligned} -3a - 4 &> 2 \\ -3a - 4 + 4 &> 2 + 4 \quad \leftarrow \text{add } 4 \text{ to each side} \\ -3a &> 6 \\ \frac{-3a}{-3} &< \frac{6}{-3} \quad \leftarrow \text{divide each side by } -3 \text{ and} \\ &\quad \text{reverse the inequality symbol} \\ a &< -2 \end{aligned}$$

### Example 3

Solve by first *subtracting*, then *multiplying* by a *negative number*, and *reversing* the inequality sign.

$$\begin{aligned} \frac{y}{-2} + 5 &\leq 0 \\ \frac{y}{-2} + 5 - 5 &\leq 0 - 5 \quad \leftarrow \text{subtract } 5 \text{ from each side} \\ \frac{y}{-2} &\leq -5 \\ \frac{(-2)y}{-2} &\geq (-5)(-2) \quad \leftarrow \text{multiply each side by } -2 \text{ and} \\ &\quad \text{reverse the inequality symbol} \\ y &\geq 10 \end{aligned}$$



### Example 4

Solve by first subtracting, then *multiplying* by a **positive number**. Do **not** *reverse* the inequality sign.

$$\begin{aligned}\frac{n}{2} + 5 &\leq 2 \\ \frac{n}{2} + 5 - 5 &\leq 2 - 5 \quad \leftarrow \text{subtract 5 from each side} \\ \frac{n}{2} &\leq -3 \\ \frac{(2)n}{2} &\leq -3(2) \quad \leftarrow \text{multiply each side by 2, but} \\ n &\leq -6 \quad \text{do not reverse the inequality symbol because} \\ &\quad \text{we multiplied by a positive number}\end{aligned}$$

**When multiplying or dividing both sides of an inequality by the same positive number, do not reverse the inequality symbol—leave it alone.**

### Example 5

Solve by first adding, then *dividing* by a positive number. Do **not** *reverse* the inequality sign.

$$\begin{aligned}7x - 3 &> -24 \\ 7x - 3 + 3 &> -24 + 3 \quad \leftarrow \text{add 3 to each side} \\ 7x &> -21 \quad \leftarrow \text{divide each side by 7, but} \\ \frac{7x}{7} &> \frac{-21}{7} \quad \text{do not reverse the inequality symbol because} \\ x &> -3 \quad \text{we divided by a positive number}\end{aligned}$$



## Practice

Solve each **inequality** on the following page. Use the examples below and pages 340-343 for reference. Show **essential steps**.



**Remember: Reverse the inequality symbol every time we multiply or divide both sides of the inequality by a negative number.** See the example below.

Example:

$$\begin{array}{rcl} 7 - 3x > 13 & & \\ 7(-7) - 3x > 13(-7) & \longleftarrow & \text{subtract 7 from both sides} \\ -3x > 6 & & \\ \frac{-3x}{-3} > \frac{6}{-3} & \longleftarrow & \text{divide both sides by -3 and} \\ x < -2 & & \text{reverse the inequality} \\ & & \text{symbol} \end{array}$$

Notice in the example above that we first subtracted 7 from both sides of the sentence. Then we solved for  $x$ , we divided both sides by  $-3$ , and the  $>$  symbol became a  $<$  symbol.

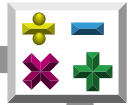
Check your answer by choosing a number that fits your answer. Replace the variable in the original sentence with the chosen number. Check to see if it makes a *true* statement.

In the example above, choose a number that makes  $x < -2$  a *true* statement. For example, let's try  $-3$ .

Now put  $-3$  in place of the variable in the original problem and see what happens.

$$\begin{array}{rcl} 7 - 3x > 13 & \longleftarrow & \text{original sentence} \\ 7 - 3(-3) > 13 & \longleftarrow & \text{replace } x \text{ with } -3 \\ 7 - (-9) > 13 & \swarrow & \\ 7 + 9 > 13 & & \\ 16 > 13 & \longleftarrow & \text{This is a true statement, so} \\ & & \text{the answer } (x < -2) \text{ is} \\ & & \text{correct.} \end{array}$$





See **directions** and **examples** on previous page.

1.  $6x - 7 > 17$

2.  $13x + 20 < x - 4$

3.  $\frac{x}{5} \geq 1.5$

4.  $\frac{2x}{5} > 4.8$



5.  $5(x - 4) < 20$

6.  $3(4x - 7) \geq 15$

7.  $3(x - 7) - x > -9$

8.  $\frac{-1}{2}x - \frac{3}{4} \leq 6$



9.  $2x - 9 < -21$

10.  $\frac{-12}{x} < 8$

11.  $4(x - 7) - x > -7$

12.  $\frac{2}{3}x > 10$



## Practice

Solve each **inequality**. Show **essential steps**.

1.  $5x - 3 \leq 12$

2.  $2a + 7 \geq 5a - 5$

3.  $\frac{2x}{5} > 2.4$

4.  $5(x - 5) < 20$



5.  $-2(x + 6) > 14$

6.  $8x - 12x > 48$

7.  $5x - 3 \geq 2x + 18$

8.  $-2x + 4 < -4x - 12$



9.  $2(3x - 4) + 6 \leq 16$

10.  $-5x - \frac{3}{2} \geq \frac{11}{2}$

11.  $-3x > \frac{-33}{7}$

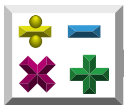
12.  $\frac{2}{3}x > 11$



## Practice

Write **True** if the statement is correct. Write **False** if the statement is not correct.

- \_\_\_\_\_ 1. An equation is a mathematical sentence that uses an equal sign to show that two quantities are equal.
- \_\_\_\_\_ 2. A product is the result of dividing two numbers.
- \_\_\_\_\_ 3. A quotient is the result of multiplying two numbers.
- \_\_\_\_\_ 4. An expression is a collection of numbers, symbols, and/or operation signs that stand for a number.
- \_\_\_\_\_ 5. To simplify an expression, perform as many indicated operations as possible.
- \_\_\_\_\_ 6. A common multiple is a number that is a multiple of two or more numbers.
- \_\_\_\_\_ 7. The smallest of the common multiples of two or more numbers is called the least common multiple (LCM).
- \_\_\_\_\_ 8. A number that is the result of subtraction is called the sum.
- \_\_\_\_\_ 9. A number that is the result of adding numbers together is called the difference.
- \_\_\_\_\_ 10. When solving an inequality, every time you add or subtract both sides of the inequality by a negative number, you will have to reverse the inequality symbol.



## Unit Review

*Simplify each expression.*

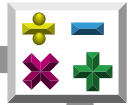
1.  $\frac{5x - 10}{x - 2}$

2.  $\frac{6x - 9y}{3}$

3.  $\frac{12a^2b^5 + 18a^3b^4 - 24a^4b^3}{-6a^2b^3}$

4.  $\frac{12x - 6}{10x - 5}$





5.  $\frac{x^2 - 4}{x^2 + x - 6}$

6.  $\frac{x^2 + 3x - 10}{x + 5}$

Write each **sum** or **difference** as a single fraction in **lowest terms**.

7.  $\frac{3a}{8} + \frac{a}{8} - \frac{6}{8}$

8.  $\frac{x + 3}{6} - \frac{x - 3}{6}$



9.  $\frac{x-2}{4} + \frac{x+2}{4}$

10.  $\frac{2}{3x+1} + \frac{5}{x-3}$

11.  $\frac{3}{a^2-9} - \frac{6}{a^2+a-6}$



Write each **product** or **quotient** as a single fraction in **simplest terms**.

12.  $\frac{21x^2y^3}{3xy} \cdot \frac{-9}{7xy^2}$

13.  $\frac{a}{a+4} \cdot \frac{3a+12}{6}$

14.  $\frac{-12}{x^2-x} \div \frac{4x-2}{x^2-1}$

15.  $\frac{x^2-x-20}{x^2+7x+12} \cdot \frac{x^2+9x+18}{x^2-7x+10}$



*Solve each equation.*

16.  $3(4x - 2) = 30$

17.  $7x - 2(x + 3) = 19$

18.  $5 - \frac{x}{2} = 12$

19.  $28 + 6x = 23 + 8x$



*Solve each inequality.*

20.  $5x + 4 \geq 20$

21.  $16 - 4x < 20$

22.  $5(x + 2) > 4x + 7$



## **Unit 5: How Radical Are You?**

This unit focuses on simplifying radical expressions and performing operations involving radicals.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

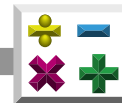
#### **Algebra Body of Knowledge**

##### **Standard 6: Radical Expressions and Equations**

- MA.912.A.6.1  
Simplify radical expressions.
- MA.912.A.6.2  
Add, subtract, multiply and divide radical expressions (square roots and higher).







## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

**coefficient** .....the number that multiplies the variable(s) in an algebraic expression

*Example:* In  $4xy$ , the coefficient of  $xy$  is 4.

If no number is specified, the coefficient is 1.

**conjugate** .....if  $x = a + b$ , then  $a - b$  is the conjugate of  $x$

*Example:* The expressions  $(a + \sqrt{b})$  and  $(a - \sqrt{b})$  are conjugates of each other.

**decimal number** .....any number written with a decimal point in the number

*Examples:* A decimal number falls between two whole numbers, such as 1.5, which falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called *decimal fractions*, such as five-tenths, or  $\frac{5}{10}$ , which is written 0.5.

**denominator** .....the bottom number of a fraction, indicating the number of equal parts a whole was divided into

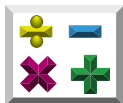
*Example:* In the fraction  $\frac{2}{3}$  the denominator is 3, meaning the whole was divided into 3 equal parts.

**digit** .....any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9

**distributive property** .....the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products

*Examples:*  $x(a + b) = ax + bx$

$$5(10 + 8) = 5 \cdot 10 + 5 \cdot 8$$



**expression** ..... a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables

*Examples:*  $4r^2$ ;  $3x + 2y$ ;  $\sqrt{25}$

An expression does *not* contain equal (=) or inequality (<, >, ≤, ≥, or ≠) signs.

**factor** ..... a number or expression that divides evenly into another number; one of the numbers multiplied to get a product

*Examples:* 1, 2, 4, 5, 10, and 20 are factors of 20 and  $(x + 1)$  is one of the factors of  $(x^2 - 1)$ .

**FOIL method** ..... a pattern used to multiply two binomials; multiply the first, outside, inside, and last terms:

**F** First terms

**O** Outside terms

**I** Inside terms

**L** Last terms.

*Example:*

$$\begin{array}{c}
 \begin{array}{ccc}
 & 2 \text{ Outside} & \\
 \curvearrowright & & \curvearrowleft \\
 1 \text{ First} & & \\
 \curvearrowright & & \curvearrowleft \\
 (a + b)(x - y) = ax - ay + bx - by \\
 \curvearrowleft & & \curvearrowright \\
 3 \text{ Inside} & & \\
 \curvearrowleft & & \curvearrowright \\
 4 \text{ Last} & & 
 \end{array}
 \end{array}
 \begin{array}{cccc}
 \mathbf{F} & \mathbf{O} & \mathbf{I} & \mathbf{L} \\
 ax & -ay & +bx & -by
 \end{array}$$

**fraction** ..... any part of a whole

*Example:* One-half written in fractional form is  $\frac{1}{2}$ .

**irrational number** ..... a real number that cannot be expressed as a ratio of two integers

*Example:*  $\sqrt{2}$

**like terms** .....terms that have the same variables and the same corresponding exponents  
*Example:* In  $5x^2 + 3x^2 + 6$ , the like terms are  $5x^2$  and  $3x^2$ .

**numerator** .....the top number of a fraction, indicating the number of equal parts being considered  
*Example:* In the fraction  $\frac{2}{3}$ , the numerator is 2.

**perfect square** .....a number whose square root is a whole number  
*Example:* 25 is a perfect square because  $5 \times 5 = 25$ .

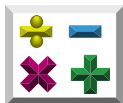
**product** .....the result of multiplying numbers together  
*Example:* In  $6 \times 8 = 48$ , the product is 48.

**radical** .....an expression that has a root (square root, cube root, etc.)  
*Example:*  $\sqrt{25}$  is a radical  
 Any root can be specified by an index number,  $b$ , in the form  $\sqrt[b]{a}$  (e.g.,  $\sqrt[3]{8}$ ).  
 A radical without an index number is understood to be a square root.

The diagram shows the expression  $\sqrt[3]{8} = 2$ . Arrows point from labels to parts of the expression:  
 - "root to be taken (index)" points to the 3.  
 - "radical sign" points to the checkmark symbol.  
 - "radicand" points to the 8.  
 - "root" points to the 2.  
 The word "radical" is written below the entire expression.

**radical expression** .....a numerical expression containing a radical sign  
*Examples:*  $\sqrt{25}$        $2\sqrt{25}$

**radical sign ( $\sqrt{\phantom{x}}$ )** .....the symbol ( $\sqrt{\phantom{x}}$ ) used before a number to show that the number is a *radicand*



## rationalizing

**the denominator** .....a method used to remove or eliminate radicals from the denominator of a fraction

**rational number** .....a number that can be expressed as a ratio  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$

**simplest radical form** .....an expression under the radical sign that contains no perfect squares greater than 1, contains no fractions, and is not in the denominator of a fraction

*Example:*  $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$

**simplify an expression** .....to perform as many of the indicated operations as possible

**square root** .....a positive real number that can be multiplied by itself to produce a given number

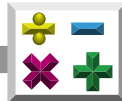
*Example:* The square root of 144 is 12 or  $\sqrt{144} = 12$ .

**term** .....a number, variable, product, or quotient in an expression

*Example:* In the expression  $4x^2 + 3x + x$ , the terms are  $4x^2$ ,  $3x$ , and  $x$ .

**variable** .....any symbol, usually a letter, which could represent a number

**whole numbers** .....the numbers in the set  $\{0, 1, 2, 3, 4, \dots\}$



## Unit 5: How Radical Are You?

### Introduction

We will see that radical expressions can be rewritten to conform to the mathematical definitions of simplest terms. We will then be able to perform the operations of addition, subtraction, multiplication and division on these reformatted expressions. We will also explore the effects of multiplying a radical expression by its conjugate.

### Lesson One Purpose

#### Reading Process Strand

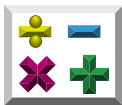
##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### Algebra Body of Knowledge

##### Standard 6: Radical Expressions and Equations

- MA.912.A.6.1  
Simplify radical expressions.



## Simplifying Radical Expressions

A **radical expression** is any mathematical **expression** that contains a **square root** symbol. Look at the following examples:

$$\sqrt{5} \qquad \frac{\sqrt{6}}{3} \qquad \frac{3}{\sqrt{6}} \qquad \frac{7}{5 + \sqrt{2}} \qquad \sqrt{36}$$

Certain numbers can be reformatted to make them easier to work with. To do so, mathematicians have rules that make working with numbers uniform. If we all play by the same rules, we should all have the same outcome.

With this in mind, here are the two basic rules for working with *square roots*.

1. Never leave a **perfect square factor** under a **radical sign** ( $\sqrt{\quad}$ ).  
Why? Because if you do, the radical expression is *not* simplified.
2. Never leave a *radical sign* in a **denominator**.  
Why? Because if you do, the radical expression is *not* simplified.

**Important!** Do **not** use your calculator with the *square roots*. It will change the numbers to **decimal** approximations. We are looking for exact answers.



Let's explore each of the rules...one at a time.

### Rule One

First, let's review the idea of *perfect squares*. Perfect squares happen whenever you multiply a number times itself. In the following examples,

$$3 \times 3 = 9$$

$$7 \times 7 = 49$$

$$9 \times 9 = 81$$

9, 49, and 81 are all perfect squares.



It will be helpful to learn the chart below. You will be asked to use these numbers many times in this unit and in real-world applications. The chart shows the perfect squares underneath the radical sign, then gives the square root of each perfect square.

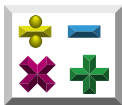
**Perfect Squares:**  
**Square Root = Whole Number**

$\sqrt{1}$	=	1
$\sqrt{4}$	=	2
$\sqrt{9}$	=	3
$\sqrt{16}$	=	4
$\sqrt{25}$	=	5
$\sqrt{36}$	=	6
$\sqrt{49}$	=	7
$\sqrt{64}$	=	8
$\sqrt{81}$	=	9
$\sqrt{100}$	=	10
$\sqrt{121}$	=	11
$\sqrt{144}$	=	12
$\sqrt{169}$	=	13
$\sqrt{196}$	=	14
$\sqrt{225}$	=	15
$\sqrt{256}$	=	16
$\sqrt{289}$	=	17
$\sqrt{324}$	=	18
$\sqrt{361}$	=	19
$\sqrt{400}$	=	20

Any time you see a perfect square under a square root symbol, simplify it by writing it as the square root.

Sometimes, perfect squares are hidden in an *expression* and we have to search for them. At first glance,  $\sqrt{45}$  looks as if it is in **simplest radical form**. However, when we realize that 45 has a *factor* that is a perfect square, we can rewrite it as

$$\sqrt{45} = \sqrt{9} \cdot \sqrt{5}.$$



From the information in the chart, we know that 9 is a perfect square and that

$$\sqrt{9} = 3. \text{ Therefore}$$

$$\sqrt{45} = 3 \cdot \sqrt{5} \text{ or } 3\sqrt{5}.$$

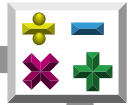
Let's look at some examples.

$$\begin{aligned}\sqrt{18} &= \sqrt{9} \cdot \sqrt{2} \\ &= 3 \cdot \sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \cdot \sqrt{5} \\ &= 2 \cdot \sqrt{5} \\ &= 2\sqrt{5}\end{aligned}$$

Now you try some in the following practices. Study the chart of perfect squares on page 367 before you start the practices.





## Practice

Simplify *each* radical expression.



**Remember:** Never leave a perfect square factor under a radical sign.

1.  $\sqrt{50}$

6.  $\sqrt{32}$

2.  $\sqrt{27}$

7.  $\sqrt{12}$

3.  $\sqrt{125}$

8.  $\sqrt{45}$

4.  $\sqrt{64}$

9.  $\sqrt{300}$

5.  $\sqrt{13}$

10.  $\sqrt{8}$



## Practice

Simplify *each* radical expression.

1.  $\sqrt{48}$

6.  $-\sqrt{200}$

2.  $2\sqrt{40}$

7.  $-\sqrt{250}$

3.  $\sqrt{60}$

8.  $\sqrt{108}$

4.  $\sqrt{242}$

9.  $\sqrt{405}$

5.  $\sqrt{28}$

10.  $5\sqrt{90}$



## Rule Two

Now it's time to work on that second rule: never leave a square root in the *denominator*. Because if a square root is left in the denominator of a radical expression, the radical expression is *not* simplified.

If a **fraction** has a denominator that is a perfect square root, just rewrite the *fraction* using that square root. Let's look at examples.

$$\frac{2}{\sqrt{36}} = \frac{2}{6} = \frac{1}{3} \qquad \frac{4}{\sqrt{81}} = \frac{4}{9}$$

Many times, however, that denominator will *not* be a perfect square root. In those cases, we have to reformat the denominator so that it is a perfect square root. This is called **rationalizing the denominator** or the bottom number of the fraction. To do this, we make it into a **rational number** by using a method to eliminate **radicals** from the denominator of a fraction. Remember, we aren't concerned about what may happen to the format of the **numerator**, just the denominator.

To reformat an **irrational** denominator (one with a square root in it), we find a number to multiply it by that will produce a perfect square root.

Follow the explanation of this example carefully.

$$\frac{2}{\sqrt{7}}$$

← Yikes! This denominator is irrational!  
I need to *rationalize* it.

$$\frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

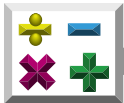
← Look what happens if I multiply the denominator by itself. (Since,  $\frac{\sqrt{7}}{\sqrt{7}} = 1$ , I have *not* changed the value of the original fraction.)

$$\frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}}$$

← Because I remember the perfect square roots from the chart on page 240, I see that  $\sqrt{49}$  is a perfect square root...and therefore rational!

$$\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}} = \frac{2\sqrt{7}}{7}$$

← This may *not* look like a simpler expression than I started with, but *it does conform to the second rule*.

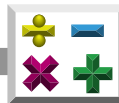


Follow along with this example.

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = \frac{2\sqrt{3}}{1} = 2\sqrt{3}$$

In the above example, notice that we reduced the “real 6” and the “real 3,” but not with the square root of 3. Do *not* mix a rational number with an irrational number, sometimes referred to as a *non-rational* number when you are reducing...they are *not like terms*!

It's time for you to practice.



## Practice

Simplify *each* radical expression.



**Remember:** Never leave a square root in the denominator.

*Example:*  $\frac{6}{\sqrt{5}} = \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{\sqrt{25}} = \frac{6\sqrt{5}}{5}$

Show all your steps.

1.  $\frac{7}{\sqrt{2}}$

6.  $\frac{4}{\sqrt{3}}$

2.  $\frac{5}{\sqrt{6}}$

7.  $\frac{7}{\sqrt{10}}$

3.  $\frac{1}{\sqrt{3}}$

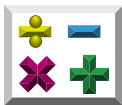
8.  $\frac{3}{\sqrt{7}}$

4.  $\frac{3}{\sqrt{5}}$

9.  $\frac{4}{\sqrt{11}}$

5.  $\frac{5}{\sqrt{18}}$

10.  $\frac{\sqrt{2}}{\sqrt{15}}$



## Practice

Simplify *each* radical expression.

*Example:*  $\frac{10}{\sqrt{6}} = \frac{10}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{10\sqrt{6}}{\sqrt{36}} = \frac{10\sqrt{6}}{6} = \frac{5\sqrt{6}}{3}$

1.  $\frac{9}{\sqrt{6}}$

6.  $\frac{3}{\sqrt{18}}$

2.  $\frac{-2}{\sqrt{8}}$

7.  $\sqrt{\frac{1}{3}}$

3.  $\frac{2}{\sqrt{7}}$

8.  $\frac{-5}{\sqrt{20}}$

4.  $\frac{5}{\sqrt{5}}$

9.  $\frac{3\sqrt{5}}{\sqrt{6}}$

5.  $\sqrt{\frac{2}{3}}$

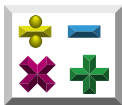
10.  $\frac{7\sqrt{3}}{\sqrt{5}}$



## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |       |   |                          |
|-------|---|--------------------------|
| _____ | 1. a number whose square root is a whole number   | A. factor                |
| _____ | 2. an expression under the radical sign that contains no perfect squares greater than 1, contains no fractions, and is not in the denominator of a fraction | B. irrational number     |
| _____ | 3. the symbol ( $\sqrt{\quad}$ ) used before a number to show that the number is a <i>radicand</i>  | C. like terms            |
| _____ | 4. terms that have the same variables and the same corresponding exponents  | D. perfect square        |
| _____ | 5. a real number that cannot be expressed as a ratio of two integers  | E. radical expression    |
| _____ | 6. a number or expression that divides evenly into another number   | F. radical sign          |
| _____ | 7. a numerical expression containing a radical sign   | G. rational number       |
| _____ | 8. a number that can be expressed as a ratio $\frac{a}{b}$ , where $a$ and $b$ are integers and $b \neq 0$  | H. simplest radical form |
| _____ | 9. a positive real number that can be multiplied by itself to produce a given number  | I. square root           |



## Lesson Two Purpose

### Algebra Body of Knowledge

#### Standard 6: Radical Expressions and Equations

- MA.912.A.6.2  
Add, subtract, multiply and divide radical expressions (square roots and higher).

### Add and Subtract Radical Expressions

We can add or subtract radical expressions only when those radical expressions match. For instance,

$$5\sqrt{2} + 6\sqrt{2} = 11\sqrt{2}.$$

Notice that we did not change the  $\sqrt{2}$ 's. We simply added the **coefficients** because they had matching radical parts.



**Remember:** *Coefficients* are any factor in a **term**. Usually, but not always, a coefficient is a number instead of a **variable** or a *radical*.

The same is true when we subtract radical expressions.

$$5\sqrt{7} - 3\sqrt{7} = 2\sqrt{7}$$





At first glance, it may sometimes appear that there are no matching numbers under the radical sign. But, if we **simplify** the expressions, we often find radical expressions that we can add or subtract.

Look at this example.

$$3\sqrt{8} + 5\sqrt{2} - 4\sqrt{32}$$

Notice that  $\sqrt{8}$  and  $\sqrt{32}$  each have perfect square factors and can be simplified. Follow the simplification process step by step and see what happens.

$$3\sqrt{8} + 5\sqrt{2} - 4\sqrt{32} =$$

$$3\sqrt{4}\sqrt{2} + 5\sqrt{2} - 4\sqrt{16}\sqrt{2} =$$

↖ We *found* the perfect square factors of  $\sqrt{8}$  and  $\sqrt{32}$  and rewrote the problem.

$$3 \cdot 2\sqrt{2} + 5\sqrt{2} - 4 \cdot 4\sqrt{2} =$$

← Next, we *simplified* the perfect square roots.

$$6\sqrt{2} + 5\sqrt{2} - 16\sqrt{2} =$$

↖ We *multiplied* the new factors for each coefficient.

$$-5\sqrt{2}$$

← Finally, we *add and subtract* matching radical expressions, in order, from *left to right*.



## When Radical Expressions Don't Match or Are Not in Radical Form

What happens when radical expressions don't match, or there is a number that is not in radical form? Just follow the steps on the previous pages and leave your answer, with appropriate terms in descending order. Watch this!

$$\begin{aligned}\sqrt{75} + \sqrt{27} - \sqrt{16} + \sqrt{80} &= \\ \sqrt{25}\sqrt{3} + \sqrt{9}\sqrt{3} - 4 + \sqrt{16}\sqrt{5} &= \\ 5\sqrt{3} + 3\sqrt{3} - 4 + 4\sqrt{5} &= \\ 8\sqrt{3} - 4 + 4\sqrt{5} &= \\ 8\sqrt{3} + 4\sqrt{5} - 4 &\longleftarrow \text{rewritten in descending order}\end{aligned}$$



## Practice

**Simplify** each of the following. Refer to pages 376-378 as needed.

1.  $4\sqrt{7} + 10\sqrt{7}$

2.  $-5\sqrt{2} + 7\sqrt{2} - 4\sqrt{2}$

3.  $3\sqrt{7} + 5 - \sqrt{7}$

4.  $2\sqrt{27} - 4\sqrt{12}$



5.  $\sqrt{2} + \sqrt{18} - \sqrt{16}$

6.  $\sqrt{3} + 5\sqrt{3} - \sqrt{27}$

7.  $\sqrt{50} + \sqrt{18}$

8.  $\sqrt{27} + \sqrt{12} - \sqrt{48}$



## Practice

**Simplify** each of the following. Refer to pages 376-378 as needed.

1.  $-3\sqrt{5} + 4\sqrt{2} - \sqrt{5} + \sqrt{8}$

2.  $\sqrt{81} + \sqrt{24} - \sqrt{9} + \sqrt{54}$

3.  $\sqrt{50} - \sqrt{45} + \sqrt{32} - \sqrt{80}$

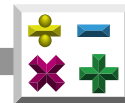
4.  $5\sqrt{7} + 2\sqrt{3} - 4\sqrt{7} - \sqrt{27}$



5.  $\sqrt{200} - \sqrt{8} + 3\sqrt{72} - 6$

6.  $12 - 3\sqrt{5} + 2\sqrt{144} - \sqrt{20}$

7.  $\sqrt{18} + \sqrt{48} - \sqrt{32} - \sqrt{27}$



## Lesson Three Purpose

### Algebra Body of Knowledge

#### Standard 6: Radical Expressions and Equations

- MA.912.A.6.2  
Add, subtract, multiply and divide radical expressions (square roots and higher).

### Multiply and Divide Radical Expressions

Radical expressions *don't* have to match when we multiply or divide them. The following examples show that we simply multiply or divide the **digits** under the radical signs and then simplify our results, if possible.

#### Example 1

$$\sqrt{5} \times \sqrt{6} = \sqrt{30}$$

#### Example 2

$$\sqrt{8} \times \sqrt{3} = \sqrt{24} = \sqrt{4} \sqrt{6} = 2\sqrt{6}$$

#### Example 3

$$\sqrt{18} \times \sqrt{2} = \sqrt{36} = 6$$

#### Example 4

$$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4} = 2$$

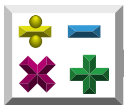
#### Example 5

$$\frac{\sqrt{20}}{\sqrt{10}} = \sqrt{2}$$

#### Example 6

$$\frac{\sqrt{8}}{\sqrt{24}} = \frac{\sqrt{1}}{\sqrt{3}} \quad (\text{we must simplify this}) \rightarrow \frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$$

After studying the examples above, try the following practice.



## Practice

**Simplify** each of the following. Refer to the **examples** on the previous page as needed.

1.  $\sqrt{5} \cdot \sqrt{10}$

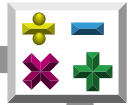
2.  $\sqrt{2} \cdot \sqrt{50}$

3.  $\sqrt{75} \cdot \sqrt{3}$

4.  $\sqrt{6} \cdot \sqrt{10}$

5.  $\frac{\sqrt{30}}{\sqrt{2}}$





6.  $\frac{\sqrt{8}}{\sqrt{32}}$

7.  $\frac{\sqrt{6}}{\sqrt{10}}$

8.  $\frac{\sqrt{75}}{\sqrt{3}}$

9.  $\frac{\sqrt{72}}{\sqrt{18}}$

10.  $\frac{\sqrt{5}}{\sqrt{10}}$



## Working with a Coefficient for the Radical

What happens when there is a coefficient for the *radical*? It is important to multiply or divide the radical numbers together separately from the coefficients. Then simplify each answer. Look at the following examples.

### Example 1

$$\begin{array}{l} \text{multiply coefficients} \\ 3 \cdot 5 = 15 \\ 3\sqrt{7} \cdot 5\sqrt{2} = 15\sqrt{14} \\ \text{multiply radicands} \\ \sqrt{7} \cdot \sqrt{2} = \sqrt{14} \end{array}$$

### Example 2

$$6\sqrt{3} \cdot \sqrt{3} = 6\sqrt{9} = 6 \cdot 3 = 18$$



**Remember:** If there is *no* written coefficient, then it is understood to be a 1.

### Example 3

$$\frac{2\sqrt{14}}{6\sqrt{7}} = \frac{1\sqrt{2}}{3} = \frac{\sqrt{2}}{3}$$

### Example 4

$$\frac{12\sqrt{5}}{6\sqrt{10}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

### Example 5

$$\frac{\sqrt{6} - \sqrt{12}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{3}} - \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{2} - \sqrt{4} = \sqrt{2} - 2$$

Now it's time to practice on the following page.



## Practice

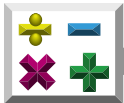
**Simplify** each of the following. Refer to the **examples** on the previous page as needed.

1.  $5\sqrt{3} \cdot 6\sqrt{5}$

2.  $2\sqrt{5} \cdot 4\sqrt{2}$

3.  $8\sqrt{2} \cdot 5\sqrt{3}$

4.  $2\sqrt{7} \cdot \sqrt{7}$



5.  $\frac{3\sqrt{10}}{6\sqrt{5}}$

6.  $\frac{4\sqrt{6}}{2\sqrt{12}}$

7.  $\frac{9\sqrt{5}}{3\sqrt{10}}$

8.  $2\sqrt{7} \cdot 5\sqrt{7}$

9.  $5\sqrt{6} \cdot 4\sqrt{2}$



## Practice

**Simplify** each of the following. Refer to the **examples** on page 386 as needed.

1.  $\frac{\sqrt{15} - \sqrt{20}}{\sqrt{5}}$

2.  $\frac{\sqrt{8} - \sqrt{12}}{\sqrt{2}}$

3.  $\frac{\sqrt{30} - \sqrt{50}}{\sqrt{10}}$

4.  $\frac{3\sqrt{18}}{\sqrt{3}}$



5.  $\frac{4\sqrt{6}}{6\sqrt{2}}$

6.  $\frac{10\sqrt{8}}{12\sqrt{12}}$

7.  $\frac{\sqrt{75} - \sqrt{50}}{\sqrt{25}}$



## Lesson Four Purpose

### Reading Process Strand

#### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.

### Algebra Body of Knowledge

#### Standard 6: Radical Expressions and Equations

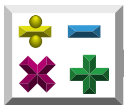
- MA.912.A.6.2  
Add, subtract, multiply and divide radical expressions (square roots and higher).

## Multiple Terms and Conjugates

Sometimes it is necessary to multiply or divide radical expressions with more than one *term*. To multiply radicals with multiple terms by a single term, we use the old reliable **distributive property**. See how the *distributive property* works for these examples.

### Example 1

$$\begin{array}{c} \curvearrowright \quad \curvearrowright \\ 6(\sqrt{5} + \sqrt{3}) = \\ 6\sqrt{5} + 6\sqrt{3} \end{array}$$



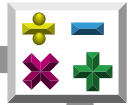
### Example 2

$$\begin{aligned}\sqrt{3}(2\sqrt{5} - 4\sqrt{3}) &= \\ 2\sqrt{15} - 4\sqrt{9} &= \\ 2\sqrt{15} - 4 \cdot 3 &= \\ 2\sqrt{15} - 12\end{aligned}$$

### Example 3

$$\begin{aligned}6\sqrt{3}(2\sqrt{2} + 5\sqrt{6}) &= \\ 12\sqrt{6} + 30\sqrt{18} &= \\ 12\sqrt{6} + 30\sqrt{9\sqrt{2}} &= \\ 12\sqrt{6} + 30 \cdot 3\sqrt{2} &= \\ 12\sqrt{6} + 90\sqrt{2}\end{aligned}$$





## Practice

**Simplify** each of the following. Refer to the **examples** on the previous pages as needed.

1.  $2(\sqrt{6} + \sqrt{5})$

2.  $\sqrt{2}(\sqrt{6} + \sqrt{5})$

3.  $3\sqrt{2}(5\sqrt{3} - 4\sqrt{2})$

4.  $6(3\sqrt{8} - 5\sqrt{2})$

5.  $\sqrt{6}(3\sqrt{8} - 5\sqrt{2})$



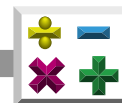
6.  $-2(\sqrt{5} + 7)$

7.  $2\sqrt{3}(\sqrt{7} + \sqrt{10})$

8.  $4(2\sqrt{3} - 5\sqrt{2})$

9.  $4\sqrt{3}(2\sqrt{3} - 5\sqrt{2})$

10.  $8\sqrt{6}(2\sqrt{6} + 5\sqrt{8})$



## The FOIL Method

Another reliable method we can use when multiplying two radical expressions with multiple terms is the **FOIL method**: multiplying the **f**irst, **o**utside, **i**nside, and **l**ast terms. We use that same process in problems like these.

### Example 1

$$(\sqrt{6} - 5)(\sqrt{3} + 4) =$$

$$\begin{array}{cccc} \mathbf{F} & & \mathbf{O} & \mathbf{I} & \mathbf{L} \\ \sqrt{6} \cdot \sqrt{3} + \sqrt{6} \cdot 4 - 5 \cdot \sqrt{3} - 5 \cdot 4 = \end{array}$$

← Multiply the **first** terms, the **outside** terms, the **inside** terms, and then the **last** terms.

$$\sqrt{18} + 4\sqrt{6} - 5\sqrt{3} - 20 =$$

← Carefully write out the **products**.

$$3\sqrt{2} + 4\sqrt{6} - 5\sqrt{3} - 20$$

← Simplify each term and combine like terms (if needed).

### Example 2

$$(\sqrt{3} + \sqrt{2})(\sqrt{7} - \sqrt{11}) =$$

$$\sqrt{3}\sqrt{7} - \sqrt{3}\sqrt{11} + \sqrt{2}\sqrt{7} - \sqrt{2}\sqrt{11} =$$

$$\sqrt{21} - \sqrt{33} + \sqrt{14} - \sqrt{22}$$

← Notice that no term has a perfect square as a factor. Therefore, there is no further simplifying to be done.

Time to try the following practice.



## Practice

**Simplify** each of the following. Refer to the **examples** on page 395 as needed.

1.  $(\sqrt{6} - 2)(\sqrt{5} + 7)$

2.  $(5 - \sqrt{3})(2 + \sqrt{7})$

3.  $(4 + 5\sqrt{2})(2 - \sqrt{2})$

4.  $(2\sqrt{5} - 3)(\sqrt{5} + 6)$



5.  $(4 - 3\sqrt{10})(2 - \sqrt{10})$

6.  $(2\sqrt{7} - 3)(5\sqrt{7} + 1)$

7.  $(\sqrt{5} - 7)(3\sqrt{5} + 7)$

8.  $(\sqrt{10} - \sqrt{6})(\sqrt{7} - \sqrt{13})$

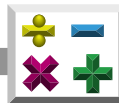


9.  $(3\sqrt{6} + 2\sqrt{3})(\sqrt{5} - 2)$

10.  $(4\sqrt{3} - \sqrt{5})(3\sqrt{3} - \sqrt{5})$

11.  $(3 + \sqrt{10})(3 - \sqrt{10})$

12.  $(6\sqrt{5} + 4)(6\sqrt{5} - 4)$



## Two-Term Radical Expressions

At the beginning of this unit, we learned that there are two rules we must remember when simplifying a radical expression. Rule one requires that we never leave a perfect square factor under a radical sign. Rule two insists that we never leave a radical in the denominator. With that in mind, let's see what to do with two-term radical expressions.

In a problem like  $\frac{2+\sqrt{7}}{5-\sqrt{6}}$ , we see that we must rationalize the denominator (reformat it without using a square root). At first glance, it may seem to you that multiplying that denominator by itself makes the square roots disappear. But when we try that, we realize that new square roots appear as a result of the FOILing.

$$\begin{aligned} & (5 - \sqrt{6})(5 - \sqrt{6}) = \\ & 25 - 5\sqrt{6} - 5\sqrt{6} + \sqrt{6}\sqrt{6} = \\ & 25 - 10\sqrt{6} + 6 \end{aligned}$$

So there must be a better way to rationalize this denominator. Try multiplying  $(5 - \sqrt{6})$  by its **conjugates**  $(5 + \sqrt{6})$ . These numbers are *conjugates* because they match, except for the signs between the terms. Notice that one has a "+" and the other has a "-".

$$\begin{aligned} & (5 - \sqrt{6})(5 + \sqrt{6}) = \\ & 25 + 5\sqrt{6} - 5\sqrt{6} - \sqrt{6}\sqrt{6} = \\ & 25 - \sqrt{36} = \\ & 25 - 6 = \\ & 19 \end{aligned}$$



Remember, we only need to rationalize the denominator. It is acceptable to leave simplified square roots in the numerator. Now, let's take a look at the entire problem.

$$\frac{2 + \sqrt{7}}{5 - \sqrt{6}} \cdot \frac{5 + \sqrt{6}}{5 + \sqrt{6}} =$$

← reformat the fraction by multiplying it by 1

$$\frac{5 + \sqrt{6}}{5 + \sqrt{6}} = 1$$

$$\frac{(2)(5) + 2\sqrt{6} + 5\sqrt{7} + \sqrt{42}}{(5)(5) + 5\sqrt{6} - 5\sqrt{6} - \sqrt{6}\sqrt{6}} =$$

← FOIL the numerator and denominator

$$\frac{10 + 2\sqrt{6} + 5\sqrt{7} + \sqrt{42}}{25 - \sqrt{36}} =$$

← simplify

$$\frac{10 + 2\sqrt{6} + 5\sqrt{7} + \sqrt{42}}{25 - 6} =$$

← simplify again

$$\frac{10 + 2\sqrt{6} + 5\sqrt{7} + \sqrt{42}}{19}$$

← and again, if necessary





Follow along with this one!

$$\frac{3 + \sqrt{2}}{4 + \sqrt{8}} \cdot \frac{4 - \sqrt{8}}{4 - \sqrt{8}} =$$

← reformat the fraction by multiplying it by 1

$$\frac{4 - \sqrt{8}}{4 - \sqrt{8}} = 1$$

$$\frac{(3)(4) - 3\sqrt{8} + 4\sqrt{2} - \sqrt{16}}{(4)(4) - 4\sqrt{8} + 4\sqrt{8} - \sqrt{8}\sqrt{8}} =$$

← FOIL the numerator and denominator

$$\frac{12 - 3\sqrt{4}\sqrt{2} + 4\sqrt{2} - \sqrt{16}}{16 - \sqrt{64}} =$$

← simplify

$$\frac{12 - 3 \cdot 2\sqrt{2} + 4\sqrt{2} - 4}{16 - 8} =$$

← simplify again

$$\frac{12 - 6\sqrt{2} + 4\sqrt{2} - 4}{8} =$$

← and again

$$\frac{8 - 2\sqrt{2}}{8} = \frac{2(4 - \sqrt{2})}{8} =$$

← and again

$$\frac{4 - \sqrt{2}}{4}$$

← and again, if necessary

With more practice, you will be able to mentally combine some of those simplifying steps and finish sooner.

So let's practice on the following page.



## Practice

**Simplify** *each of the following.*

1.  $\frac{\sqrt{5} + 2}{\sqrt{3} - 1}$

2.  $\frac{\sqrt{6} + 5}{3\sqrt{6} - 2}$

3.  $\frac{5\sqrt{2} + 7}{\sqrt{2} - 3}$

4.  $\frac{\sqrt{7} - \sqrt{5}}{\sqrt{5} + \sqrt{7}}$



5.  $\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}+\sqrt{3}}$

6.  $\frac{\sqrt{2}+\sqrt{3}}{2\sqrt{2}-5}$

7.  $\frac{\sqrt{5}+7}{\sqrt{5}-3}$

8.  $\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-3\sqrt{5}}$



## Practice

**Simplify** *each of the following.*

1.  $\frac{4 - \sqrt{7}}{3 + \sqrt{7}}$

2.  $\frac{4\sqrt{2} - \sqrt{3}}{\sqrt{2} + 3\sqrt{3}}$

3.  $\frac{6\sqrt{5} - 2}{\sqrt{5} + \sqrt{2}}$



4.  $\frac{6\sqrt{2} + 5}{1 + \sqrt{5}}$

5.  $\frac{5 + 3\sqrt{2}}{1 - \sqrt{2}}$

6.  $\frac{\sqrt{6} + 2}{2\sqrt{6} + 1}$

7.  $\frac{\sqrt{5} + 2\sqrt{7}}{\sqrt{5} + \sqrt{7}}$



## Practice

Match each **symbol or expression** with the appropriate **description**.

- |   |  |
|---|--|
| _____ 1. 7  | A. coefficient in the expression $5\sqrt{x}$ |
| _____ 2. $\sqrt{\quad}$   | B. conjugate of $x + 4$                      |
| _____ 3. $\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}} = \frac{2\sqrt{7}}{7}$ | C. perfect square of 11                      |
| _____ 4. $x - 4$  | D. radical expression                        |
| _____ 5. 5  | E. radical sign                              |
| _____ 6. $3x\sqrt{6}$   | F. rationalizing the denominator             |
| _____ 7. 121  | G. square root of 49                         |



## Unit Review

**Simplify** *each of the following.*

1.  $\sqrt{75}$

5.  $\frac{5}{\sqrt{8}}$

2.  $-\sqrt{40}$

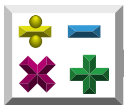
6.  $\frac{1}{\sqrt{7}}$

3.  $5\sqrt{27}$

7.  $\sqrt{\frac{3}{8}}$

4.  $\frac{3}{\sqrt{36}}$

8.  $\frac{5\sqrt{6}}{\sqrt{5}}$



9.  $6\sqrt{3} - 8\sqrt{3}$

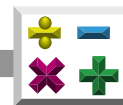
10.  $4\sqrt{8} - 5\sqrt{2} + 3\sqrt{32}$

11.  $\sqrt{75} - \sqrt{45} - \sqrt{80}$

12.  $2\sqrt{50} - 3\sqrt{45} + \sqrt{32} + \sqrt{80}$

13.  $\sqrt{5} + \sqrt{2} + \sqrt{8} + \sqrt{125}$





14.  $\sqrt{2} \times \sqrt{10}$

18.  $\sqrt{\frac{6}{30}}$

15.  $\frac{\sqrt{18}}{\sqrt{3}}$

19.  $\frac{\sqrt{60}}{3\sqrt{5}}$

16.  $\sqrt{6} \cdot \sqrt{2}$

20.  $8\sqrt{3} \cdot 2\sqrt{8}$

17.  $\frac{2\sqrt{6}}{\sqrt{24}}$

21.  $\frac{\sqrt{18} - \sqrt{12}}{\sqrt{6}}$



22.  $(3 + 5\sqrt{6})(3 - 5\sqrt{6})$

23.  $\frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} - \sqrt{6}}$

24.  $\frac{5 + 2\sqrt{3}}{2 + \sqrt{5}}$

25.  $\frac{\sqrt{6} - 1}{2\sqrt{6} + 2}$

## **Unit 6: Extreme Fractions**

This unit will illustrate the difference between shape and size as they relate to the concepts of congruency and similarity.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

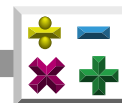
- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Algebra Body of Knowledge**

##### **Standard 5: Radical Expressions and Equations**

- MA.912.A.5.1  
Simplify algebraic ratios.
- MA.912.A.5.4  
Solve algebraic proportions.

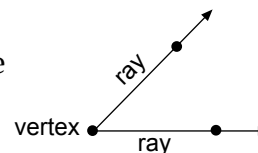




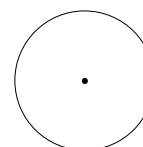
## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

**angle ( $\angle$ )** .....two rays extending from a common endpoint called the vertex; measures of angles are described in degrees ( $^{\circ}$ )



**circle** .....the set of all points in a plane that are all the same distance from a given point called the center



**congruent ( $\cong$ )** .....having exactly the same shape and size

**corresponding** .....in the same location in their respective figures

**corresponding angles and sides** .....the matching angles and sides in similar figures

**cross multiplication** .....a method for solving and checking proportions; a method for finding a missing numerator or denominator in equivalent fractions or ratios by making the cross products equal

*Example:* Solve this proportion by doing the following.

$$\frac{n}{9} = \frac{8}{12}$$

$$\frac{n}{9} \quad \frac{8}{12}$$

$$12 \times n = 9 \times 8$$

$$12n = 72$$

$$n = \frac{72}{12}$$

$$n = 6$$

Solution:

$$\frac{6}{9} = \frac{8}{12}$$



**degree (°)** .....common unit used in measuring angles

**denominator** .....the bottom number of a fraction, indicating the number of equal parts a whole was divided into

*Example:* In the fraction  $\frac{2}{3}$  the denominator is 3, meaning the whole was divided into 3 equal parts.

**distributive property** .....the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products

*Examples:*  $x(a + b) = ax + bx$   
 $5(10 + 8) = 5 \cdot 10 + 5 \cdot 8$

**equation** .....a mathematical sentence stating that the two expressions have the same value

*Example:*  $2x = 10$

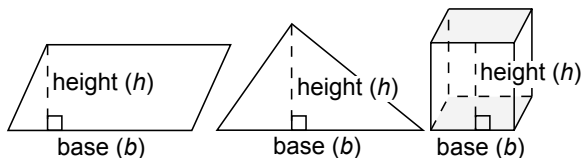
**equiangular** .....a figure with all angles congruent

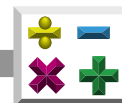
**equilateral** .....a figure with all sides congruent

**fraction** .....any part of a whole

*Example:* One-half written in fractional form is  $\frac{1}{2}$ .

**height ( $h$ )** .....a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or plane that contains the base





- integers** ..... the numbers in the set  
 $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
- length ( $l$ )** ..... a one-dimensional measure that is the measurable property of line segments
- numerator** ..... the top number of a fraction, indicating the number of equal parts being considered  
*Example:* In the fraction  $\frac{2}{3}$ , the numerator is 2.
- perimeter ( $P$ )** ..... the distance around a figure
- polygon** ..... a closed-plane figure, having at least three sides that are line segments and are connected at their endpoints  
*Examples:* triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides); concave, convex
- 
- proportion** ..... a mathematical sentence stating that two ratios are equal  
*Example:* The ratio of 1 to 4 equals 25 to 100, that is  $\frac{1}{4} = \frac{25}{100}$ .
- ratio** ..... the comparison of two quantities  
*Example:* The ratio of  $a$  and  $b$  is  $a:b$  or  $\frac{a}{b}$ , where  $b \neq 0$ .



**regular polygon** .....a polygon that is both *equilateral* (all sides congruent) and *equiangular* (all angles congruent)

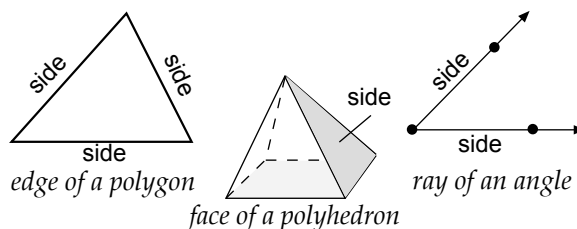
**rounded number** .....a number approximated to a specified place  
*Example:* A commonly used rule to round a number is as follows.

- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place (461 rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place (441 rounded to the nearest hundred is 400).

**scale factor** .....the constant that is multiplied by the lengths of each side of a figure that produces an image that is the same shape as the original figure

**side** .....the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle

*Example:* A triangle has three sides.



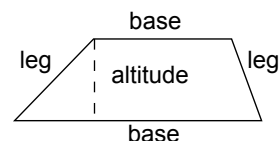




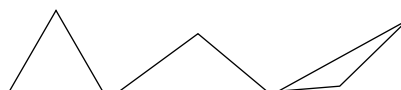
**similar figures ( $\sim$ )** .....figures that are the same shape, have corresponding congruent angles, and have corresponding sides that are proportional in length

**solve** .....to find all numbers that make an equation or inequality true

**trapezoid** .....a quadrilateral with just one pair of opposite sides parallel



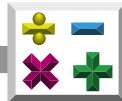
**triangle** .....a polygon with three sides



**value (of a variable)** .....any of the numbers represented by the variable

**variable** .....any symbol, usually a letter, which could represent a number





## Unit 6: Extreme Fractions

### Introduction

We should be able to see that changing the size of a geometric figure can occur without changing the shape of a figure. Working with ratios and proportions will help us understand the relationship between congruence and similarity.

### Lesson One Purpose

#### Reading Process Strand

##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.

#### Algebra Body of Knowledge

##### Standard 5: Radical Expressions and Equations

- MA.912.A.5.1  
Simplify algebraic ratios.
- MA.912.A.5.4  
Solve algebraic proportions.

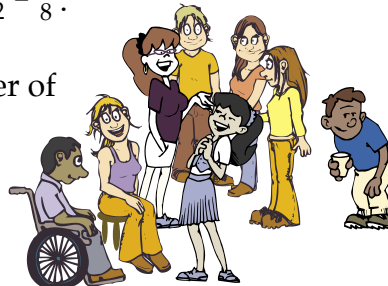


## Ratios and Proportions

**Ratio** is another word for a **fraction**. It is the comparison of two quantities: the **numerator** (top number of a *fraction*) and the **denominator** (bottom number of a fraction). For instance, if a classroom has 32 students and 20 of them are girls, we can say that the *ratio* of the number of girls to the number of students in the class is  $\frac{20}{32} = \frac{5}{8}$  or 5:8. There are several other comparisons we can make using the information. We could compare the number of boys to the number of students,  $\frac{12}{32} = \frac{3}{8}$ .

What about the number of boys to the number of girls?  $\frac{12}{20} = \frac{3}{5}$

Or, the number of girls to the number of boys?  $\frac{20}{12} = \frac{5}{3}$



When two ratios are equal to each other, we have formed a **proportion**. A *proportion* is a mathematical sentence stating that two ratios are equal.

$$\frac{6}{9} = \frac{2}{3}$$

There are several properties of proportions that will be useful as we continue through this unit.

- We could *switch* the 6 with the 3 and still have a *true* proportion (Example 1).
- We could *switch* the 2 with the 9 and still have a *true* proportion (Example 2).
- We could even *flip* both fractions over and still have a *true* proportion (Example 3).

### Example 1

$$\frac{6}{9} = \frac{2}{3}$$

$$\frac{3}{9} = \frac{2}{6}$$

### Example 2

$$\frac{6}{9} = \frac{2}{3}$$

$$\frac{6}{2} = \frac{9}{3}$$

### Example 3

$$\frac{6}{9} = \frac{2}{3}$$

$$\frac{9}{6} = \frac{3}{2}$$

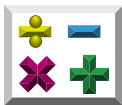


Proportions are also very handy to use for problem solving. We use a process that involves **cross multiplying**, then **solve** the resulting **equation**. Look at the example below as we *solve* the *equation* and find the **value of the variable**.

$$\begin{aligned}\frac{3}{5} &= \frac{x}{x+6} \\ \frac{3}{5} &\swarrow \nwarrow \frac{x}{x+6} && \leftarrow \text{cross multiply} \\ 3(x+6) &= 5x && \leftarrow \text{distribute} \\ 3x+18 &= 5x && \leftarrow \text{(distributive property)} \\ 3x-3x+18 &= 5x-3x && \leftarrow \text{subtract } 3x \text{ from each side} \\ 18 &= 2x \\ \frac{18}{2} &= \frac{2x}{2} && \leftarrow \text{divide each side by } 2 \\ 9 &= x\end{aligned}$$

Check your answer. Does  $\frac{9}{9+6} = \frac{3}{5}$ ? Yes,  $\frac{9}{15} = \frac{3}{5}$ , so 9 is the correct *value* for  $x$ .

Try the following practice.



## Practice

Find the **value of the variable** in each of the following. Refer to previous pages as needed. Check your answers. **Show all your work.**

1.  $\frac{2}{x+1} = \frac{4}{x}$

2.  $\frac{6}{z-2} = \frac{12}{4}$

3.  $\frac{3}{2x-1} = \frac{7}{3x+1}$

4.  $\frac{2}{x-9} = \frac{9}{x+12}$



5.  $\frac{6}{x-1} = \frac{5}{x+2}$

6.  $\frac{x-3}{18} = \frac{x+1}{30}$

7.  $\frac{x-8}{x} = \frac{5}{7}$

8.  $\frac{x+12}{2x+3} = \frac{5}{3}$

9.  $\frac{2x}{x+3} = \frac{3}{2}$



## Using Proportions Algebraically

We can use proportions in word problems as well. Here's an example.

In Coach Coffey's physical education class, the ratio of boys to girls is 3 to 4. If there are 12 boys in the class, how many girls are there?

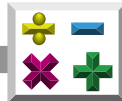
When setting up proportions, you must have a plan and be consistent when you write the ratios. If you set up one ratio as  $\frac{\text{boys}}{\text{girls}}$ , then you must set up the other ratio in the same order, as  $\frac{\text{boys}}{\text{girls}}$ .

$$\begin{array}{ll} \frac{3}{4} = \frac{12}{x} & \swarrow \text{notice that both fractions indicate } \frac{\text{boys}}{\text{girls}} \\ \frac{3}{4} \times \frac{12}{x} & \swarrow \text{cross multiply} \\ 3x = 4 \times 12 & \swarrow \text{simplify} \\ 3x = 48 & \swarrow \text{divide each side by 3} \\ \frac{3x}{3} = \frac{48}{3} & \\ x = 16 & \end{array}$$

Check your answer. Does  $\frac{12}{16} = \frac{3}{4}$ ? Yes, so 16 is the correct answer.

Now it is your turn to practice on the following page.





## Practice

Use **proportions** to solve the following. Refer to the previous pages as needed.  
Check your answers. **Show all your work.**

1. The ratio of two **integers**  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$  is 13:6. The smaller integer is 54. Find the *larger* integer.

Answer: \_\_\_\_\_

2. The ratio of two integers is 7:11. The larger integer is 187. Find the *smaller* integer.

Answer: \_\_\_\_\_

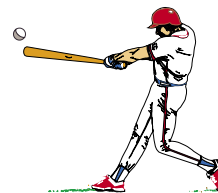
3. A shopkeeper makes \$85 profit when he sells \$500 worth of clothing. At the same rate of profit, what will he make on a \$650 sale?

Answer: \$ \_\_\_\_\_



4. A baseball player made 43 hits in 150 times at bat. At the same rate, how many hits can he expect in 1,050 times at bat?

Answer: \_\_\_\_\_



5. The cost of a 1,600-mile bus trip is \$144. At the same rate per mile, what will be the cost of a 650-mile trip?

Answer: \$ \_\_\_\_\_



6. On a map, 19 inches represent 250 miles. What **length** on the map will represent 600 miles?

Answer: \_\_\_\_\_ miles

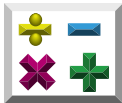




## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |   |                          |
|---|--------------------------|
| _____ 1. the comparison of two quantities   | A. cross multiplication  |
| _____ 2. the numbers in the set<br>$\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$  | B. denominator           |
| _____ 3. a mathematical sentence stating that the two expressions have the same value   | C. distributive property |
| _____ 4. to find all numbers that make an equation or inequality true   | D. equation              |
| _____ 5. the bottom number of a fraction, indicating the number of equal parts a whole was divided into   | E. fraction              |
| _____ 6. the top number of a fraction, indicating the number of equal parts being considered  | F. integers              |
| _____ 7. a mathematical sentence stating that two ratios are equal  | G. length ( $l$ )        |
| _____ 8. $x(a + b) = ax + bx$<br>$5(10 + 8) = 5 \cdot 10 + 5 \cdot 8$   | H. numerator             |
| _____ 9. any part of a whole  | I. proportion            |
| _____ 10. a one-dimensional measure that is the measurable property of line segments  | J. ratio                 |
| _____ 11. a method for solving and checking proportions; a method for finding a missing numerator or denominator in equivalent fractions or ratios by making the cross products equal | K. solve                 |



## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Algebra Body of Knowledge**

#### **Standard 5: Radical Expressions and Equations**

- MA.912.A.5.4  
Solve algebraic proportions.

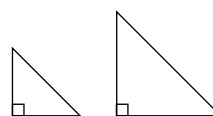


## Similarity and Congruence

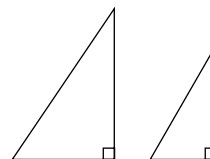
Geometric figures that are exactly the same shape, but not necessarily the same size, are called **similar figures** ( $\sim$ ). In *similar figures*, all the pairs of **corresponding angles** are the same measure, and all the pairs of **corresponding sides** are in the same ratio. This ratio, in its reduced form, is called the **scale factor**. When all pairs of *corresponding sides* are in the same ratio as the *scale factor*, we say that the **sides** are in proportion.

Some geometric figures are always similar.

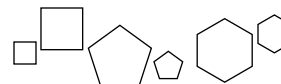
1. All **triangles** whose **angles'** ( $\angle$ ) measures of **degree** ( $^\circ$ ) are  $45^\circ$ ,  $45^\circ$ , and  $90^\circ$  are similar to each other.



2. All *triangles* whose *angles'* ( $\angle$ ) measures of *degree* ( $^\circ$ ) are  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$  are similar to each other.



3. All **regular polygons** with the same number of *sides* are similar to each other.



**Remember:** A *regular polygon* is a **polygon** that is **equilateral** and **equiangular**. Therefore, all its sides are **congruent** ( $\cong$ ) and all angles are *congruent* ( $\cong$ ).

**Note:** **Circles** seem to be similar, but since they have no angle measures, we don't include them in this group.

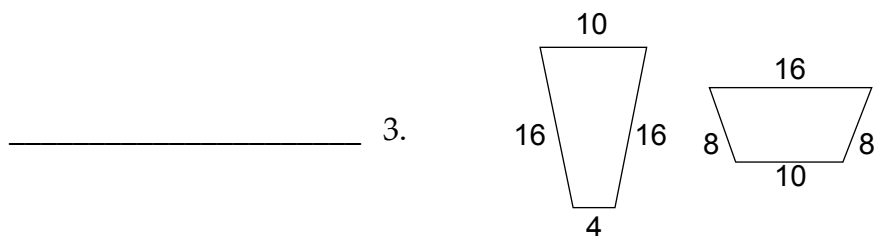
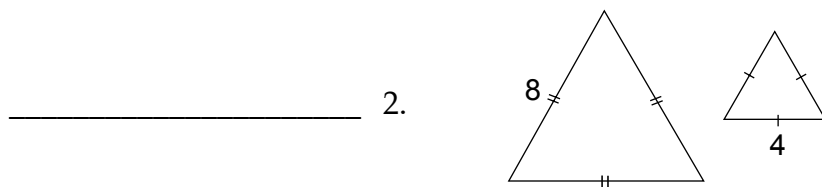
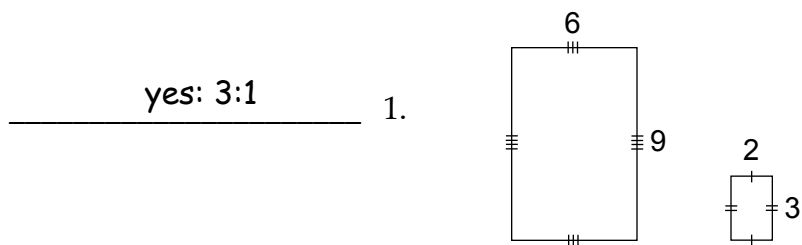


## Practice

Look at each **pair of figures** below. Determine if they are **similar** or **not** to each other.

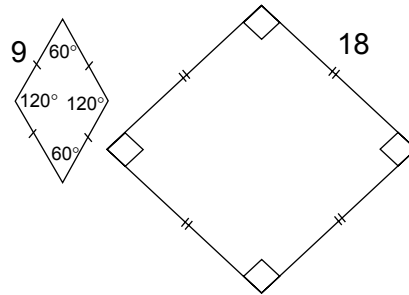
- Write **yes** if they are similar.
- Write **no** if they are not similar.
- If they are similar, write the **scale factor**.

The first one has been done for you.

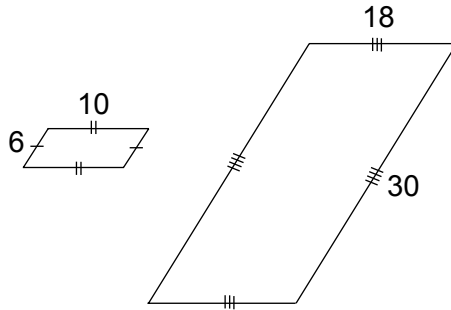




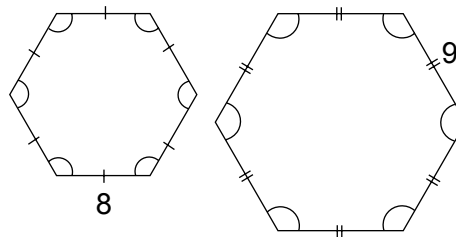
\_\_\_\_\_ 4.

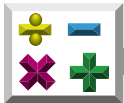


\_\_\_\_\_ 5.

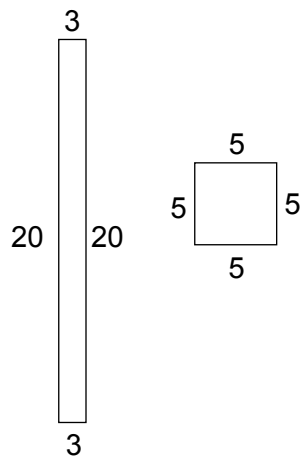


\_\_\_\_\_ 6.

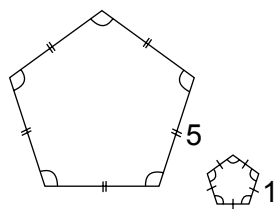




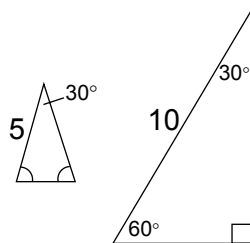
\_\_\_\_\_ 7.



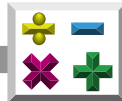
\_\_\_\_\_ 8.



\_\_\_\_\_ 9.



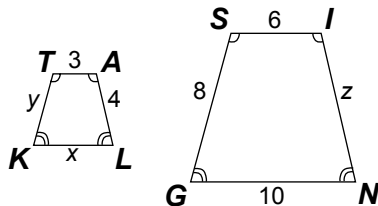




## Using Proportions Geometrically

If we know two shapes are similar, and we know some of the lengths, we often can find some of the other measures of those shapes. Look at the two similar figures below. We have labeled the trapezoids *TALK* and *SING*.

**Trapezoids *TALK* and *SING***



By locating the corresponding angles, we can say that

**Trapezoid *TALK* ~ Trapezoid *SING*.**

**Note:** ~ is the symbol for similar.

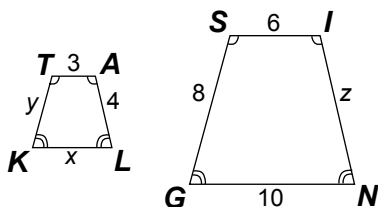
To find the values of  $x$ ,  $y$ , and  $z$ , we must first find a pair of corresponding sides with lengths given.

- Side  $TA$  and side  $SI$  are a pair of corresponding sides.
- It is given that  $TA = 3$  and  $SI = 6$ .
- So, we can set up a ratio  $\frac{TA}{SI} = \frac{3}{6}$ .
- When we reduce the ratio, we get the scale factor, which is  $\frac{1}{2}$ .
- This means that every length in *TALK* is one-half the **corresponding** length in *SING*.



Now we can use the scale factor to make proportions and find  $x$ ,  $y$ , and  $z$ . Remember to be consistent as you set up the proportions. Since my scale factor was determined by a comparison of  $TALK$  to  $SING$ , I will continue in that order:  $(\frac{TALK}{SING})$ .

### Trapezoids $TALK$ and $SING$



$$\frac{1}{2} = \frac{x}{10}$$

$$2x = 10$$

$$x = 5$$

$$\frac{1}{2} = \frac{y}{8}$$

$$2y = 8$$

$$y = 4$$

$$\frac{1}{2} = \frac{4}{z}$$

$$8 = 1z$$

$$8 = z$$

What is the **perimeter** ( $P$ ), or distance around the *polygon*, of  $TALK$ ?

Did you get 16?

Can you guess the *perimeter* of  $SING$ ?

If you guessed 32, you are correct.

Does it make sense that the perimeters should be in the same ratio as the scale factor?

Yes, because the perimeters of  $TALK$  and  $SING$  are corresponding lengths. In addition, all *corresponding* lengths in similar figures are in proportion!

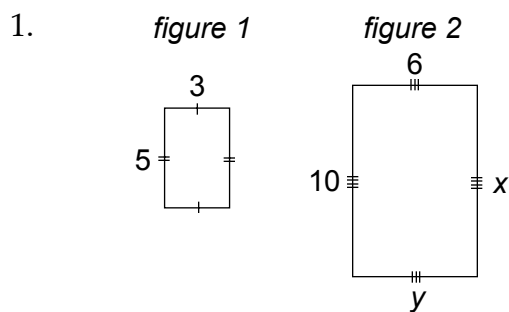


## Practice

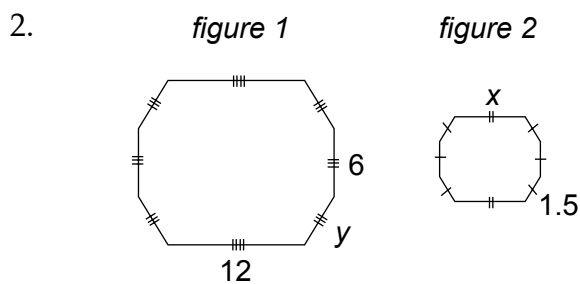
Find the following for each pair of **similar figures** below.

- scale factor (SF)
- $x =$
- $y =$
- $P_1 =$  perimeter of figure 1
- $P_2 =$  perimeter of figure 2

Refer to the previous pages as needed. **The first one has been done for you.**



SF 1:2 ;  $x =$  10 ;  $y =$  6  $P_1$  16



SF \_\_\_\_\_ ;  $x =$  \_\_\_\_\_ ;  $y =$  \_\_\_\_\_  $P_1$  \_\_\_\_\_



3.

figure 1

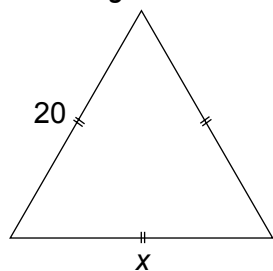


figure 2



SF \_\_\_\_\_ ;  $x =$  \_\_\_\_\_ ;  $y =$  \_\_\_\_\_  $P_2$  \_\_\_\_\_

4.

figure 1

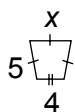
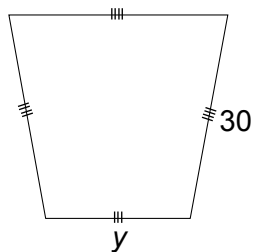


figure 2



SF \_\_\_\_\_ ;  $x =$  \_\_\_\_\_ ;  $y =$  \_\_\_\_\_  $P_1$  \_\_\_\_\_

5.

figure 1

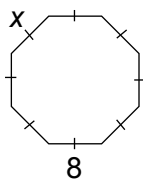
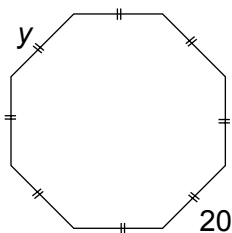


figure 2



SF \_\_\_\_\_ ;  $x =$  \_\_\_\_\_ ;  $y =$  \_\_\_\_\_  $P_1$  \_\_\_\_\_



6.

figure 1

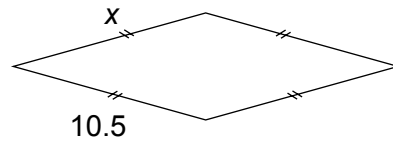
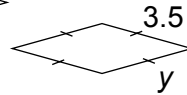


figure 2



SF \_\_\_\_\_ ;  $x =$  \_\_\_\_\_ ;  $y =$  \_\_\_\_\_  $P_2$  \_\_\_\_\_

7.

figure 1

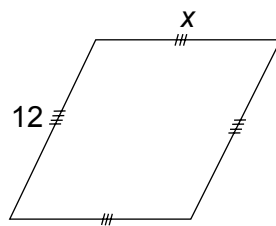
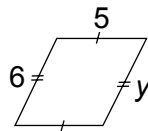


figure 2



SF \_\_\_\_\_ ;  $x =$  \_\_\_\_\_ ;  $y =$  \_\_\_\_\_  $P_2$  \_\_\_\_\_

8.

figure 1

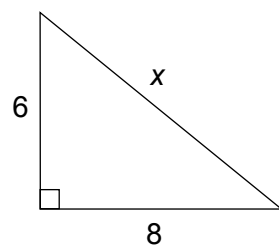
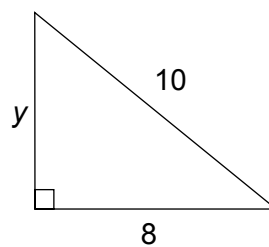


figure 2

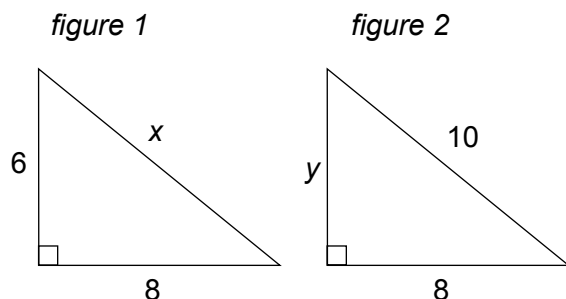


SF \_\_\_\_\_ ;  $x =$  \_\_\_\_\_ ;  $y =$  \_\_\_\_\_  $P_1$  \_\_\_\_\_



## Using Proportions to Find Heights

Look at the figures below. They are from number 8 in the previous practice.

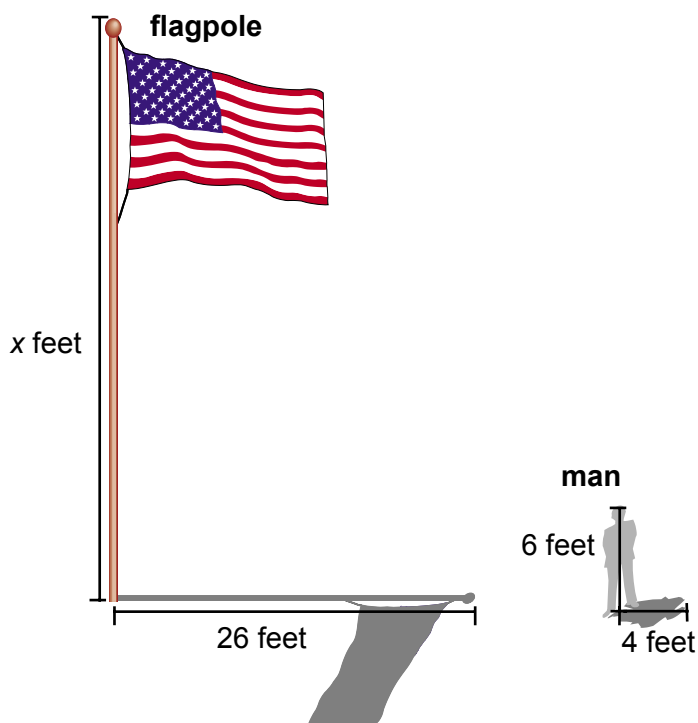


Here is what we know about *figure 1* and *figure 2* above.

- Their scale factor is  $\frac{1}{1}$ . This makes all the pairs of corresponding sides the same length.
- We already knew that their corresponding angles were the same measure because we knew that they were similar. This makes the triangles identical to each other.

Geometric figures that are *exactly* the same *shape* and *exactly* the same *size* are *congruent* to each other. The symbol for congruence,  $\cong$ , is a lot like the symbol for similar, but the equal sign,  $=$ , underneath it tells us that two things are *exactly* the same *size*.

We can use proportions to find the lengths of some items that would be difficult to measure. For instance, if we needed to know the height of a flagpole without having to inch our way up, we could use proportions. See the example on the following page.



A 6-foot man casts a 4-foot shadow at the same time a flagpole casts a 26-foot shadow. Find the **height (*h*)** of the flagpole.

To solve a problem like this, set up a proportion comparing corresponding parts.

$$\frac{\text{man's height}}{\text{man's shadow}} = \frac{\text{flagpole's height}}{\text{flagpole's shadow}}$$

$$\frac{6}{4} = \frac{x}{26}$$

$$4x = 6 \times 26$$

$$4x = 156$$

$$\frac{4x}{4} = \frac{156}{4}$$

$$x = 39 \text{ feet}$$

↙ cross  
multiply

↙ divide both  
sides by 4

Now try the following practice.



## Practice

Use **proportions** to solve the following. Refer to the previous pages as needed.

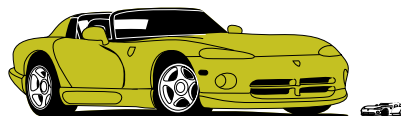
**Round to the nearest tenth. Show all your work.**

1. A tree casts a 50-foot shadow at the same time a 4-foot fence post casts a 3-foot shadow. How tall is the tree?

Answer: \_\_\_\_\_ feet

2. If the scale factor for a miniature toy car and a real car is 1 to 32 and the windshield on the toy car is 2 inches long, how long is the windshield on the real car?

Answer: \_\_\_\_\_ inches



$$\text{scale factor} = \frac{1}{32}$$



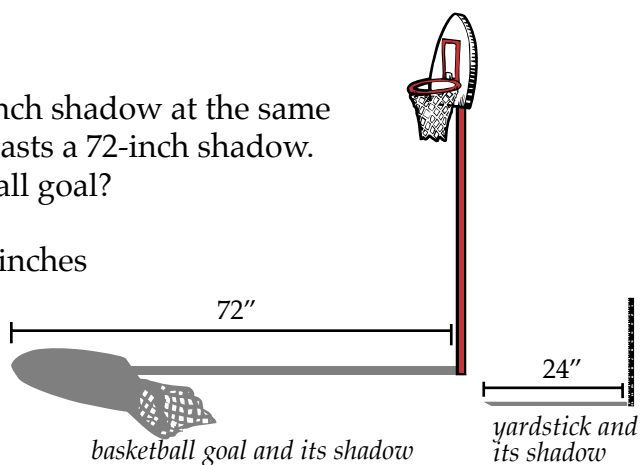


3. The goal post on the football field casts an 18-foot shadow. The 4-foot water cooler casts a 5-foot shadow. How tall is the goal post?

Answer: \_\_\_\_\_ feet

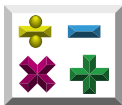
4. A yardstick casts a 24-inch shadow at the same time a basketball goal casts a 72-inch shadow. How tall is the basketball goal?

Answer: \_\_\_\_\_ inches



5. A photo that is 4 inches by 6 inches needs to be enlarged so that the shorter sides are 6 inches. What will be the length of the enlargement?

Answer: \_\_\_\_\_ inches



## Practice

Use the list below to complete the following statements.

<b>congruent (<math>\cong</math>)</b>	<b>perimeter (<math>P</math>)</b>	<b>regular polygon</b>
<b>equiangular</b>	<b>proportion</b>	<b>scale factor</b>
<b>equilateral</b>	<b>ratio</b>	

1. A figure with all angles congruent is called \_\_\_\_\_ .
2. The comparison of two quantities is a \_\_\_\_\_ .
3. Figures or objects that are exactly the same shape and size are said to be \_\_\_\_\_ .
4. The \_\_\_\_\_ is the distance around a figure.
5. A figure with all sides congruent is called \_\_\_\_\_ .
6. A(n) \_\_\_\_\_ is a mathematical sentence stating that two ratios are equal.
7. The constant that is multiplied by the lengths of each side of a figure that produces an image that is the same shape as the original figure is the \_\_\_\_\_ .
8. A polygon that is both equilateral and equiangular is called a \_\_\_\_\_ .



## Unit Review

Find the **value of the variable** in the following. Check your answers. **Show all your work.**

1.  $\frac{x+2}{x} = \frac{5}{3}$

2.  $\frac{4}{3x+1} = \frac{7}{5x-2}$

3.  $\frac{3x}{x+7} = \frac{2}{3}$

4.  $\frac{9x-1}{7} = \frac{3x-11}{2}$



Use **proportions** to solve the following. Check your answers. **Show all your work.**

5. The ratio of two integers is 9:7. The smaller integer is 448. Find the *larger* integer.

Answer: \_\_\_\_\_

6. The ratio of two integers is 6:11. The larger integer is 88. Find the *smaller* integer.

Answer: \_\_\_\_\_

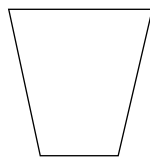
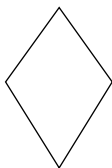
7. The cost of 24 pounds of rice is \$35. At the same rate, what would 5 pounds of rice cost? **Round to the nearest whole cent.**

Answer: \$ \_\_\_\_\_

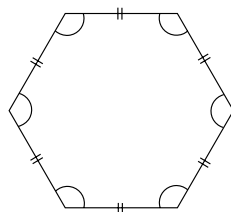
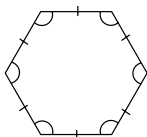


Look at each **pair of figures** below. Determine if they are **similar to each other**. Write **yes** if they are similar. Write **no** if they are not similar.

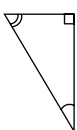
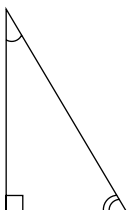
\_\_\_\_\_ 8.



\_\_\_\_\_ 9.

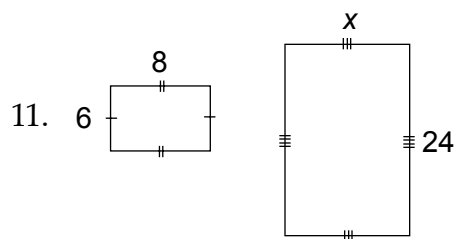


\_\_\_\_\_ 10.

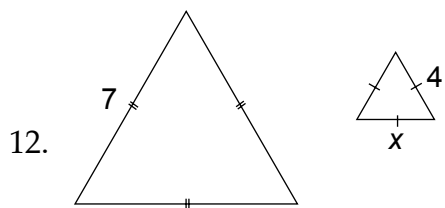




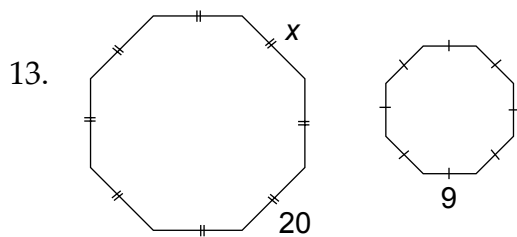
Each pair of figures below is **similar**. Find the **scale factor** and **value of the variable**.



SF = \_\_\_\_\_ ;  $x$  = \_\_\_\_\_



SF = \_\_\_\_\_ ;  $x$  = \_\_\_\_\_



SF = \_\_\_\_\_ ;  $x$  = \_\_\_\_\_



Use **proportions** to solve the following. **Show all your work.**

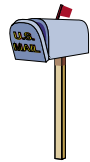
14. A tree casts a 40-foot shadow at the same time a 6-foot post casts an 8-foot shadow. How tall is the tree?

Answer: \_\_\_\_\_ feet



15. A 3.5-foot-tall mailbox casts a shadow of 5 feet at the same time a light pole casts a 20-foot shadow. How tall is the light pole?

Answer: \_\_\_\_\_ feet







## **Unit 7: Exploring Relationships with Venn Diagrams**

This unit introduces the concept of set theory and operations involving sets. It will also explore the relationship between sets and Venn diagrams, in addition to using set theory to solve problems.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Writing Process Strand**

##### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

#### **Algebra Body of Knowledge**

##### **Standard 7: Quadratic Equations**

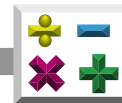
- MA.912.A.7.1  
Graph quadratic equations with and without graphing technology.

- MA.912.A.7.2  
Solve quadratic equations over the real numbers by factoring, and by using the quadratic formula.
- MA.912.A.7.8  
Use quadratic equations to solve real-world problems.
- MA.912.A.7.10  
Use graphing technology to find approximate solutions of quadratic equations.

### **Discrete Mathematics Body of Knowledge**

#### **Standard 7: Set Theory**

- MA.912.D.7.1  
Perform set operations such as union and intersection, complement, and cross product.
- MA.912.D.7.2  
Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.



## Vocabulary

*Use the vocabulary words and definitions below as a reference for this unit.*

**braces { }** .....grouping symbols used to express sets

**Cartesian cross product**.....a set of ordered pairs found by taking the  $x$ -coordinate from one set and the  $y$ -coordinate from the second set

**complement** .....the set of elements left over when the elements of one set are deleted from another

**coordinate grid or plane** ...a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced; especially designed for locating points, displaying data, or drawing maps

**counting numbers**

**(natural numbers)** .....the numbers in the set  $\{1, 2, 3, 4, 5, \dots\}$

**element or member** .....one of the objects in a set

**empty set or null set ( $\emptyset$ )** ...a set with no elements or members

**even integer**.....any integer divisible by 2; any integer with the digit 0, 2, 4, 6, or 8 in the units place; any integer in the set  $\{\dots, -4, -2, 0, 2, 4, \dots\}$

**expression** .....a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables  
*Examples:*  $4r^2$ ;  $3x + 2y$ ;  $\sqrt{25}$   
An expression does *not* contain equal ( $=$ ) or inequality ( $<$ ,  $>$ ,  $\leq$ ,  $\geq$ , or  $\neq$ ) signs.



**finite set** ..... a set in which a whole number can be used to represent its number of elements; a set that has bounds and is limited

**infinite set** ..... a set that is not finite; a set that has no boundaries and no limits

**integers**..... the numbers in the set  
 $\{ \dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots \}$

**intersection** ( $\cap$ ) ..... those elements that two or more sets have in common

**member or element** ..... one of the objects in a set

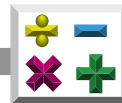
**natural numbers**

**(counting numbers)** ..... the numbers in the set  $\{1, 2, 3, 4, 5, \dots\}$

**null set ( $\emptyset$ ) or empty set** ... a set with no elements or members

**ordered pair**..... the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the  $x$ -axis and  $y$ -axis, respectively  
*Examples:  $(x, y)$  or  $(3, -4)$*

**pattern (relationship)** ..... a predictable or prescribed sequence of numbers, objects, etc.; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions)  
*Example: 2, 5, 8, 11 ... is a pattern. Each number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule,  $3n - 1$ , by using the set of counting numbers for  $n$ .*



- point** .....a specific location in space that has no discernable length or width
- positive integers** .....integers greater than zero
- relation** .....a set of ordered pairs  $(x, y)$
- roster** .....a list of all the elements in a set
- rule** .....a description of the elements in a set
- set** .....a collection of distinct objects or numbers
- union** ( $\cup$ ) .....combination of the elements in two or more sets
- Venn diagram** .....overlapping circles used to illustrate relationships among sets
- $x$ -coordinate** .....the first number of an ordered pair
- $y$ -coordinate** .....the second number of an ordered pair





## Unit 7: Exploring Relationships with Venn Diagrams

### Introduction

We will become more familiar with Venn diagrams as a mathematical tool while learning to use operations relative to set theory.

### Lesson One Purpose

#### Reading Process Strand

##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### Writing Process Strand

##### Standard 3: Prewriting

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.



## Discrete Mathematics Body of Knowledge

### Standard 7: Set Theory

- MA.912.D.7.1  
Perform set operations such as union and intersection, complement, and cross product.

### Sets

Unit 1 discussed **sets**. A *set* is a collection of distinct objects or numbers. Each item in the set is called an **element** or **member** of the set. Sets are indicated by grouping symbols called **braces** { }.

A set can have a few *elements*, lots of elements, or *no* elements—called a **null set** ( $\emptyset$ ) or **empty set**. Sets like the **counting numbers**, also called the **natural numbers**—{1, 2, 3, 4, 5, ...}—are **infinite sets** because they continue in the **pattern** and *never* end. *Patterns* are predictable. They have a prescribed sequence of numbers or objects.

Other sets with a *specified* number of elements are called **finite sets**. Some *finite sets* are *very* large; however, even very large sets with bounds and limits are finite sets.

Sets can usually be written in two different ways. One way is by **roster**. A *roster* is a list. You have probably heard of a football roster—a list of players on the team—or a class roster—a list of students in the class. Look at this set expressed in roster format.

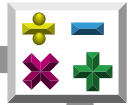
{red, orange, yellow, blue, green, indigo, violet}

We could also name this set using the **rule** format. That means describing the set.

{the colors in the rainbow}

This is another way to indicate the set of colors listed above. So you see, there are two ways to express the same set.





Let's look at some more examples.

{the set of vowels in the alphabet} means {a, e, i, o, u}

{2, 4, 6, 8, ...} is the same as {the set of positive **even integers**}



**Remember: Integers** are the numbers in the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$  and **positive integers** are integers greater than zero.



## Practice

Use the list below to complete the following statements.

braces	infinite	roster
element or member	null ( $\emptyset$ ) or empty	rule
finite	pattern	

1. The set of counting numbers is \_\_\_\_\_ because it has *no boundaries*.
2. A set with *no elements* is called a(n) \_\_\_\_\_ set.
3. *Each item* in the set is called a(n) \_\_\_\_\_ of the set.
4. The *grouping symbols* used to indicate sets are called \_\_\_\_\_ .
5. A set whose elements are *described* is in \_\_\_\_\_ format.
6. A set with a *specified* number of elements, and a whole number can be used to represent its number of elements, is a \_\_\_\_\_ set.
7. A *list* of all the elements in a set, like the list of students in one class, is in \_\_\_\_\_ format.
8. A \_\_\_\_\_ is predictable, or it has a prescribed sequence of numbers or objects.



## Practice

Express the following as sets in **roster format**.

1. integers *greater than 3* and *less than 11*

---

2. counting numbers *less than 6*

---

3. colors in the American flag

---

4. planets in the solar system

---

5. courses on your schedule

---

Express the following as sets in **rule format**.

6. breakfast, lunch, dinner

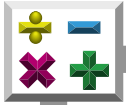
---

7. Chevrolet, Ford, Chrysler, Buick

---

8. fork, spoon, knife, plate, glass

---



9. shoulder, wrist, elbow, hand, finger

---

10. table of contents, chapter, glossary, index, page

---



## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

### **Discrete Mathematics Body of Knowledge**

#### **Standard 7: Set Theory**

- MA.912.D.7.1  
Perform set operations such as union and intersection, complement, and cross product.
- MA.912.D.7.2  
Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.



## Unions and Intersections

When you combine all the elements in one set with all the elements in another set, we call this the **union** ( $\cup$ ). A *union* is like a “marriage” of elements in a set. The symbol for union looks a bit like the letter “u.”

A problem involving a union looks like the following.

$$\{2, 3, 4, 5\} \cup \{2, 4, 6, 8\}$$

This means that you should combine everything in the first set with all new elements from the second set.

$$\{2, 3, 4, 5\} \cup \{2, 4, 6, 8\} = \{2, 3, 4, 5, 6, 8\}$$

Look at other examples.

### Example 1

$$\{6, 7, 8, 10\} \cup \{5, 7, 8, 9\} = \{5, 6, 7, 8, 9, 10\}$$

**Note:** You do *not* repeat any element even though it may have been in both sets.

### Example 2

$$\{\dots, -3, -2, -1, 0\} \cup \{0, 1, 2, 3, \dots\} = \{\text{the integers}\}$$

This result can be expressed in rule format.

### Example 3

$$\{5, 7, 9, 11\} \cup \{\} = \{5, 7, 9, 11\}$$

The *empty set* had nothing to add, so the answer is the same as the first set.



The intersection of two streets is the place where the streets cross each other. The intersection of two lines is also the **point** where they cross, or the *point(s)* they have in common. Likewise, when we take the intersection of two sets, we take only those elements that the two sets have in common. The symbol for **intersection** ( $\cap$ ) looks like an upside-down union symbol.

An *intersection* problem would look like the following.

$$\{2, 3, 4, 5\} \cap \{2, 4, 6, 8\}$$

This means that you should include only those elements that the sets have in common.

$$\{2, 3, 4, 5\} \cap \{2, 4, 6, 8\} = \{2, 4\}$$

Look at these examples.

#### Example 1

$$\{6, 7, 8, 10\} \cap \{5, 7, 8, 9\} = \{7, 8\}$$

**Note:** The only elements that appears in both sets are 7 and 8.

#### Example 2

$$\{\dots, -3, -2, -1, 0\} \cap \{0, 1, 2, 3, \dots\} = \{0\}$$

The only element the sets have in common is 0.

#### Example 3

$$\{5, 7, 9, 11\} \cap \{\} = \{\}$$

Since the empty set has no elements, it cannot have any elements in common with another set.

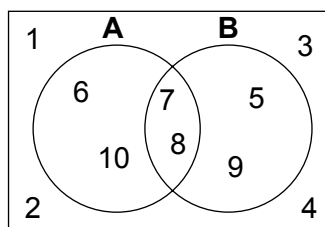


We can also use **Venn diagrams** to illustrate the union and intersection of sets. Unit 1 had a *Venn diagram* showing the relationships between sets of numbers.

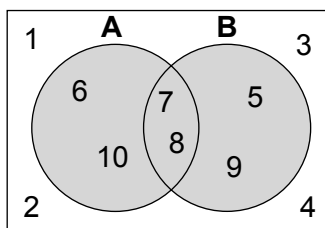
### Example 1

Look at the examples below. The sets illustrated are using Venn diagrams.

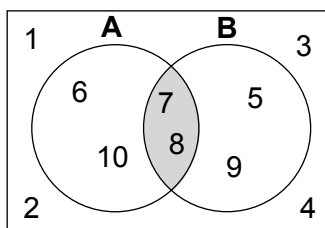
Set A = {6, 7, 8, 10} and set B = {5, 7, 8, 9}



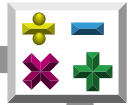
The union of A and B ( $A \cup B$ ) is both circles. Notice that there are numbers outside of set A and set B. Those are *not* part of the union *or* intersection.



The intersection of A and B ( $A \cap B$ ) is only the football shape in the middle where the numbers that A and B have in common are located.







Look at these examples as well.

**Example 2**

$$A = \{\dots, -3, -2, -1, 0\}, B = \{0, 1, 2, 3, \dots\}$$

$$A \cup B = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

$$A \cap B = \{0\}$$

**Example 3**

$$A = \{5, 7, 9, 11\}, B = \{\}$$

$$A \cup B = \{5, 7, 9, 11\}$$

$$A \cap B = \{\}$$

Your turn to try some.



## Practice

*Answer the following.*

1. What is the union of  $\{6, 7, 13\}$  and  $\{5, 6, 15\}$ ?

---

2. What is the union of  $\{6, 7, 10\}$  and  $\{2\}$ ?

---

3. What is the union of  $\{5, 7, 9\}$  and  $\{3, 7, 9\}$ ?

---

4.  $\{2, 3, 4\} \cup \{1, 3, 5, 7\}$

---

5.  $\{2, 4, 6, 8\} \cup \{1, 3, 5, 7\}$

---

6.  $\{\} \cup \{5, 12, 15\}$

---

7.  $\{1, 2, 3, 4\} \cup \{\}$

---

8.  $\{1, 2, 3, \dots, 10\} \cup \{2, 4, 6, 8, 10\}$

---



9. What is the intersection of  $\{2, 8\}$  and  $\{1, 3, 9, 13\}$ ?

---

10. What is the intersection of  $\{6, 7, 13\}$  and  $\{5, 6, 15\}$ ?

---

11. What is the intersection of  $\{3, 5, 9\}$  and  $\{3, 6, 9\}$ ?

---

12.  $\{2, 3, 4\} \cap \{1, 3, 5, 7\}$

---

13.  $\{2, 4, 6, 8\} \cap \{1, 3, 5, 7\}$

---

14.  $\{\} \cap \{5, 12, 15\}$

---

15.  $\{1, 2, 3, 4\} \cap \{\}$

---

16.  $\{1, 2, 3, \dots, 10\} \cap \{2, 4, 6, 8, 10\}$

---

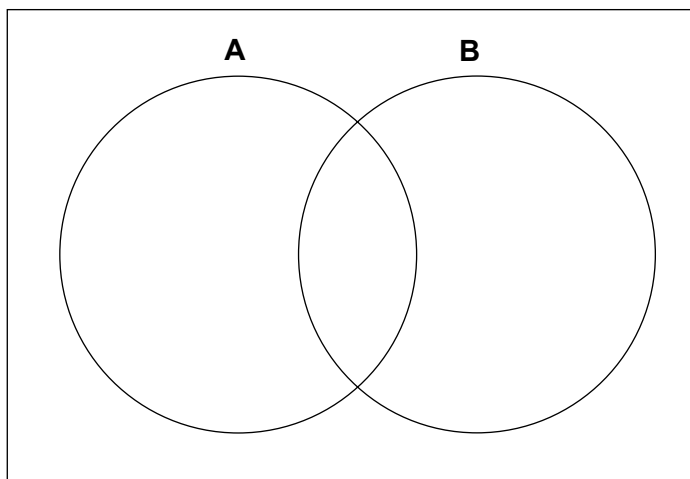


## Practice

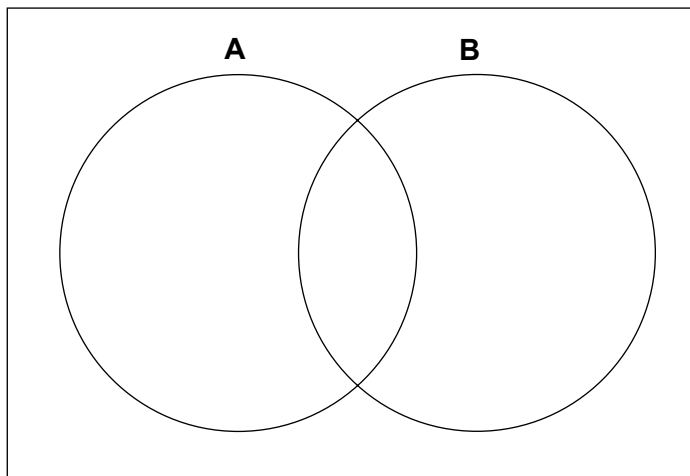
Use the **Venn diagrams** below to illustrate the following **sets**.

1.  $A = \{1, 2, 3, 4, 5, 6\}$   
 $B = \{4, 5, 6, 7, 8\}$

a.  $A \cup B$



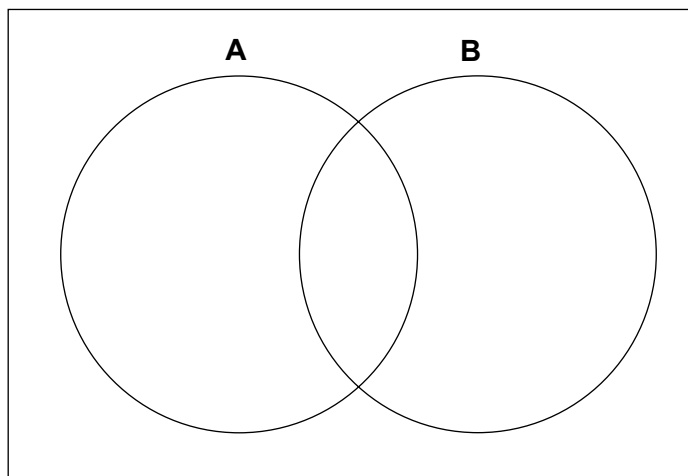
b.  $A \cap B$



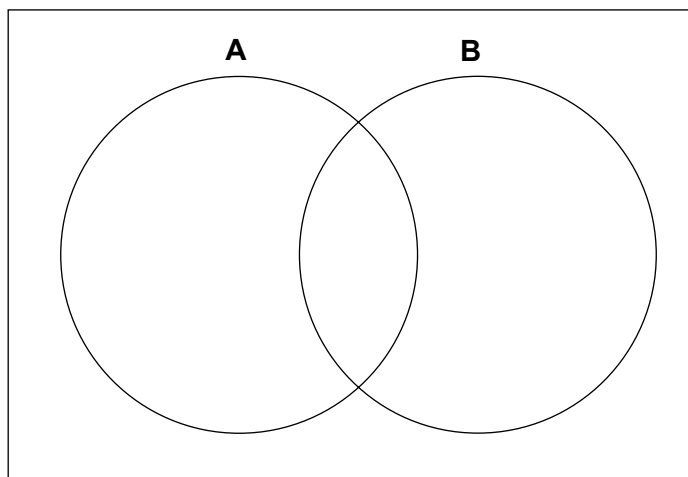


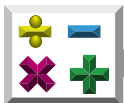
2.  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$   
 $B = \{2, 4, 6, 8, 10\}$

a.  $A \cup B$



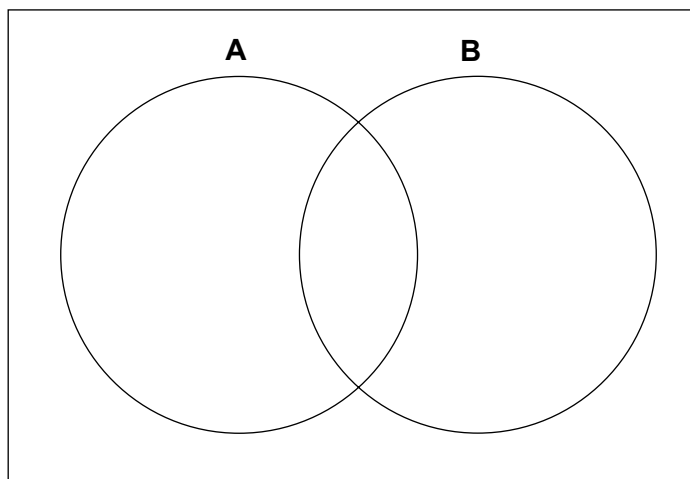
b.  $A \cap B$



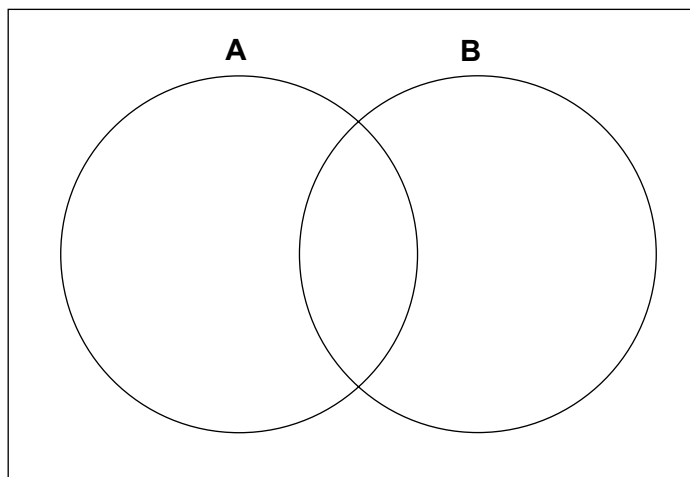


3.  $A = \{ \}$   
 $B = \{2, 4, 6, 8, 10\}$

a.  $A \cup B$



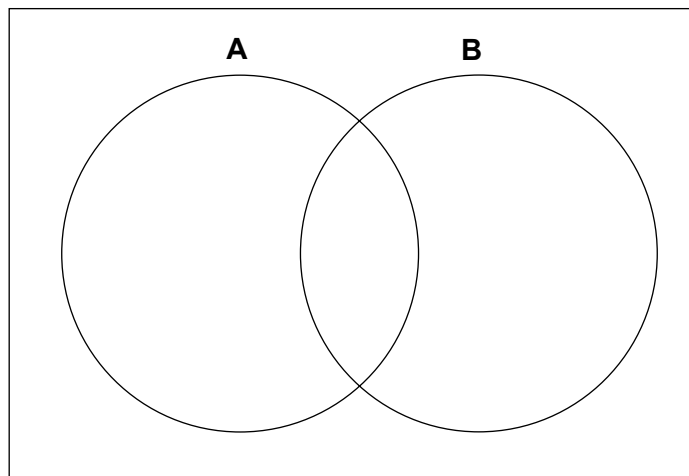
b.  $A \cap B$



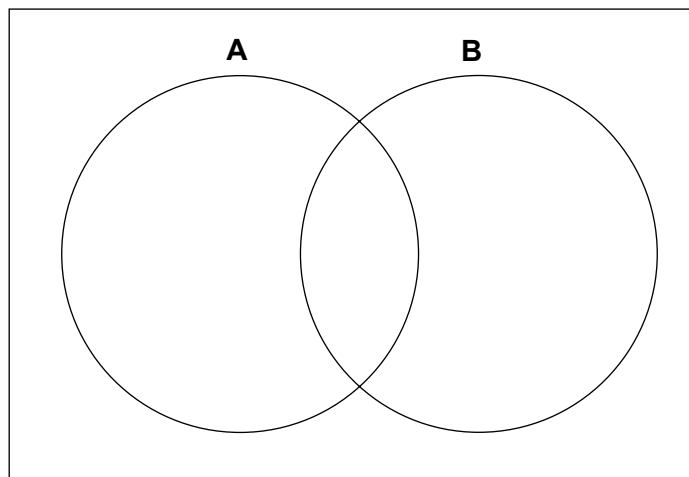


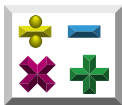
4.  $A = \{3, 6, 9, 12, 15\}$   
 $B = \{2, 4, 6, 8, 10, 12\}$

a.  $A \cup B$



b.  $A \cap B$





## **Lesson Three Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Discrete Mathematics Body of Knowledge**

### **Standard 7: Set Theory**

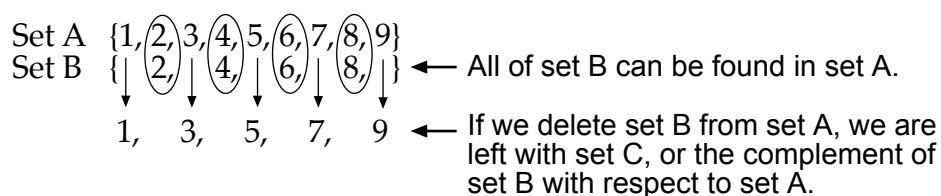
- MA.912.D.7.1  
Perform set operations such as union and intersection, complement, and cross product.
- MA.912.D.7.2  
Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.





## Complements

Look at set A {1, 2, 3, 4, 5, 6, 7, 8, 9} and set B {2, 4, 6, 8}. Do you see that all of the elements from set B can be found in set A? If we delete set B from set A we are left with the elements 1, 3, 5, 7, 9.



We could place these in a set and call it by another name, perhaps set C. We call set C the **complement** of set B with respect to set A. In other words, when we delete the elements of set B from set A we end up with set C.

Let's look at another example.

With respect to set R {red, orange, yellow, green, blue, indigo, violet}, find the *complement* of set S {red, yellow, blue}. We would delete red, yellow, and blue from set R and end up with a new set T {orange, green, indigo, violet}.

In symbols, this example looks like the following.

$$R - S = T.$$

The symbol for complement looks like a minus sign (-).



## Practice

*Answer the following.*

1. With respect to  $A \{1, 3, 6, 9, 12, 15, 18\}$  find the complement of  $B \{3, 12, 15\}$ .

---

2. With respect to  $A \{1, 3, 6, 9, 12, 15, 18\}$  find the complement of  $C \{6, 12, 18\}$ .

---

3.  $\{2, 4, 6, 7, 8\} - \{2, 6, 8\}$

---

4.  $\{2, 4, 6, 8, 9, 13, 14, 16\} - \{6, 9, 14\}$

---

5.  $\{\text{integers}\} - \{\text{odd integers}\}$

---

6.  $\{\text{letters of the alphabet}\} - \{\text{vowels}\}$

---

7. Find the complement of  $\{\text{animals with four feet}\}$  with respect to  $\{\text{dogs, cats, fish, birds, mice, rabbits}\}$ .

---

8.  $\{6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\} - \{\text{multiples of 3}\}$

---



9.  $\{\text{integers}\} - \{\text{positive numbers}\}$

---

10.  $\{2, 4, 6, 8, 10\} - \{2, 4, 6, 8, 10\}$

---

11.  $\{2, 4, 6, 8, 10\} - \{ \}$

---

12.  $\{\text{students in your class}\} - \{\text{male students in your class}\}$

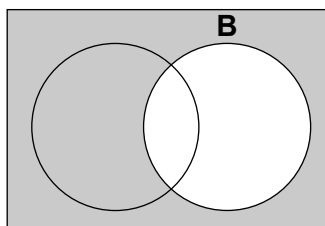
---



## Complements in Venn Diagrams

When talking about complements in Venn diagrams, we use a slightly different notation.

The figure below represents the complement of B.



We use the symbol  $\overline{B}$  to indicate that we are deleting all the elements of set B from the diagram and shading everything except what is in set B.



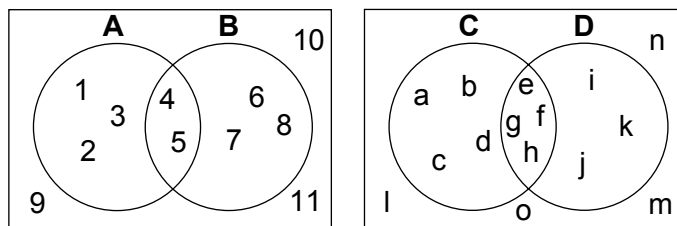
## Practice

Use the Venn diagrams below to give each set in **roster format**.



**Remember:** *Roster format* is a list of all the elements in a set.

**Note:** Elements listed outside the circles but inside the rectangles are part of the sets.



1.  $\bar{A}$

---

2.  $\bar{B}$

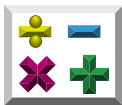
---

3.  $\bar{C}$

---

4.  $\bar{D}$

---



## **Lesson Four Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Discrete Mathematics Body of Knowledge**

### **Standard 7: Set Theory**

- MA.912.D.7.1  
Perform set operations such as union and intersection, complement, and cross product.



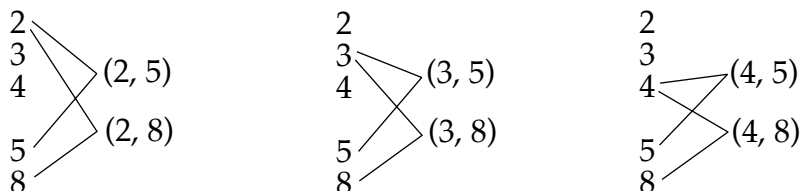
## Cartesian Cross Products

Another operation we can do with sets involves **Cartesian cross products**. A *Cartesian cross product* is a set of **ordered pairs** found by taking the ***x*-coordinate** from one set and the ***y*-coordinate** from the second set. The Cartesian coordinate system is named after the mathematician René Descartes (1596-1650). We use his work every time we graph on a **coordinate grid** or **plane**. Keeping that in mind, you will find it no surprise that Cartesian cross products have something to do with graphing.

To find a Cartesian cross product we must have two sets.

Let's let  $A = \{2, 3, 4\}$  and  $B = \{5, 8\}$ .

The **expression** in symbols looks like  $A \times B$ . The  $\times$  almost looks like a large multiplication sign. However, don't be fooled. We are not going to multiply. We are going to create a **relation**, which is another name for a set of *ordered pairs*.



So,  $A \times B = \{(2, 5), (2, 8), (3, 5), (3, 8), (4, 5), (4, 8)\}$

Notice that in the newly created set, every element is an ordered pair  $(x, y)$ . Also see that each number in the  $x$  position came from set A and each number in the  $y$  position came from set B.

Let's look at another one.

$$\{3, 5\} \times \{1, 2, 3\} = \{(3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3)\}$$

Notice that the resulting set is a *relation* because every element is an ordered pair.

It's time for you to try.



## Practice

*Answer the following.*

1.  $\{1, 2\} \times \{4, 6\}$

---

2.  $\{3, 4, 7, 8\} \times \{2, 5\}$

---

3.  $\{1, 5, 9\} \times \{3, 6, 9\}$

---

4.  $\{2, 4\} \times \{1, 3\}$

---

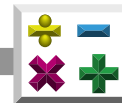
5.  $\{2, 4\} \times \{2, 4\}$

---

6.  $\{6, 8\} \times \{4, 5, 7\}$

---



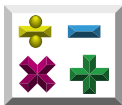


## Practice

Use the list below to complete the following statements.

<b>braces</b>	<b>intersection (<math>\cap</math>)</b>	<b>roster</b>	<b>set</b>
<b>element or member</b>	<b>null (<math>\emptyset</math>) or empty set</b>	<b>rule</b>	<b>union (<math>\cup</math>)</b>
<b>finite</b>	<b>relation</b>		

1. The combining of the elements in two or more sets is called the \_\_\_\_\_ of the sets.
2. A set of ordered pairs is called a \_\_\_\_\_.
3. A \_\_\_\_\_ is a list of the elements in a set.
4. The set of elements that two or more sets have in common is called the \_\_\_\_\_ of the sets.
5. A \_\_\_\_\_ is a collection of distinct objects or numbers.
6. A description of the elements in a set is called a \_\_\_\_\_.
7. The symbols used to express a set are called \_\_\_\_\_.
8. A set with no elements is called a(n) \_\_\_\_\_.
9. An item in a set is called a(n) \_\_\_\_\_.
10. A set with a specified number of elements is called a \_\_\_\_\_ set.



## **Lesson Five Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

### **Discrete Mathematics Body of Knowledge**

#### **Standard 7: Set Theory**

- MA.912.D.7.1  
Perform set operations such as union and intersection, complement, and cross product.
- MA.912.D.7.2  
Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.



## Using Venn Diagrams for Three Categories

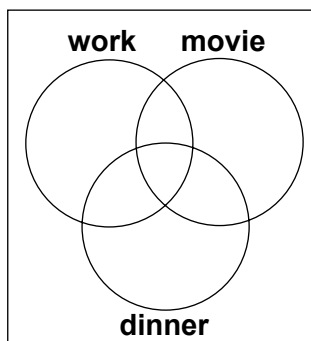
We can use Venn diagrams to solve problems that might otherwise seem impossible. Here is an example.

A group of Leon High School seniors answered a questionnaire about their plans for the weekend. In the group, 20 planned to work, 19 planned to see a movie, while 28 were planning to go out to dinner. Exactly 7 seniors planned to do all three. Another 12 seniors were planning to do dinner only. There are 2 seniors who were going to work and go to a movie but not go out to dinner, and 15 seniors were going to work and go out to dinner.

Next, we will use Venn diagrams to answer the following.

1. How many seniors answered the questionnaire?
2. How many seniors were going to dinner and a movie, but not work?
3. How many seniors were going to dinner or a movie?
4. How many seniors were only planning to work?

The first thing we will do is set up a Venn diagram for the *three* categories of plans. Notice that the three circles overlap.

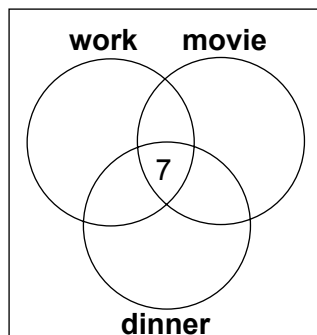


Then we read back through the statements to fill in the different sections of the diagram. Try to find the *middle* information first and then work your way to the outside.

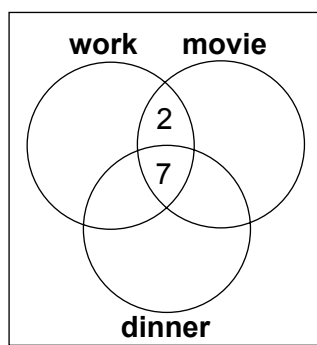
Pay careful attention to the wording. When the word “and” is used that indicates the *intersection* of two sets. The word “or” means *union*.



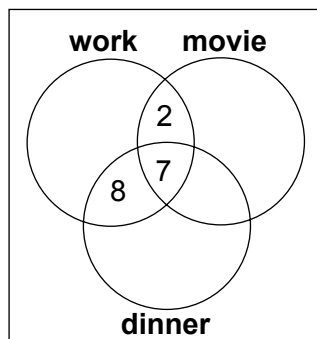
Let's fill this in step by step.



Exactly 7 seniors planned to do *all three* activities. The 7 goes in the *middle*.

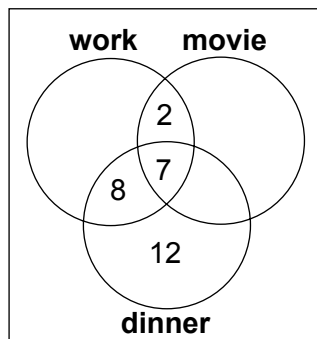


There are 2 seniors who were going to work *and* to a movie, but **not** dinner.



There are 15 who were going to work **and** going out to dinner. *And* means *intersection*, which means the football shape where work and dinner *overlap*.

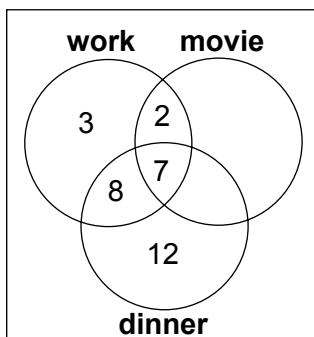
Since that football shape already contains 7, we subtract  $15 - 7 = 8$  to fill in the rest of the football shape.



There are 12 who were planning to do dinner *only*. That means they are in the dinner circle but *not* in any of the *overlapping* parts.



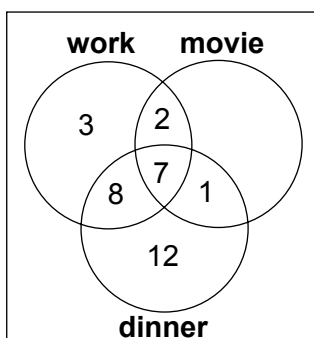
Now go back to the *broader clues*.



There are **20** who planned to work. This means the *entire work circle must contain 20 people*.

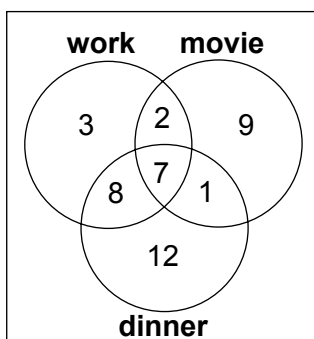
There are already  $7 + 8 + 2$  in the work circle.

So  $20 - (7 + 8 + 2) = 20 - 17 = 3$ . There are **3** who are *only* going to work.



There are **19** who planned to see a *movie*, but there are two spaces for the rest of the students. So let's look at those who were going to dinner. There are **28** *dinner folks*. Our diagram shows  $12 + 8 + 7$  already in the dinner circle.

So the empty space in the dinner circle will be  $28 - (12 + 8 + 7) = 1$ .



Now we can go back to the *moviegoers*. There are **19** who planned to see a movie.

So  $19 - (2 + 7 + 1) = 9$ .



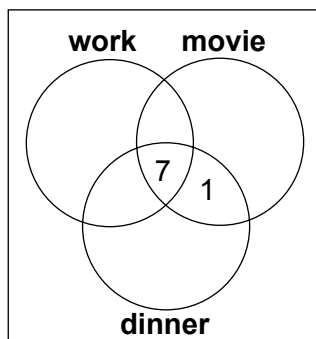
Now we have everything filled in and can answer the questions.

1. How many seniors answered the questionnaire?

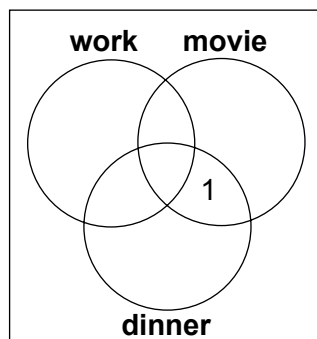
Count each number in the diagram only once and add them together.

$$3 + 2 + 9 + 8 + 7 + 1 + 12 = 42$$

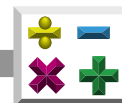
2. How many seniors were going to dinner and a movie, but *not* work?



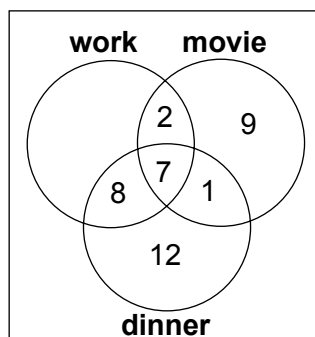
Dinner **and** movie =  $7 + 1 = 8$   
(in football shape)



Delete those in the work part of the  
football shape  
 $8 - 7 = 1$



3. How many seniors were going to dinner **or** a movie?



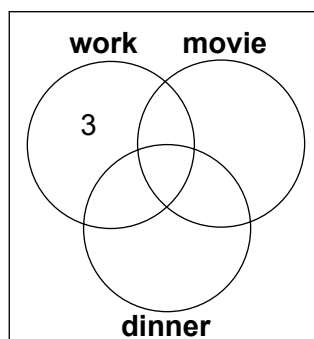
Dinner **or** a movie



**Remember:** This means union. Be careful *not* to count anyone twice!

$$2 + 9 + 8 + 7 + 1 + 12 = 39$$

4. How many seniors were only planning to work?



There were 3 students who were in the work circle *without* overlapping into the other circles.

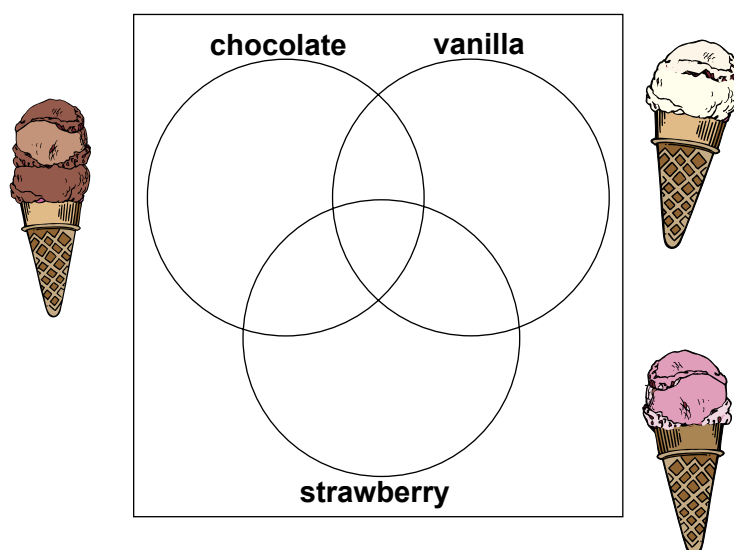
So 3 students planned to work only.



## Practice

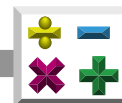
Use the **Venn diagram** below to answer the following.

Jen and Berry's Ice Cream store had 110 customers yesterday. There were 62 customers who bought chocolate ice cream, 38 who chose vanilla, and 41 who chose strawberry ice cream. Another 13 chose chocolate and strawberry. Then 20 chose strawberry *only*, 16 chose chocolate and vanilla, and 7 chose *all three*.



1. How many bought *no* ice cream?  
\_\_\_\_\_
2. How many chose vanilla *only*?  
\_\_\_\_\_
3. How many chose chocolate *or* vanilla or both?  
\_\_\_\_\_
4. How many chose strawberry *or* vanilla or both, but *not* chocolate?  
\_\_\_\_\_



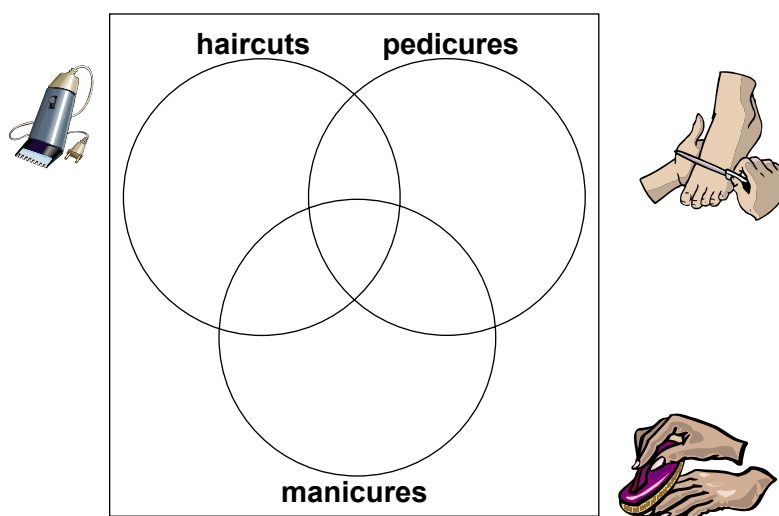


## Practice

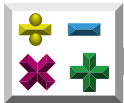
Use the **Venn diagram** below to answer the following.

Svetlana owns a day spa. At the end of the day, the tabulation indicated that clients visited for the following reasons: Haircuts, 62; pedicures, 28; manicures, 41; *all three*, 5; haircut *and* pedicure, 13; manicure *and* haircut *only*, 6; manicure *only*, 25.

**Note:** Assume everyone who visited the spa had one of the procedures.



1. How many had manicures *and* pedicures?  
\_\_\_\_\_
2. How many had manicures *or* pedicures or both?  
\_\_\_\_\_
3. How many had haircuts *and* manicures?  
\_\_\_\_\_
4. How many had haircuts *and* pedicures but *not* manicures?  
\_\_\_\_\_



5. How many had *only* a pedicure?

---

6. How many clients visited the salon on this day?

---



## Unit Review

Answer the following.

1. Express the set of integers *greater than 5 and less than 12* in *roster* format.

---

2. Express the set containing eyes, eyebrows, nose, mouth, and chin in *rule* format.

---

3.  $\{6, 8, 10\} \cup \{10, 12, 14\}$

---

4.  $\{6, 8, 10\} \cap \{10, 12, 14\}$

---

5.  $\{1, 3, 6\} \cup \{1, 2, 3, 4\}$

---

6.  $\{1, 3, 6\} \cap \{1, 2, 3, 4\}$

---

7.  $\{5, 7, 8\} \cup \{\}$

---

8.  $\{5, 7, 8\} \cap \{\}$

---

9.  $\{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$

---



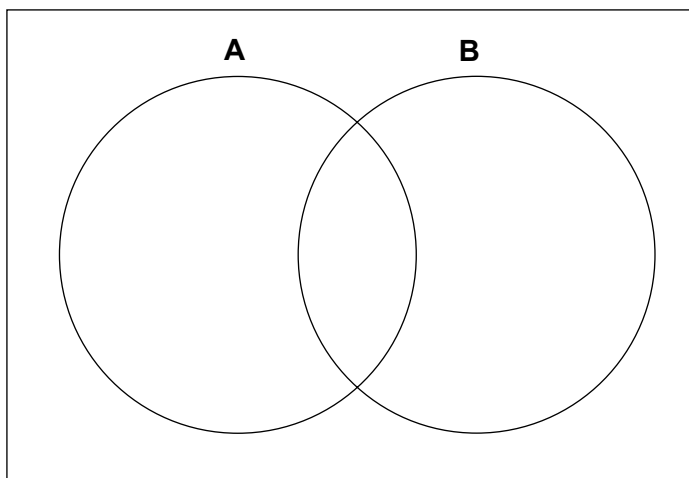
10.  $\{1, 2, 3, 4\} \cap \{5, 6, 7, 8\}$

---

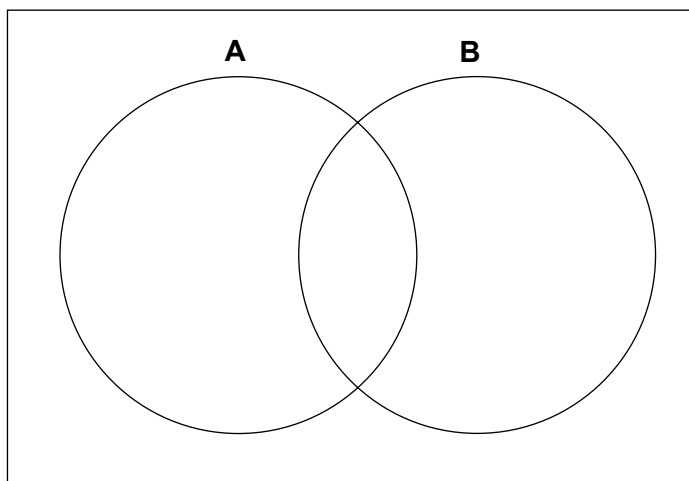
Use the **Venn diagrams** below to illustrate the following **sets**.

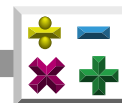
$$A = \{2, 4, 6, 9, 12\} \quad B = \{2, 4, 5, 6, 7, 8, 10\}$$

11.  $A \cup B$



12.  $A \cap B$





*Answer the following.*

13. With respect to  $A \{5, 10, 15, 20, 25, 30\}$  find the complement of  $B \{10, 20, 30\}$

---

14.  $\{2, 4, 6, 8, 10\} - \{2, 4\}$

---

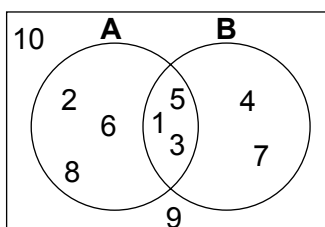
15.  $\{1, 2, 3, 4, 5\} - \{ \}$

---

16.  $\{1, 2, 3, 4\} - \{1, 2, 3, 4\}$

---

17. Use the Venn diagram below to give a set in roster format for  $\bar{A}$ .



---



*Answer the following.*

18.  $\{3, 5\} \times \{2, 4, 6\}$

---

19.  $\{1, 2, 5\} \times \{0, 4\}$

---

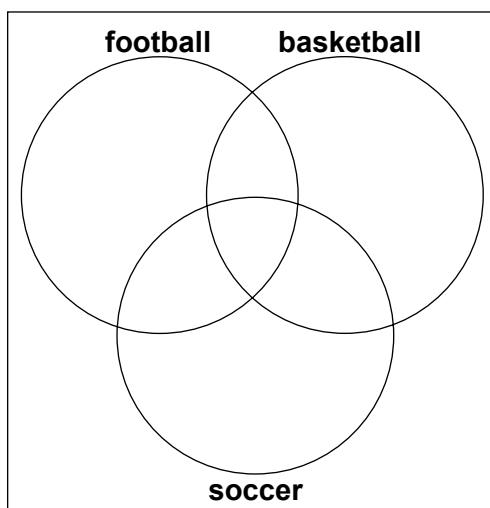
20.  $\{4, 8\} \times \{2, 6, 8\}$

---



Use the **Venn diagram** below to answer the following.

Of 74 boys in a school, the numbers out for a sport or sports were as follows: football, 48; basketball, 20; soccer, 30; football and soccer, 10; basketball and football, 11; soccer and basketball, 8; all three, 3.



21. How many were not out for any sport?
- \_\_\_\_\_
22. How many were out for football but not soccer?
- \_\_\_\_\_
23. How many were out for soccer and basketball but not football?
- \_\_\_\_\_
24. How many play basketball only?
- \_\_\_\_\_
25. How many play football or soccer?
- \_\_\_\_\_





## **Unit 8: Is There a Point to This?**

This unit uses algebraic concepts along with the rules related to radical expressions to explore the coordinate plane.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Writing Process Strand**

##### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

### **Algebra Body of Knowledge**

##### **Standard 1: Real and Complex Number Systems**

- MA.912.A.1.8  
Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.7  
Rewrite equations of a line into slope-intercept form and standard form.
- MA.912.A.3.8  
Graph a line given any of the following information: a table of values, the  $x$ - and  $y$ -intercepts, two points, the slope and a point, the equation of the line in slope-intercept form, standard form, or point-slope form.
- MA.912.A.3.9  
Determine the slope,  $x$ -intercept, and  $y$ -intercept of a line given its graph, its equation, or two points on the line.
- MA.912.A.3.10  
Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line.

### **Standard 5: Rational Expressions and Equations**

- MA.912.A.5.1  
Simplify algebraic ratios.

### **Standard 6: Radical Expressions and Equations**

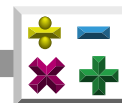
- MA.912.A.6.1  
Simplify radical expressions.
- MA.912.A.6.2  
Add, subtract, multiply and divide radical expressions (square roots and higher).

## **Geometry Body of Knowledge**

### **Standard 1: Points, Lines, Angles, and Planes**

- MA.912.G.1.4  
Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

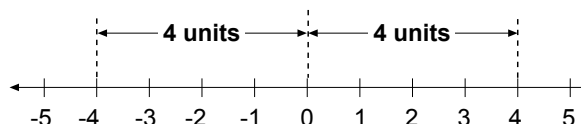




## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

**absolute value** .....a number's distance from zero (0) on a number line; distance expressed as a positive value  
*Example:* The absolute value of both 4, written  $|4|$ , and negative 4, written  $|-4|$ , equals 4.



**common denominator** .....a common multiple of two or more denominators  
*Example:* A common denominator for  $\frac{1}{4}$  and  $\frac{5}{6}$  is 12.

**constant** .....a quantity that always stays the same

**coordinate grid or plane** ...a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced; especially designed for locating points, displaying data, or drawing maps

**coordinate plane** .....the plane containing the  $x$ - and  $y$ -axes

**coordinates** .....numbers that correspond to points on a coordinate plane in the form  $(x, y)$ , or a number that corresponds to a point on a number line

**degree ( $^{\circ}$ )** .....common unit used in measuring angles

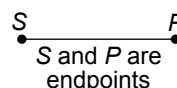


**denominator** ..... the bottom number of a fraction, indicating the number of equal parts a whole was divided into

*Example:* In the fraction  $\frac{2}{3}$  the denominator is 3, meaning the whole was divided into 3 equal parts.

**distance** ..... the length of a segment connecting two points

**endpoint** ..... either of two points marking the end of a line segment



**equation** ..... a mathematical sentence stating that the two expressions have the same value

*Example:*  $2x = 10$

**expression** ..... a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables

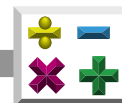
*Examples:*  $4r^2$ ;  $3x + 2y$ ;  $\sqrt{25}$

An expression does *not* contain equal (=) or inequality (<, >, ≤, ≥, or ≠) signs.

**factor** ..... a number or expression that divides evenly into another number; one of the numbers multiplied to get a product

*Examples:* 1, 2, 4, 5, 10, and 20 are factors of 20 and  $(x + 1)$  is one of the factors of  $(x^2 - 1)$ .

**formula** ..... a way of expressing a relationship using variables or symbols that represent numbers



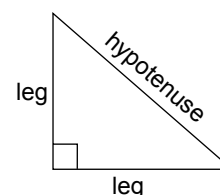
**graph** ..... a drawing used to represent data  
*Example:* bar graphs, double bar graphs, circle graphs, and line graphs

**graph of a point** ..... the point assigned to an ordered pair on a coordinate plane

**horizontal** ..... parallel to or in the same plane of the horizon



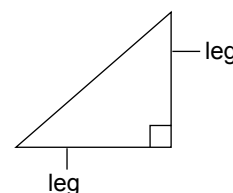
**hypotenuse** ..... the longest side of a right triangle; the side opposite the right angle



**integers** ..... the numbers in the set  
 $\{ \dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots \}$

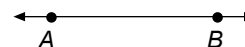
**intersect** ..... to meet or cross at one point

**leg** ..... in a right triangle, one of the two sides that form the right angle



**length ( $l$ )** ..... a one-dimensional measure that is the measurable property of line segments

**line ( $\leftrightarrow$ )** ..... a collection of an infinite number of points forming a straight path extending in opposite directions having unlimited length and no width



**linear equation** ..... an algebraic equation in which the variable quantity or quantities are raised to the zero or first power and the graph is a straight line  
*Example:*  $20 = 2(w + 4) + 2w$ ;  $y = 3x + 4$



**line segment (—)** ..... a portion of a line that consists of two defined endpoints and all the points in between  
*Example:* The line segment  $AB$  is between point  $A$  and point  $B$  and includes point  $A$  and point  $B$ .

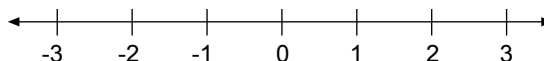


**midpoint**  
**(of a line segment)** ..... the point on a line segment equidistant from the endpoints

**negative integers** ..... integers less than zero

**negative numbers** ..... numbers less than zero

**number line** ..... a line on which ordered numbers can be written or visualized

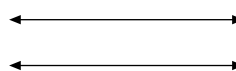


**numerator** ..... the top number of a fraction, indicating the number of equal parts being considered  
*Example:* In the fraction  $\frac{2}{3}$ , the numerator is 2.

**ordered pair** ..... the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the  $x$ -axis and  $y$ -axis, respectively  
*Examples:*  $(x, y)$  or  $(3, -4)$

**parallel (||)** ..... being an equal distance at every point so as to never intersect

**parallel lines** ..... two lines in the same plane that are a constant distance apart; lines with equal slopes









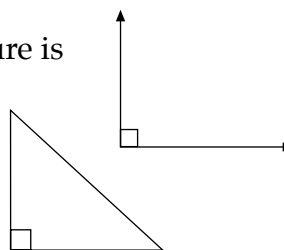
**radical sign** ( $\sqrt{\quad}$ ) ..... the symbol ( $\sqrt{\quad}$ ) used before a number to show that the number is a *radicand*

**radicand** ..... the number that appears within a radical sign  
*Example:* In  $\sqrt{25}$ , 25 is the radicand.

**reciprocals** ..... two numbers whose product is 1; also called *multiplicative inverses*  
*Examples:* 4 and  $\frac{1}{4}$  are reciprocals because  $\frac{4}{1} \times \frac{1}{4} = 1$ ;  $\frac{3}{4}$  and  $\frac{4}{3}$  are reciprocals because  $\frac{3}{4} \times \frac{4}{3} = 1$ ; zero (0) has no multiplicative inverse

**right angle** ..... an angle whose measure is exactly  $90^\circ$

**right triangle** ..... a triangle with one right angle



**rise** ..... the vertical change on a graph between two points

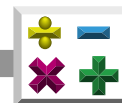
**root** ..... an equal factor of a number

*Examples:*

In  $\sqrt{144} = 12$ , 12 is the square root.

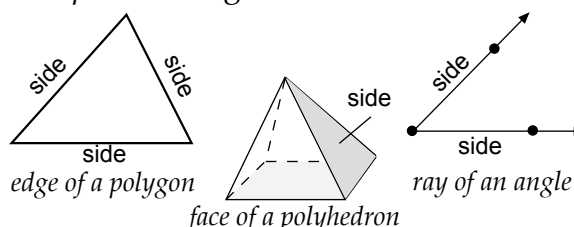
In  $\sqrt[3]{125} = 5$ , 5 is the cube root.

**run** ..... the horizontal change on a graph between two points



**side** .....the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle

*Example:* A triangle has three sides.



**simplest radical form** .....an expression under the radical sign that contains no perfect squares greater than 1, contains no fractions, and is not in the denominator of a fraction

*Example:*  $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$

**simplify a fraction** .....write fraction in lowest terms or simplest form

**slope** .....the ratio of change in the vertical axis ( $y$ -axis) to each unit change in the horizontal axis ( $x$ -axis) in the form  $\frac{\text{rise}}{\text{run}}$  or  $\frac{\Delta y}{\Delta x}$ ; the constant,  $m$ , in the linear equation for the slope-intercept form  $y = mx + b$

**slope-intercept form** .....a form of a linear equation,  $y = mx + b$ , where  $m$  is the slope of the line and  $b$  is the  $y$ -intercept

**square (of a number)** .....the result when a number is multiplied by itself or used as a factor twice  
*Example:* 25 is the square of 5.

**square root** .....a positive real number that can be multiplied by itself to produce a given number  
*Example:* The square root of 144 is 12 or  $\sqrt{144} = 12$ .



**standard form**

(of a linear equation) .....  $ax + by + c = 0$ , where  $a$ ,  $b$ , and  $c$  are integers and  $a > 0$

**sum** ..... the result of adding numbers together  
*Example:* In  $6 + 8 = 14$ , the sum is 14.

**triangle** ..... a polygon with three sides



**value (of a variable)** ..... any of the numbers represented by the variable

**variable** ..... any symbol, usually a letter, which could represent a number

**vertical** ..... at right angles to the horizon; straight up and down



**$x$ -axis** ..... the horizontal number line on a rectangular coordinate system

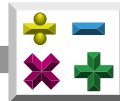
**$x$ -coordinate** ..... the first number of an ordered pair

**$x$ -intercept** ..... the value of  $x$  at the point where a line or graph intersects the  $x$ -axis; the value of  $y$  is zero (0) at this point

**$y$ -axis** ..... the vertical number line on a rectangular coordinate system

**$y$ -coordinate** ..... the second number of an ordered pair

**$y$ -intercept** ..... the value of  $y$  at the point where a line or graph intersects the  $y$ -axis; the value of  $x$  is zero (0) at this point



## Unit 8: Is There a Point to This?

### Introduction

We will explore the relationships that exist between points, segments, and lines on a coordinate plane. Utilizing the formulas for finding distance, midpoint, slope, and equations of lines, we can identify the ways in which points and lines are related to each other.

### Lesson One Purpose

#### Reading Process Strand

##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### Writing Process Strand

##### Standard 3: Prewriting

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.



## **Algebra Body of Knowledge**

### **Standard 6: Radical Expressions and Equations**

- MA.912.A.6.1  
Simplify radical expressions.
- MA.912.A.6.2  
Add, subtract, multiply and divide radical expressions (square roots and higher).

## **Geometry Body of Knowledge**

### **Standard 1: Points, Lines, Angles, and Planes**

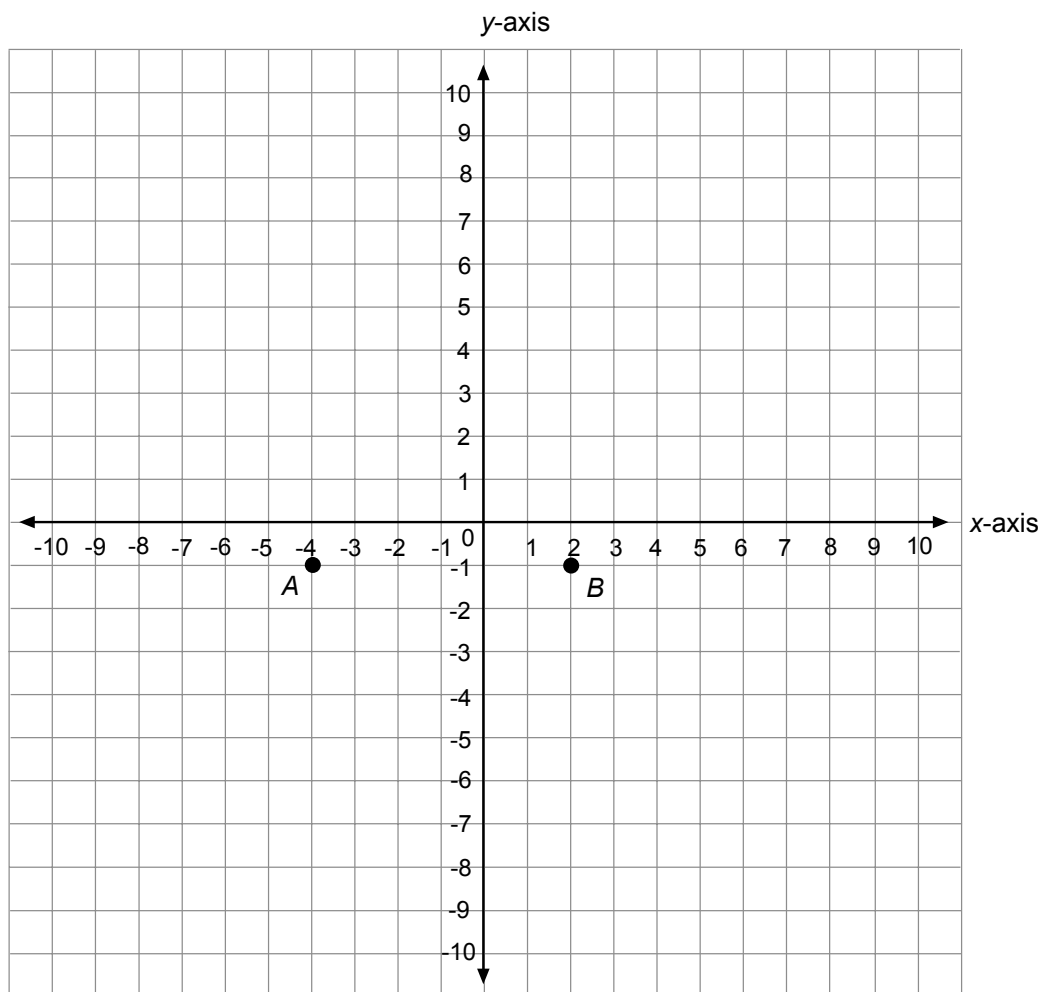
- MA.912.G.1.4  
Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.



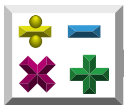
## Distance

Look at the following **coordinate grids** or **planes**. The horizontal number line on a *rectangular coordinate system* is the ***x*-axis**. The vertical line on a coordinate system is the ***y*-axis**. We can easily find the **distance** between the given **graphs of the points** below. The *graph of a point* is the **point** assigned to an **ordered pair** on a *coordinate plane*.

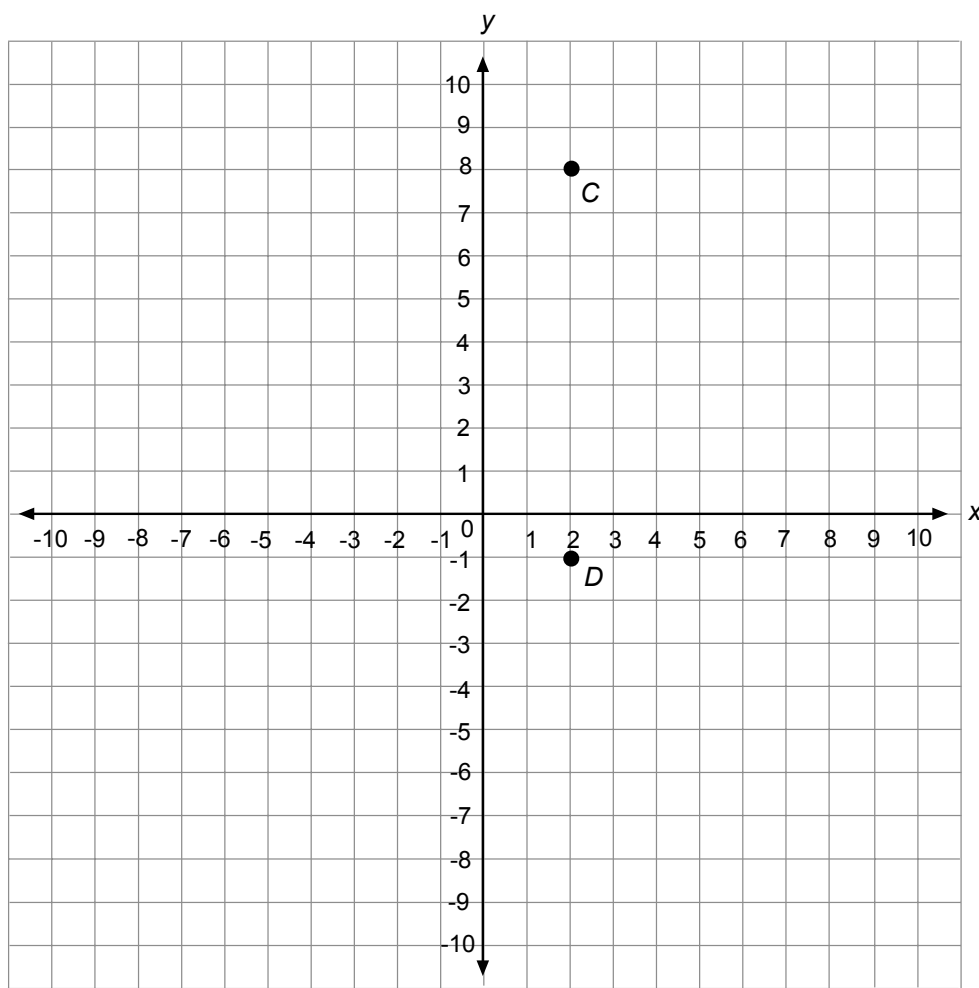
**Graph of Points A and B**



Because the *points* on the **graph** above are on the same **horizontal** ( $\leftrightarrow$ ) **line**, we can count the spaces from one *point* to the other. So, the *distance* from A to B is 6.



Graph of Points *C* and *D*



Because the points on the graph above are on the same **vertical** ( $\updownarrow$ ) *line*, we can count the spaces from one point to the other. So, the distance from *C* to *D* is 9.



**Remember:** Distance is always a **positive number**. Even when you back your car down the driveway, you have covered a *positive* distance. If you get a **negative number**, simply take the **absolute value** of the number.

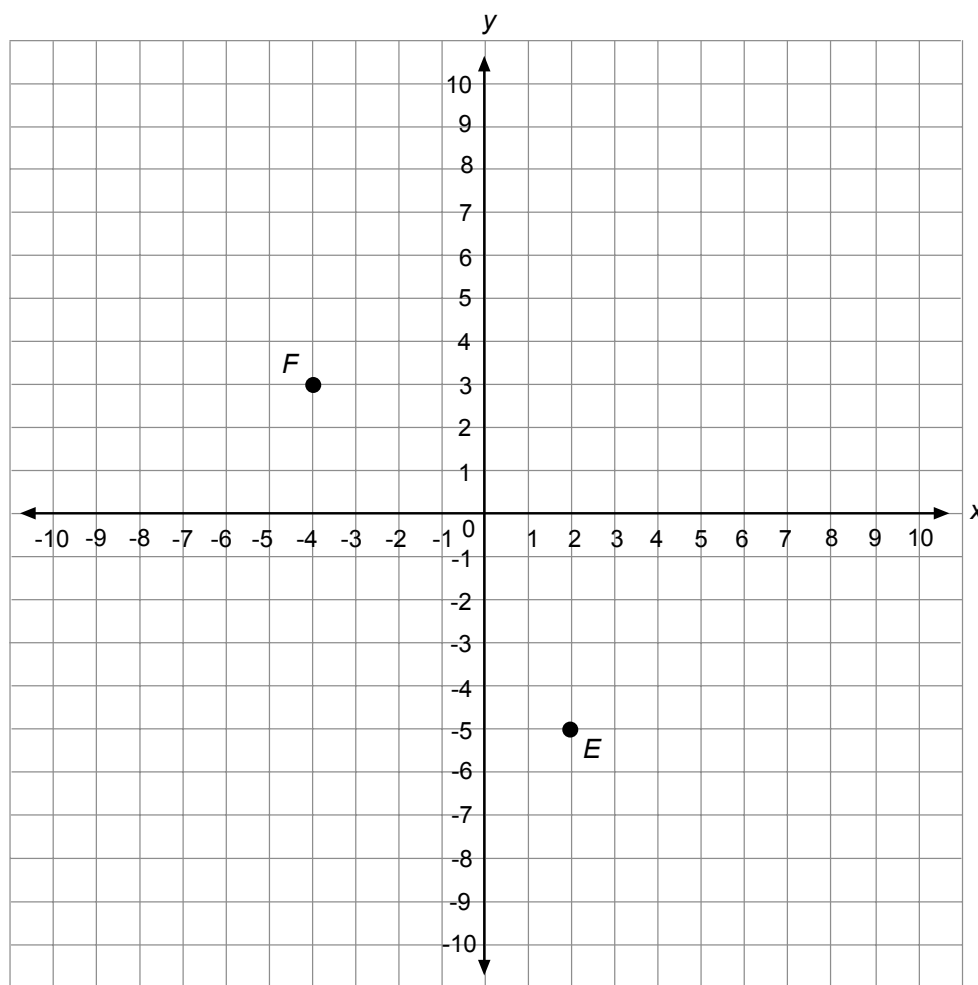




In many instances, the points we need to identify to find the distance between are not on the same *horizontal* or *vertical* line. Because we would have to count points on a *diagonal*, we would not get an accurate measure of the distance between those points. We will examine two methods to determine the distance between any two points.

Look at the graph below. We want to find the distance between point  $E$  (2, -5) and  $F$  (-4, 3).

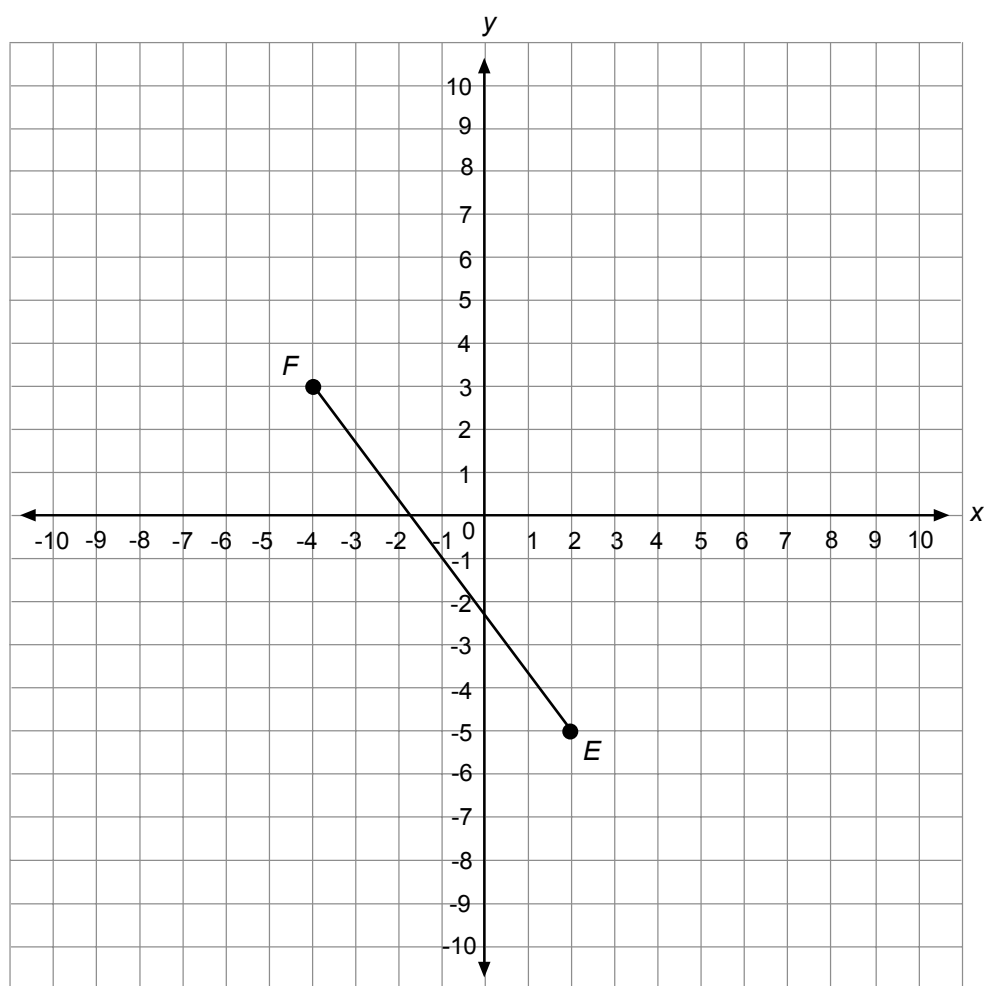
**Graph of Points  $E$  and  $F$**

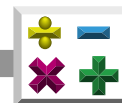




Notice that the distance between  $E$  and  $F$  looks like the **hypotenuse** of a **right triangle**.

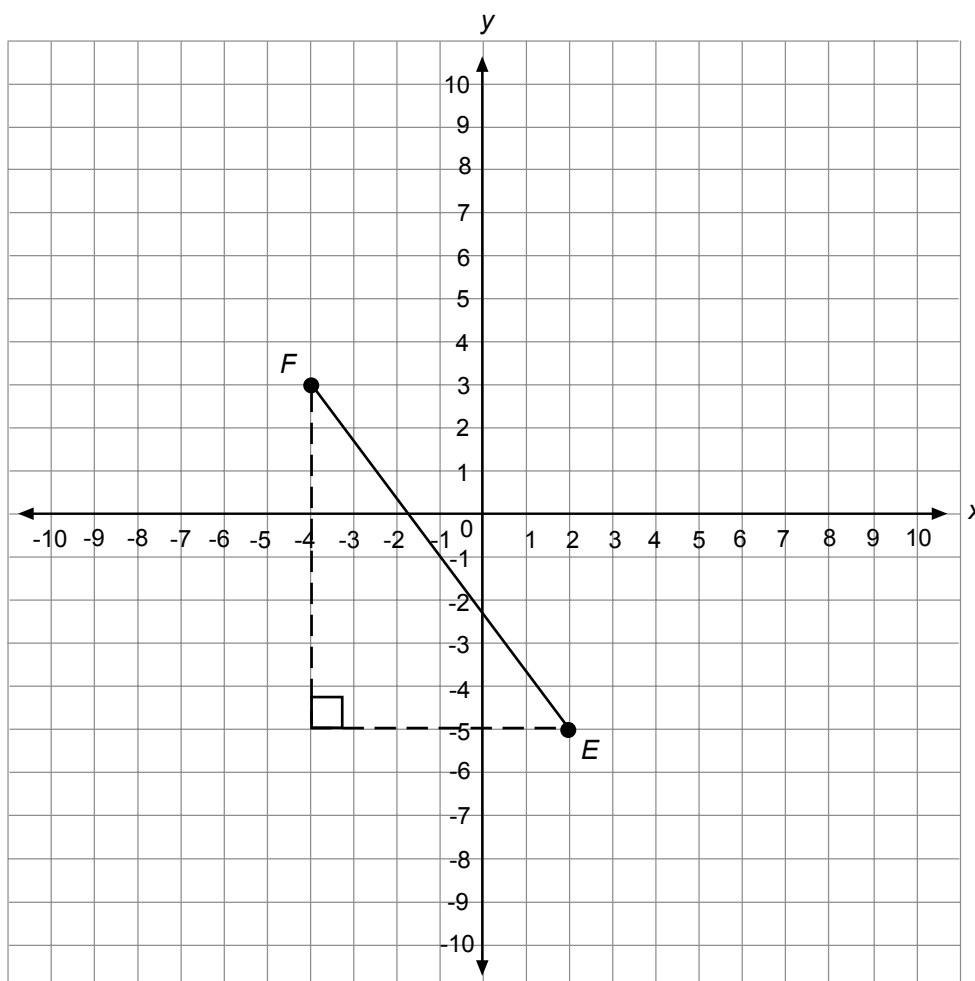
**Graph of Points  $E$  and  $F$**





Let's sketch the rest of the **triangle** and see what happens.

**Graph of Points *E* and *F***



By completing the sketch of the *triangle*, we see that the result is a *right triangle* with one horizontal **side** and one vertical *side*. We can count to find the **lengths** (*l*) of these two sides, and then use the **Pythagorean theorem** to find the distance from *E* to *F*.



**Remember:** The *Pythagorean theorem* is the **square** of the *hypotenuse* (*c*) of a right triangle and is equal to the **sum** of the *squares* of the **legs** (*a* and *b*), as shown in the **equation**  $a^2 + b^2 = c^2$ .

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2 \\ \sqrt{100} &= c \\ 10 &= c \end{aligned}$$



### Remember:

- The opposite of *squaring* a number is called *finding the square root*. For example, the *square root* of 100, or  $\sqrt{100}$ , is 10.
- The square root of a number is shown by the symbol  $\sqrt{\phantom{x}}$ , which is called a **radical sign** or *square root sign*.
- The number underneath is called a **radicand**.  

radical  
sign

 $\rightarrow \sqrt{100} \leftarrow$ 

radicand

**radical**
- The **radical** is an **expression** that has a **root**. A *root* is an equal **factor** of a number.  

$\sqrt{100} = 10$  because  $10^2 = 100$

$\sqrt{9} = 3$  because  $3^2 = 9$
- $\sqrt{100}$  is a **radical expression**. It is a numerical *expression* containing a *radical sign*.  

$\sqrt{121} = 11$  because  $11^2 = 121$

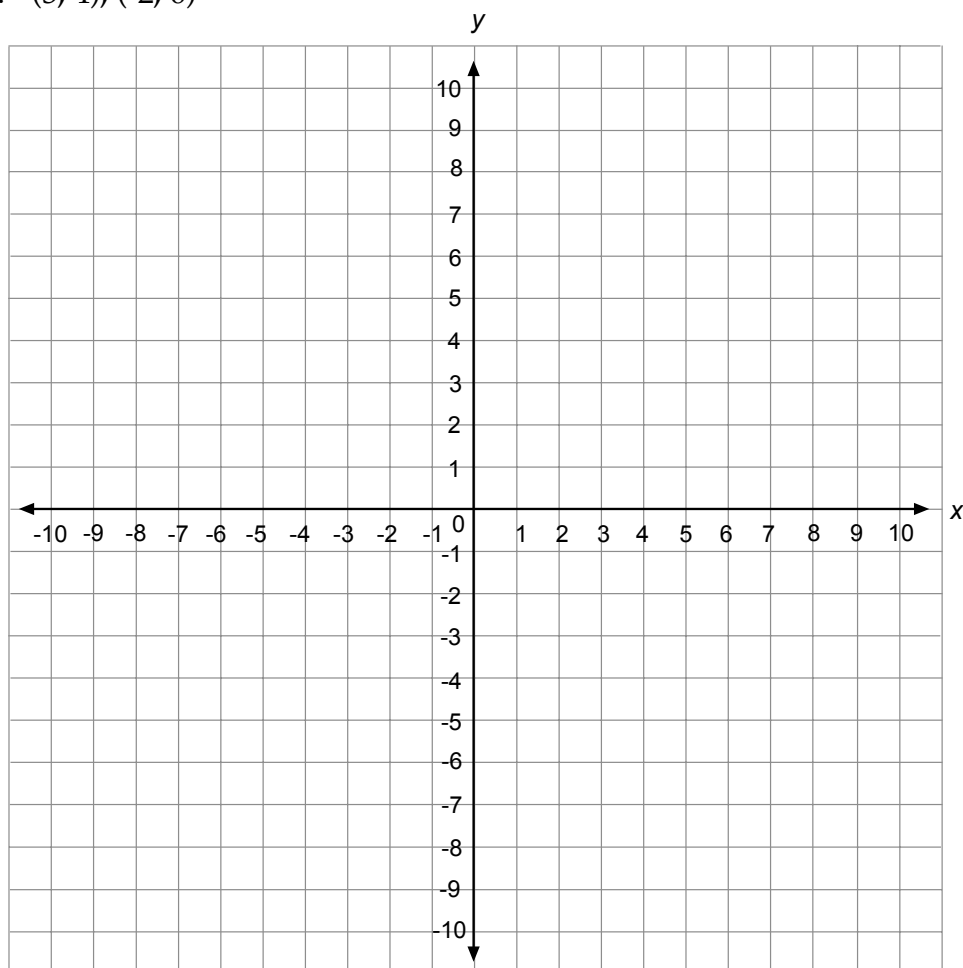


## Practice

*For the following:*

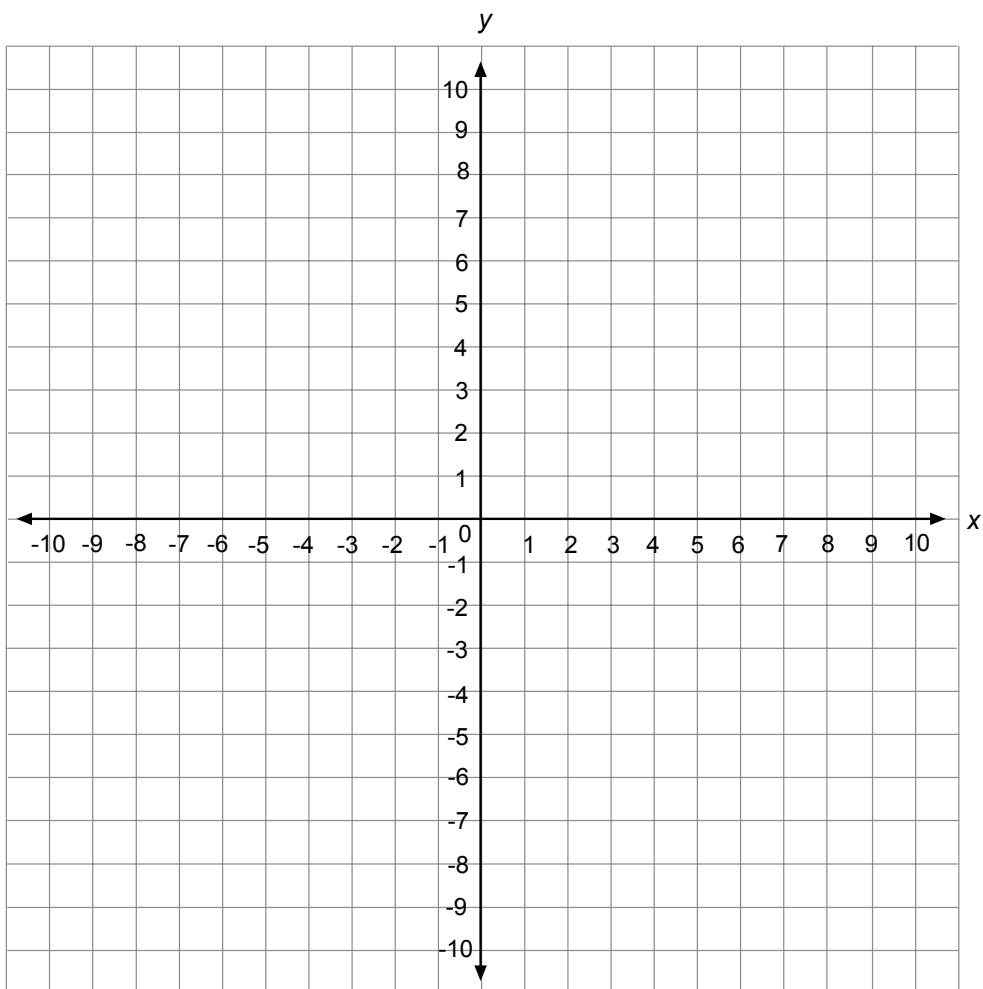
- plot the two points
- draw the hypotenuse
- complete the triangle
- use the Pythagorean theorem to find the distance between the given points
- show all your work
- leave answers in **simplest radical form**.

1.  $(3, 4)$ ,  $(-2, 6)$



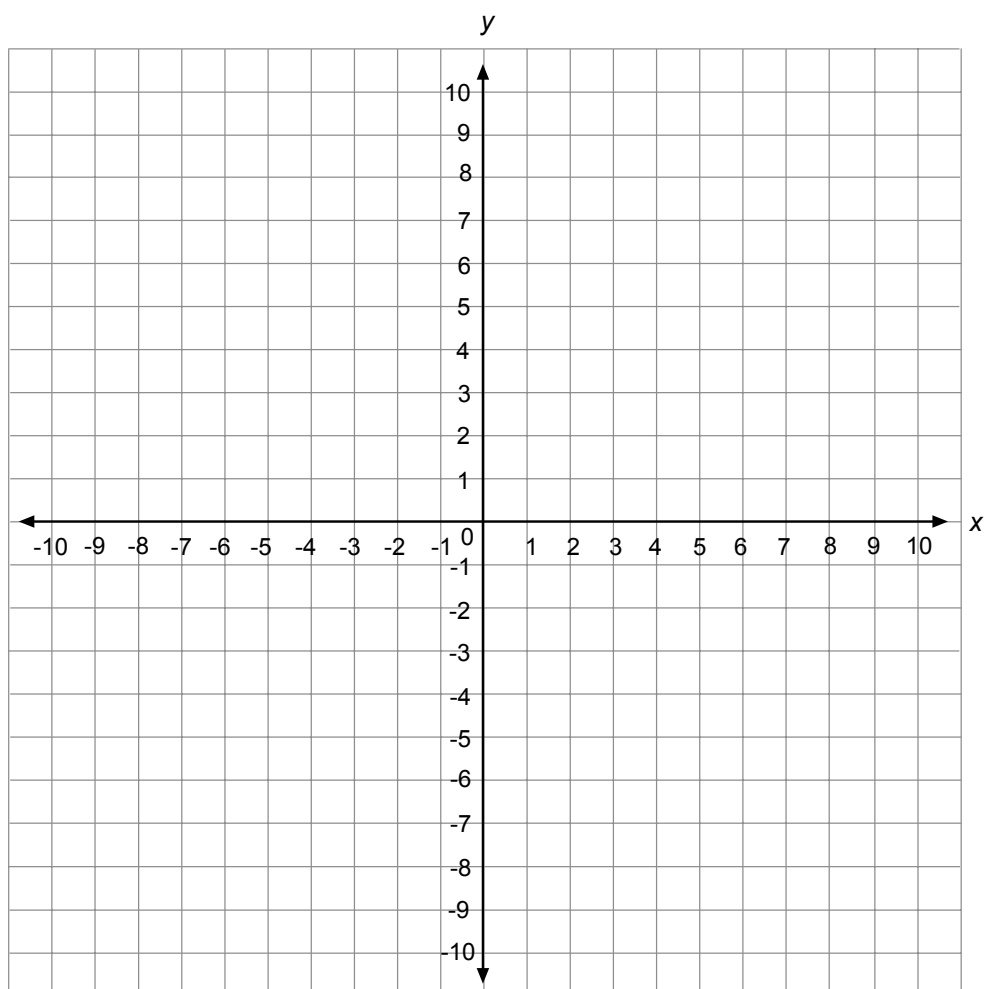


2.  $(3, -3), (6, 4)$



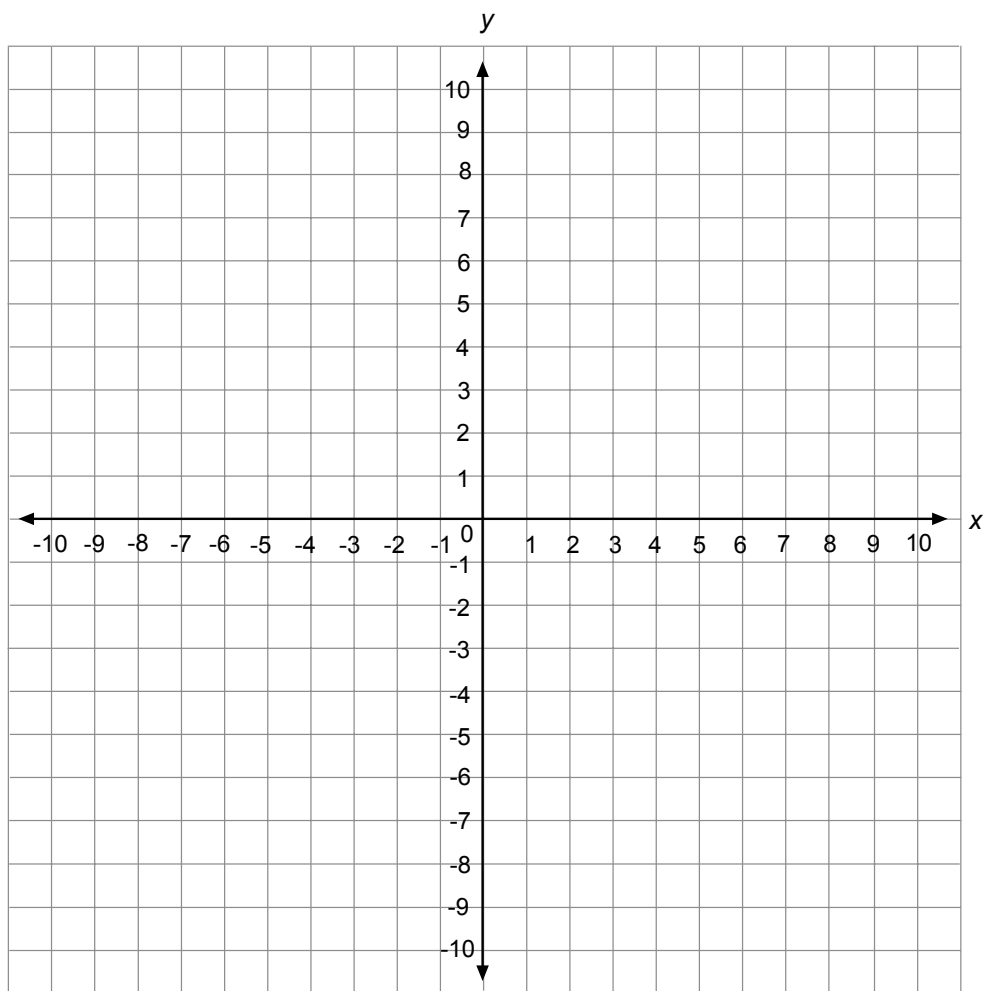


3.  $(-5, 0), (2, 3)$

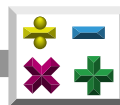




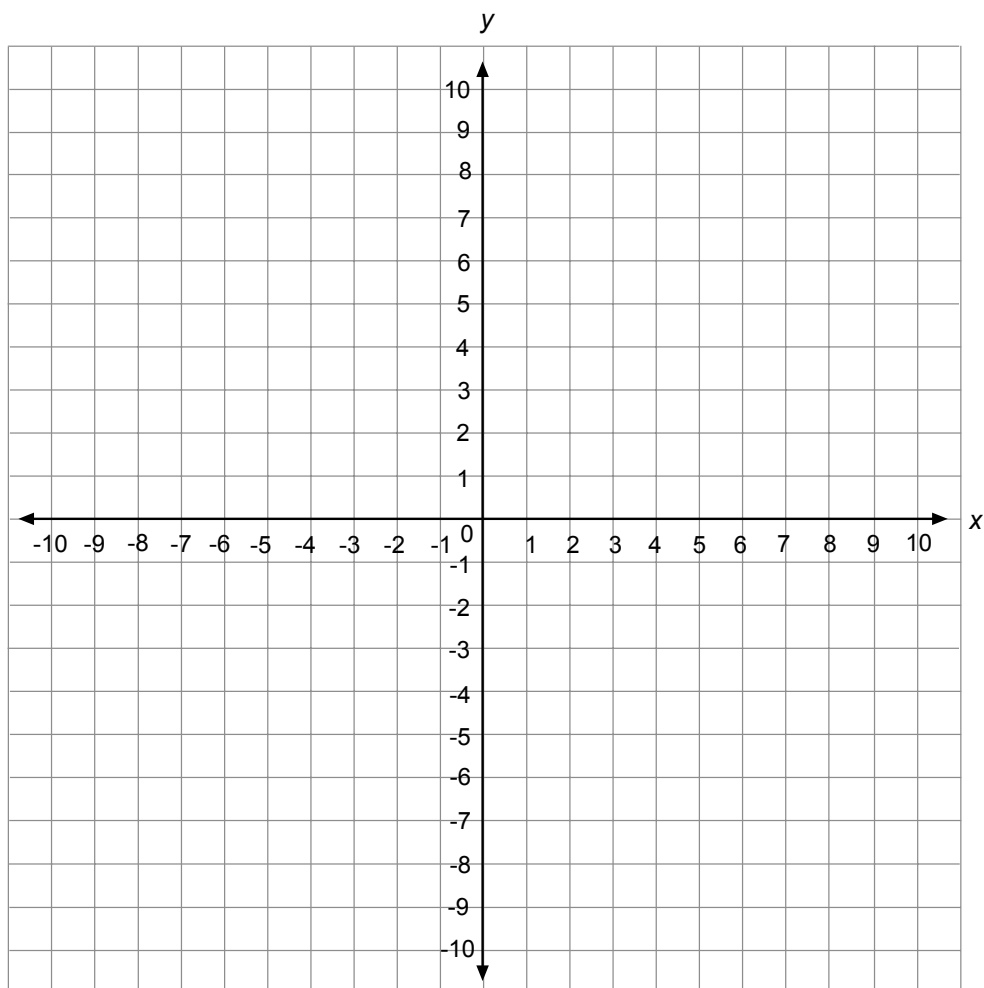
4.  $(4, -3), (-3, 4)$





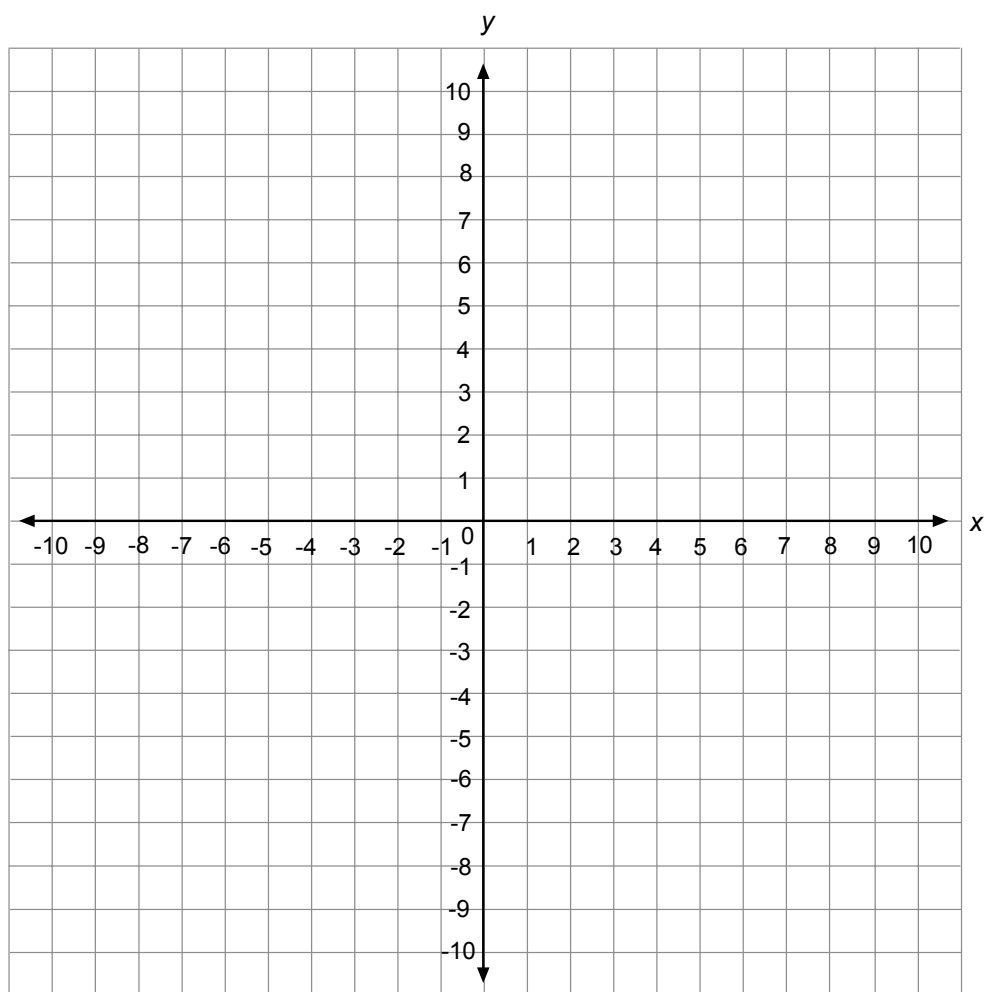


5.  $(0, 2), (-5, 7)$



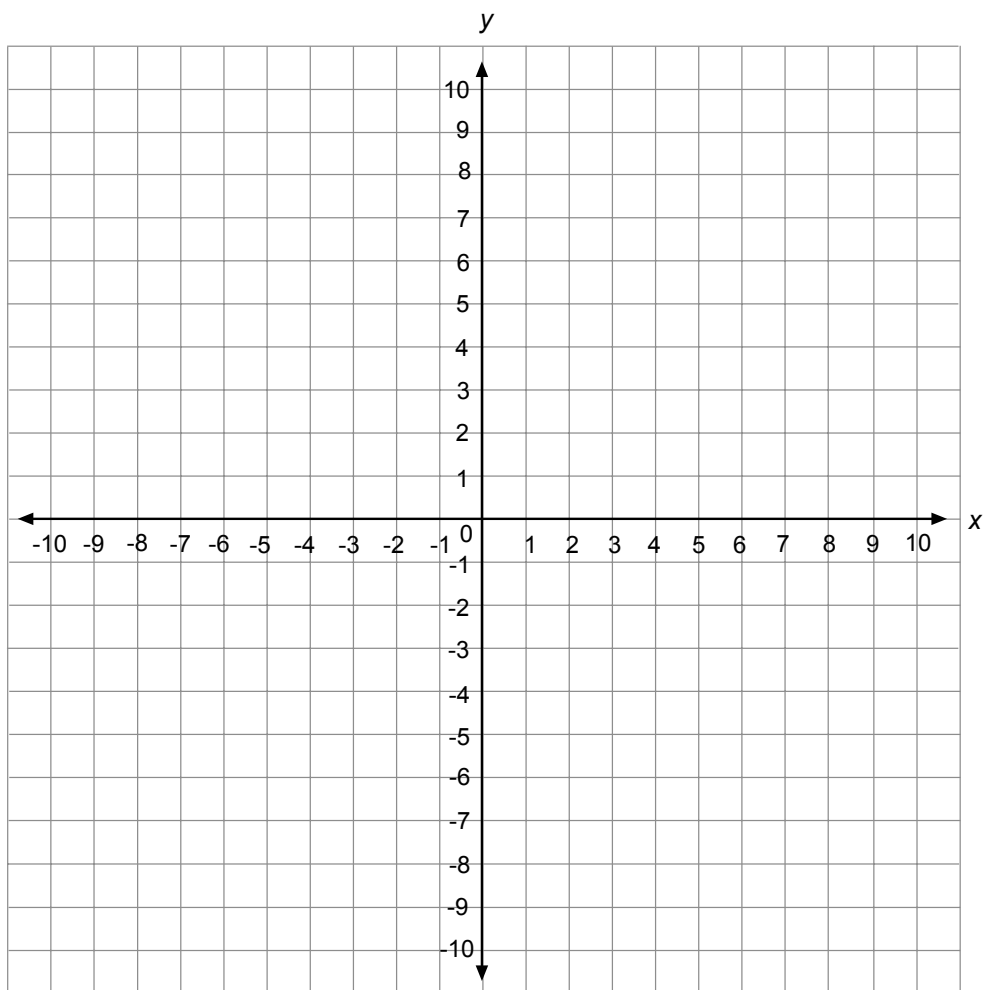


6.  $(2, 2), (-1, -2)$



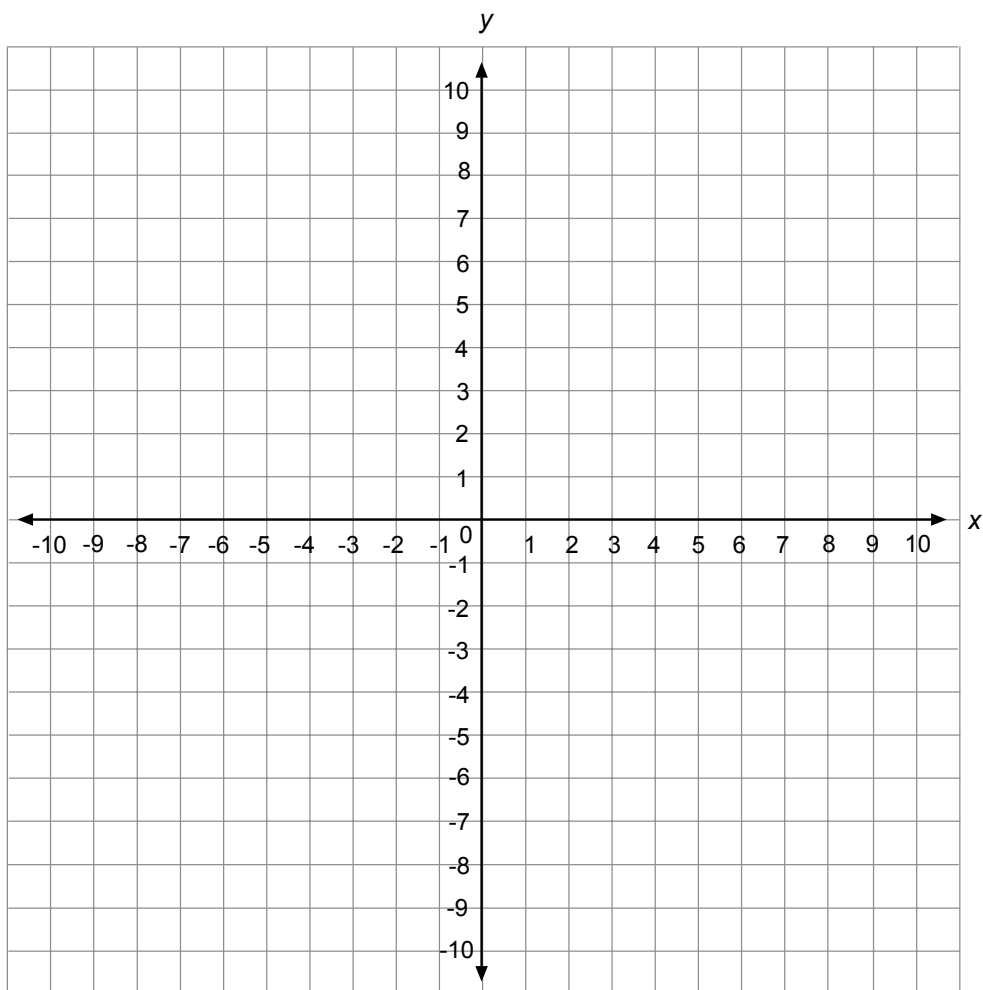


7.  $(0, 0), (-4, 4)$



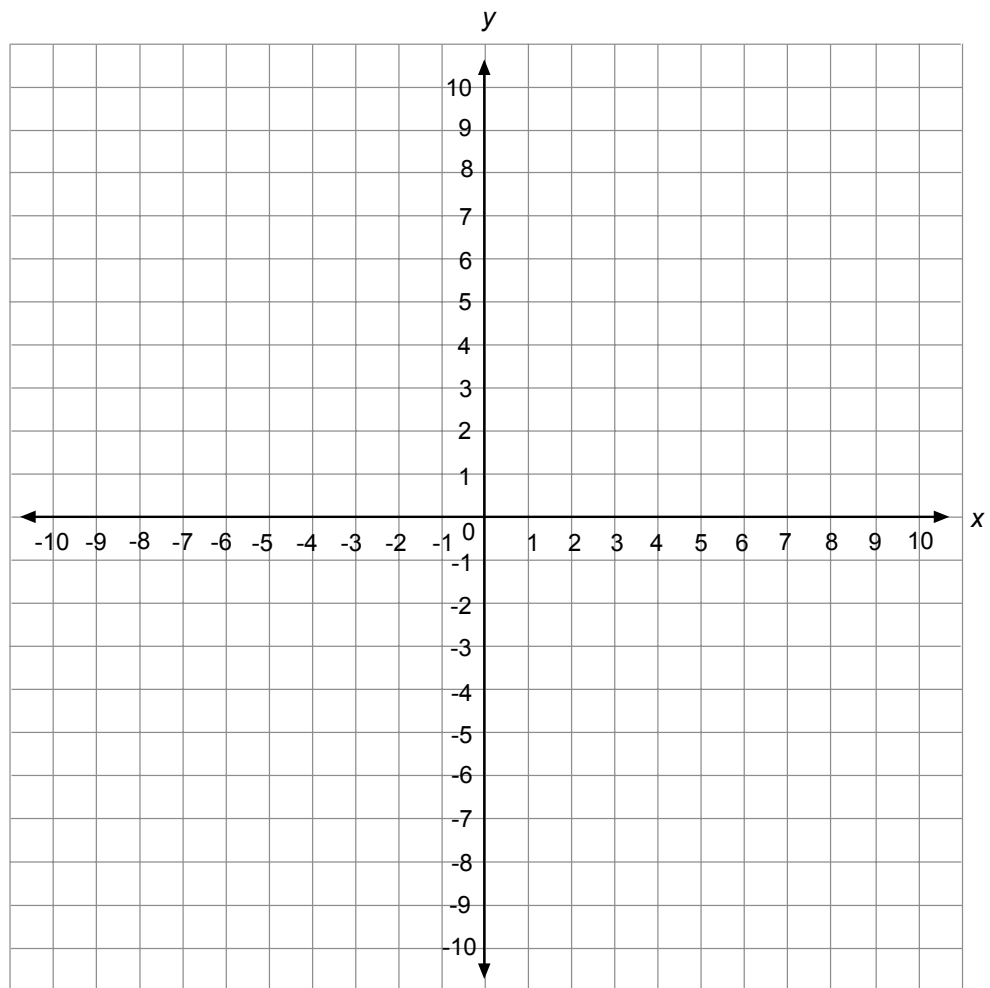


8.  $(3, 5), (-2, -7)$



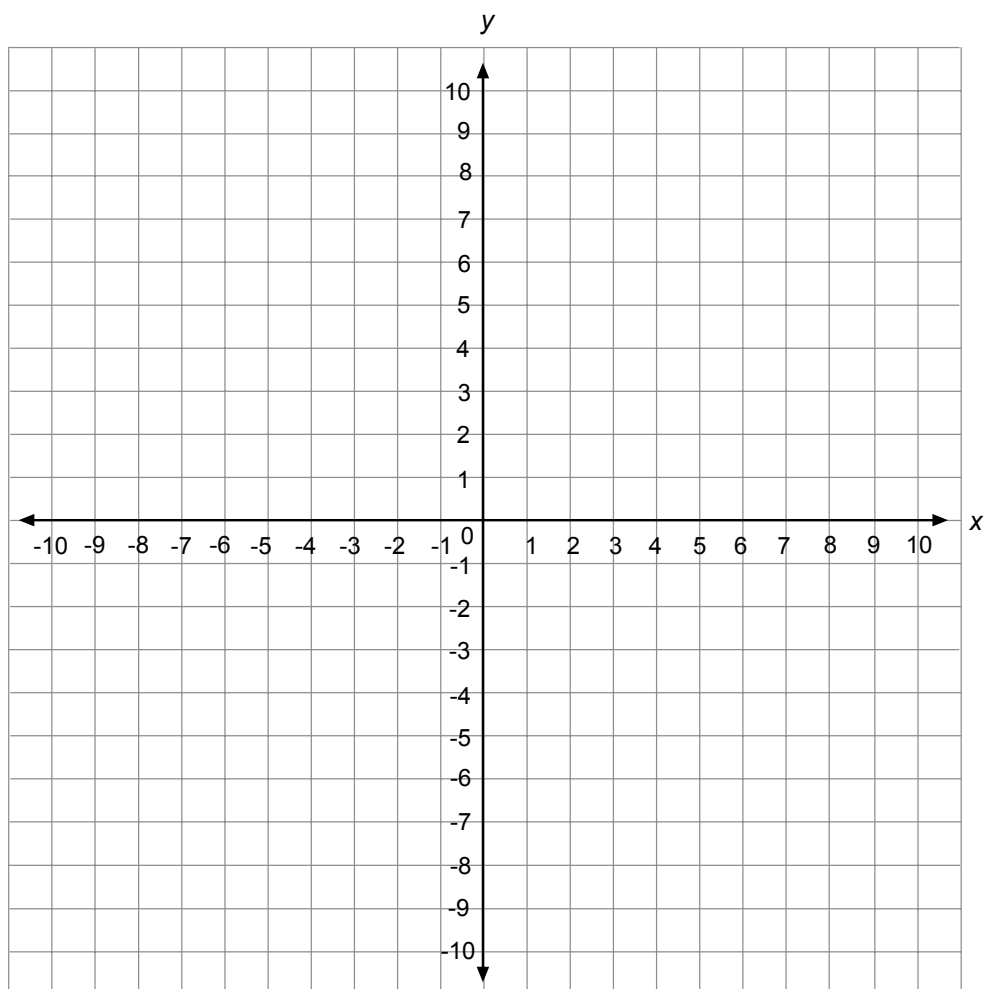


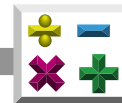
9.  $(6, -7), (-2, 8)$





10.  $(-4, 6), (5, -6)$





## Practice

Use the list below to write the correct term for each definition on the line provided.

<b>absolute value</b>	<b>horizontal</b>	<b>vertical</b>
<b>coordinate grid or plane</b>	<b>negative numbers</b>	<b><math>x</math>-axis</b>
<b>distance</b>	<b>positive numbers</b>	<b><math>y</math>-axis</b>
<b>graph (of a point)</b>		

- \_\_\_\_\_ 1. parallel to or in the same plane of the horizon
- \_\_\_\_\_ 2. the length of a segment connecting two points
- \_\_\_\_\_ 3. at right angles to the horizon; straight up and down
- \_\_\_\_\_ 4. numbers less than zero
- \_\_\_\_\_ 5. a number's distance from zero (0) on a number line
- \_\_\_\_\_ 6. numbers greater than zero
- \_\_\_\_\_ 7. the vertical number line on a rectangular coordinate system
- \_\_\_\_\_ 8. the point assigned to an ordered pair on a coordinate plane
- \_\_\_\_\_ 9. the horizontal number line on a rectangular coordinate system
- \_\_\_\_\_ 10. a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced

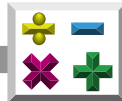


## Practice

*Match each definition with the correct term. Write the letter on the line provided.*

- |       |   |                         |
|-------|---|-------------------------|
| _____ | 1. a one-dimensional measure that is the measurable property of line segments   | A. hypotenuse           |
| _____ | 2. the longest side of a right triangle; the side opposite the right angle  | B. leg                  |
| _____ | 3. the square of the hypotenuse ( $c$ ) of a right triangle is equal to the sum of the square of the legs ( $a$ and $b$ ) | C. length ( $l$ )       |
| _____ | 4. the edge of a polygon  | D. Pythagorean theorem  |
| _____ | 5. a polygon with three sides   | E. right triangle       |
| _____ | 6. a triangle with one right angle  | F. side                 |
| _____ | 7. the result of adding numbers together  | G. square (of a number) |
| _____ | 8. in a right triangle, one of the two sides that form the right angle  | H. sum                  |
| _____ | 9. the result when a number is multiplied by itself or used as a factor twice   | I. triangle             |





## Using the Distance Formula

Sometimes, it is inconvenient to *graph* when finding the distance. So, another method we often use to find the distance between two points is the distance **formula**.

The distance *formula* is as follows.

<p style="text-align: center;"><b>distance formula</b></p> $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
---

The little 1s and 2s that are *subscripts* to the  $x$ 's and  $y$ 's signify that they come from different *ordered pairs*.

### Example

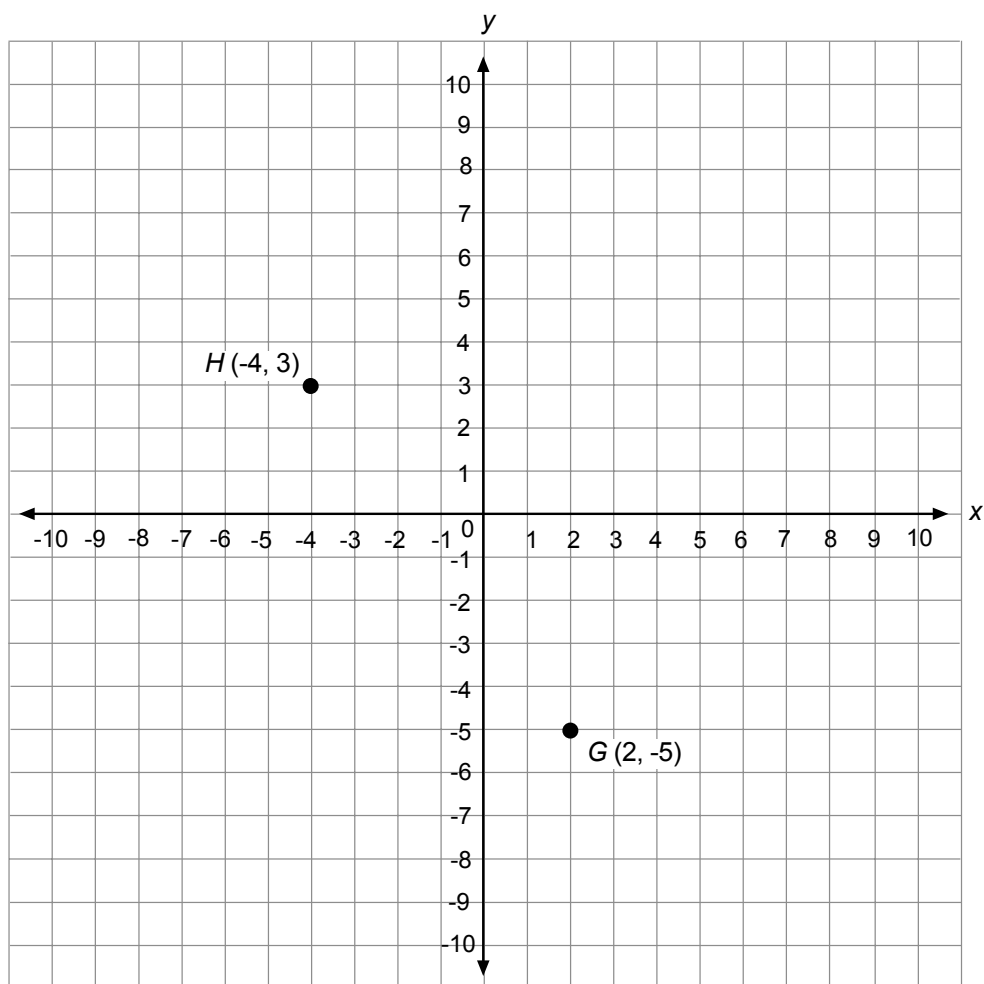
$(x_1, y_1)$  is one *ordered pair* and  $(x_2, y_2)$  is another ordered pair.

**Note:** Be consistent when putting the **values** into the formulas.

Let's look at the same example of  $G(2, -5)$  and  $H(-4, 3)$ , and use the distance formula. See the graph on the following page.



### Graph of Points *G* and *H*



$$\begin{aligned}x_1 &= 2 \\y_1 &= -5 \\x_2 &= -4 \\y_2 &= 3\end{aligned}$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(-4 - 2)^2 + (3 - (-5))^2} =$$

$$\sqrt{(-6)^2 + (8)^2} =$$

$$\sqrt{36 + 64} =$$

$$\sqrt{100} =$$

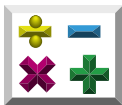
$$10$$



Compare the numbers in the distance formula to the numbers used in the *Pythagorean theorem*.

$$\begin{aligned}a^2 + b^2 &= c^2 \\6^2 + 8^2 &= c^2 \\36 + 64 &= c^2 \\100 &= c^2 \\\sqrt{100} &= c \\10 &= c\end{aligned}$$

You should always get the same answer using either method.



## Practice

Use the distance formula to solve the following. **Show all your work.** Leave answers in **simplest radical form.**

**distance formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1.  $(3, 4), (-2, 6)$

2.  $(3, -3), (6, 4)$

3.  $(-5, 0), (2, 3)$



4.  $(4, -3), (-3, 4)$

5.  $(0, 2), (-5, 7)$

6.  $(2, 2), (-1, -2)$

7.  $(0, 0), (-4, 4)$



8.  $(3, 5), (-2, -7)$

9.  $(6, -7), (-2, 8)$

10.  $(-4, 6), (5, -6)$



**Check yourself:** Compare your answers to the practice on pages 517-526. Do they match? If not, rework until both sets of practice answers match.



## Practice

Use your **favorite of the two methods** shown on pages 529-531. One method uses the **distance formula** and the other method uses the **Pythagorean theorem**. Find the **distance between each pair of points** below using either method. Refer to the examples on pages 529-531 as needed.

**Show all your work.** Leave answers in **simplest radical form**.

**distance formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

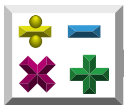
**Pythagorean theorem**

$$a^2 + b^2 = c^2$$

1.  $(0, 0), (-3, 4)$

2.  $(5, -6), (6, -5)$

3.  $(-5, -8), (3, 7)$



4.  $(-2, -8), (0, 0)$

5.  $(6, 6), (-3, -3)$

6.  $(-1, 2), (5, 10)$





## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

### **Algebra Body of Knowledge**

#### **Standard 5: Rational Expressions and Equations**

- MA.912.A.5.1  
Simplify algebraic ratios.



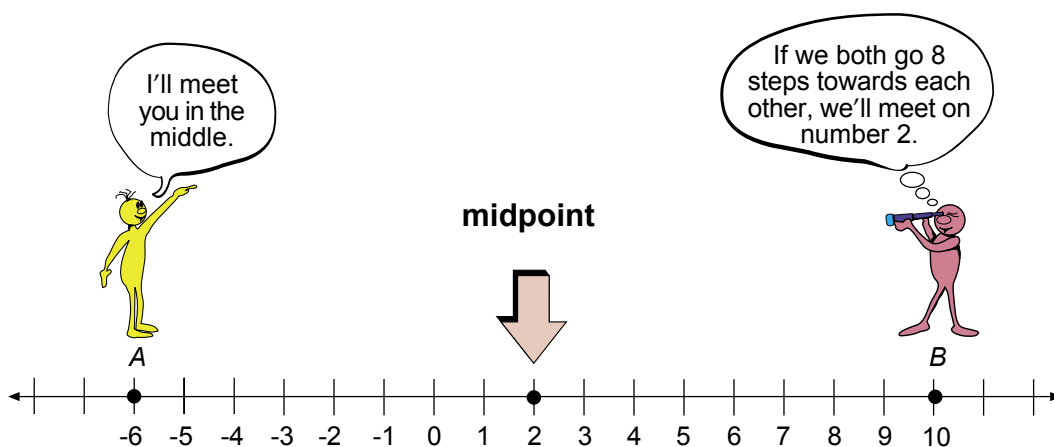
## Midpoint

Sometimes it is necessary to find the *point* that is exactly in the middle of two given **endpoints**. We call this the **midpoint (of a line segment)**. What we are actually trying to find are the **coordinates** of that point, which is like the *address* of the point, or its *location* on a coordinate plane or a **number line**.

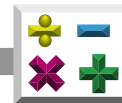
### Finding the Midpoint of a Line Segment Using a Number Line

You can find the *midpoint* of a **line segment** ( $\text{---}$ ), also called a *segment*, in a couple of different ways. One way is to use a *number line*.

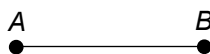
On a number line, you can find the midpoint of a *line segment* by counting in from both *endpoints* until you reach the middle.



*how to use a number line to find the midpoint of a line segment*



**Remember:** If we draw a *line segment* from one point to another, we can call it line segment  $\overline{AB}$  or *segment*  $AB$ . See a representation of line segment  $AB$  ( $\overline{AB}$ ) below. The symbol ( $\overline{\quad}$ ) drawn over the two uppercase letters describes a line segment. The symbol has no arrow because the line segment has a definite beginning and end called endpoints.  $A$  and  $B$  are endpoints of the line segment  $AB$  ( $\overline{AB}$ ).



On the other hand, the symbol ( $\longleftrightarrow$ ) drawn over two uppercase letters describes a line. The symbol has arrows because a line has no definite beginning or end.  $A$  and  $B$  are points on the line  $AB$  ( $\longleftrightarrow$ ).



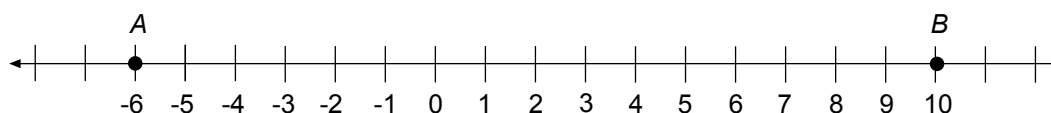
## Method One Midpoint Formula

Another way to find the midpoint of a line segment is to use the Method One midpoint formula below. To do this, add the two endpoints together and divide by two.

**Method One midpoint formula**

$$\frac{a + b}{2}$$

$$\begin{aligned} \frac{a + b}{2} &= \\ \frac{-6 + 10}{2} &= \\ \frac{4}{2} &= \\ 2 \end{aligned}$$



Therefore, for points  $A$  and  $B$  on the number line, the midpoint is

$$\frac{-6 + 10}{2} = \frac{4}{2} = 2.$$



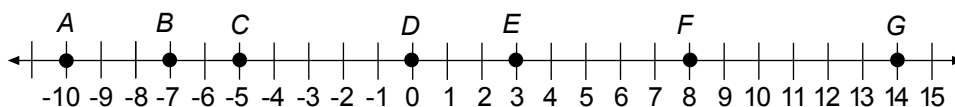
## Practice

Find the **coordinate of the midpoint** for each **pair of points** on the **number line** below. Use either of the methods below from pages 538-539.

- Use the number line and count in from both endpoints of a line segment until you reach the middle to determine the midpoint.
- Use the **Method One midpoint formula** and add the two endpoints together, then divide by two. **Show all your work.**

### Method One midpoint formula

$$\frac{a + b}{2}$$



Refer to previous pages as needed.

1. A and C

2. B and E



3. *A* and *E*

4. *D* and *G*

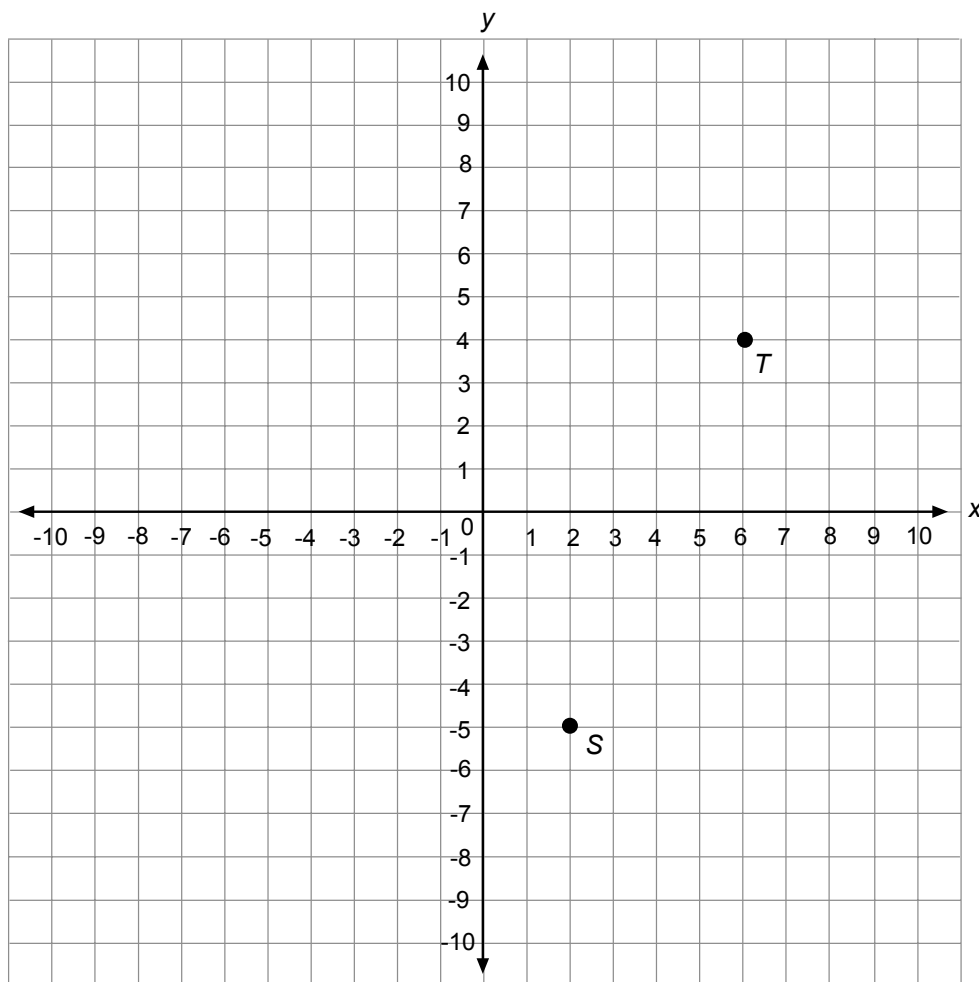
5. *A* and *G*



## Method Two Midpoint Formula

Do you think the process may change a bit when we try to find the midpoint of points  $S$  and  $T$  as seen on the graph below?

Graph of Points  $S$  and  $T$



When the points are on a **coordinate plane**, or the plane containing the  $x$ - and  $y$ -axes, we have to think in two dimensions to find the *coordinates* of the midpoint. The midpoint will have an  **$x$ -coordinate** and a  **$y$ -coordinate** ( $x, y$ ). To find the midpoint on a coordinate plane, we simply use the Method Two midpoint formula twice—once to find the  $x$ -coordinate and again to find the  $y$ -coordinate.



### Method Two midpoint formula

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Let's see how this works.

We see that point  $S$  has coordinates  $(2, -5)$ , and  $T$  is located at  $(6, 4)$ .  
Use the Method Two midpoint formula to find the exact location of the midpoint of  $\overline{ST}$ .

$$\begin{aligned} \text{midpoint of } \overline{ST} &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = && \begin{array}{l} \nearrow \text{find the } \textit{average} \text{ of the} \\ \searrow \text{x-values, then the average of} \\ \text{the y-values} \end{array} \\ \left( \frac{2+6}{2}, \frac{-5+4}{2} \right) &= && \begin{array}{l} \nearrow \text{add the x's then the y's} \\ \searrow \end{array} \\ \left( \frac{8}{2}, \frac{-1}{2} \right) &= && \begin{array}{l} \nearrow \text{now } \textbf{simplify each fraction} \\ \searrow \end{array} \\ \left( 4, \frac{-1}{2} \right) &= && \end{aligned}$$



## Practice

Find the **midpoint** of the **coordinates** for each **segment** whose endpoints are given. Use the **Method Two midpoint formula** below. **Show all your work.** Refer to pages 542-543 as needed.

**Method Two midpoint formula**

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

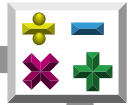
1.  $(2, 8), (-4, 2)$

2.  $(0, 0), (-3, -4)$

3.  $(1, 2), (4, 3)$

4.  $(-3, -5), (9, 0)$





5.  $(-4, 6), (3, 3)$

6.  $(5, -6), (-5, 6)$

7.  $(6, 6), (-4, -4)$

8.  $(5, 5), (-5, -5)$

9.  $(8, -4), (10, 9)$

10.  $(6, 8), (-3, 5)$



## Practice

*Match each definition with the correct term. Write the letter on the line provided.*

- |       |  |                                 |
|-------|--|---------------------------------|
| _____ | 1. a portion of a line that consists of two defined endpoints and all points in between  | A. coordinate plane             |
| _____ | 2. the plane containing the $x$ - and $y$ -axes  | B. coordinates                  |
| _____ | 3. write fraction in lowest terms or simplest form   | C. line segment ( $\text{—}$ )  |
| _____ | 4. the second number of an ordered pair  | D. midpoint (of a line segment) |
| _____ | 5. the number paired with a point on the number line   | E. number line                  |
| _____ | 6. numbers that correspond to points on a coordinate plane in the form $(x, y)$ , or a number that corresponds to a point on a number line | F. simplify a fraction          |
| _____ | 7. the point on a line segment equidistant from the endpoints  | G. $x$ -coordinate              |
| _____ | 8. a line on which ordered numbers can be written or visualized  | H. $y$ -coordinate              |



## **Lesson Three Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

### **Algebra Body of Knowledge**

#### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.9  
Determine the slope,  $x$ -intercept, and  $y$ -intercept of a line given its graph, its equation, or two points on the line.



## **Geometry Body of Knowledge**

### **Standard 1: Points, Lines, Angles, and Planes**

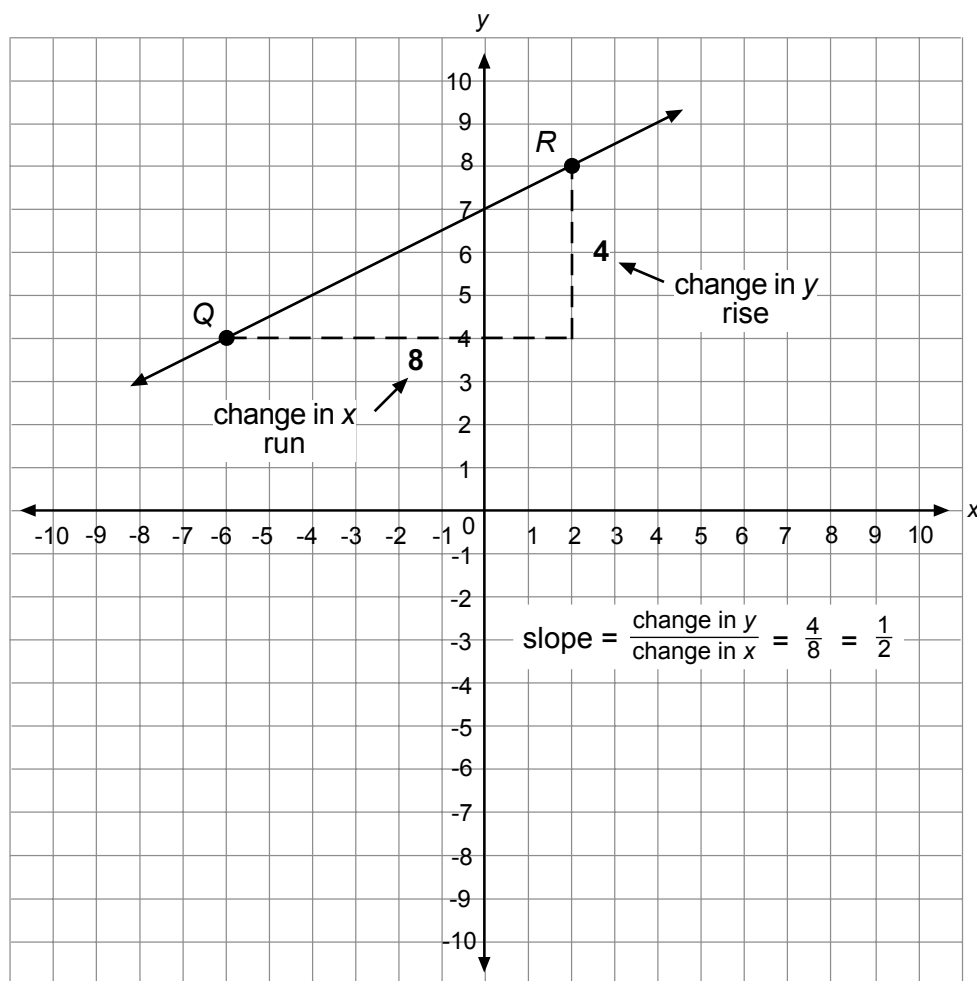
- MA.912.G.1.4  
Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.



## Slope

**Slope** can be thought of as the slant of a line. It is often defined as  $\frac{\text{rise}}{\text{run}}$ , which means the change in the  $y$ -values (**rise**) on the *vertical* axis, divided by the change in the  $x$ -values (**run**) on the *horizontal* axis. In the figure below we can count to find the *slope* between points  $Q$   $(-6, 4)$  and  $R$   $(2, 8)$ .

**Graph of Points  $Q$  and  $R$**



*slope of a line*



However, we can also use the *slope formula* to determine the slope of a line without having to see a graph of the two points of the line.

**slope formula**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



**Remember:**  $m$  is always used to represent slope.

However, we must know the coordinates of two points on a line so that we can use the formula. Refer to points  $Q$  and  $R$  on the previous page. The coordinates of  $Q$  are  $(-6, 4)$  and the coordinates of  $R$  are  $(2, 8)$ . Let's see how this works in the slope formula.

$$x_1 = -6$$

$$x_2 = 2$$

$$y_1 = 4$$

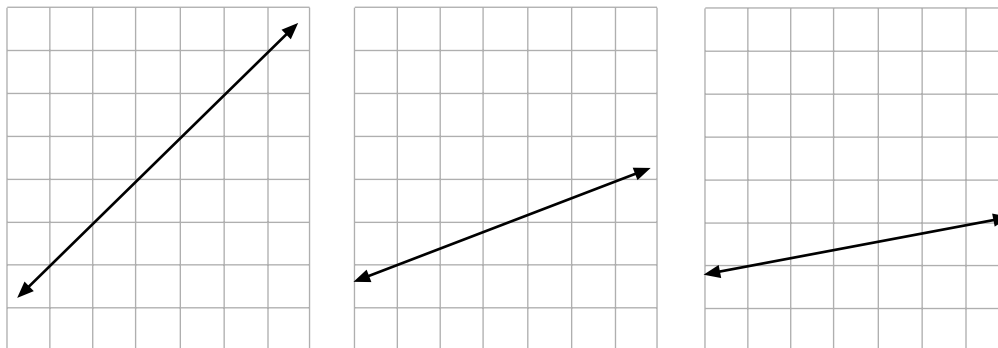
$$y_2 = 8$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{2 - (-6)} = \frac{4}{8} = \frac{1}{2}$$



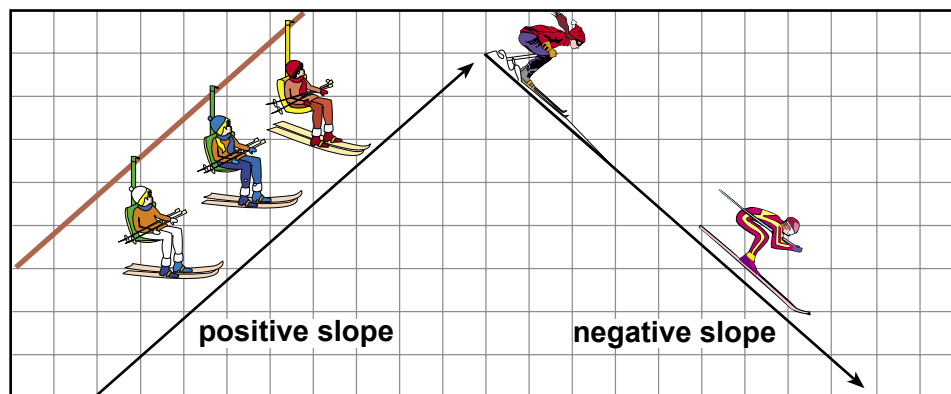
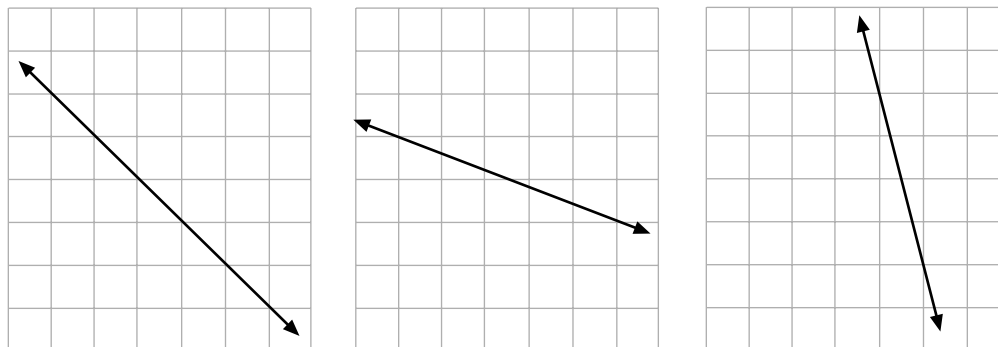
When the slope of a line is *positive*, the line will *rise* from left to right.

### Examples



When the slope of a line is *negative*, the line will *fall* from left to right.

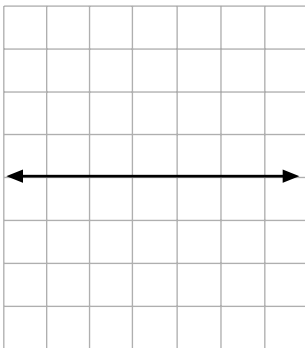
### Examples



*slope*



When the slope has a zero in the **numerator** ( $\frac{0}{x}$ ), the line will be *horizontal* and have a slope of 0.



When the slope has a zero in the **denominator** ( $\frac{y}{0}$ ), the line will be *vertical* and have *no* slope at all. We sometimes say that the slope of a vertical line is *undefined*.







## Practice

Use the **slope formula** below to find the slope of each line passing through points listed below. **Simplify the answer.** Then determine whether the line is rising, falling, horizontal, or vertical. Write the answer on the line provided. Show all your work.

**slope formula**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



**Remember:**

$\frac{0}{x}$  = a line that is horizontal with a **zero (0) slope**

$\frac{y}{0}$  = a line that is vertical with **no slope**

\_\_\_\_\_ 1. (2, 8), (-4, 2)

\_\_\_\_\_ 2. (0, 0), (-3, -4)

\_\_\_\_\_ 3. (1, 2), (4, 3)



\_\_\_\_\_ 4.  $(3, -6), (3, 4)$

\_\_\_\_\_ 5.  $(-3, -5), (9, 0)$

\_\_\_\_\_ 6.  $(-4, 6), (3, 3)$

\_\_\_\_\_ 7.  $(4, 2), (-5, 2)$



\_\_\_\_\_ 8.  $(5, -6), (-5, 6)$

\_\_\_\_\_ 9.  $(6, 6), (-4, -4)$

\_\_\_\_\_ 10.  $(6, 7), (6, -4)$

\_\_\_\_\_ 11.  $(5, 5), (-5, -5)$



\_\_\_\_\_ 12.  $(0, 4), (0, 9)$

\_\_\_\_\_ 13.  $(8, -4), (10, -9)$

\_\_\_\_\_ 14.  $(6, 5), (-3, 8)$

\_\_\_\_\_ 15.  $(4, 5), (8, 16)$



## **Lesson Four Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 1: Real and Complex Number Systems**

- MA.912.A.1.8  
Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.7  
Rewrite equations of a line into slope-intercept form and standard form.



- MA.912.A.3.8  
Graph a line given any of the following information: a table of values, the  $x$ - and  $y$ -intercepts, two points, the slope and a point, the equation of the line in slope-intercept form, standard form, or point-slope form.
- MA.912.A.3.9  
Determine the slope,  $x$ -intercept, and  $y$ -intercept of a line given its graph, its equation, or two points on the line.

## Geometry Body of Knowledge

### Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4  
Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

## Equations of Lines

An *equation* of a line can be expressed in several ways. Mathematicians sometimes use the format  $ax + by = c$ . This is called **standard form (of a linear equation)**. In the *standard form*, **linear equations** have the following three rules.

1.  $a$ ,  $b$ , and  $c$  are **integers**, or the numbers in the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
2.  $a$  cannot be a **negative integer**
3.  $a$  and  $b$  cannot both be equal to 0



### Linear Equations in Standard Form

- $ax + by = c$
- $x$  and  $y$  are **variables**
- $a$ ,  $b$ , and  $c$  are **constants** for the given equation

You can graph a line fairly easily by using standard form.

Follow this example.

$$3x + 2y = 12$$

If we replace  $x$  with 0 we get the following.

$$\begin{aligned} 3x + 2y &= 12 \\ 3(0) + 2y &= 12 \\ 0 + 2y &= 12 \\ 2y &= 12 \\ y &= 6 \end{aligned}$$

This tells us that the point  $(0, 6)$  is on the graph of the line  $3x + 2y = 12$ . In fact  $(0, 6)$  is called the  **$y$ -intercept** of the line. It is the point where the line crosses the  $y$ -axis.

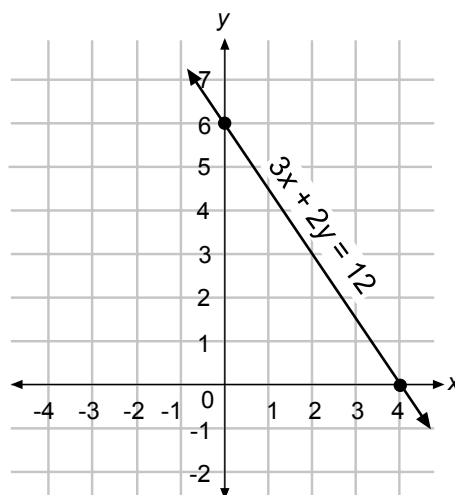
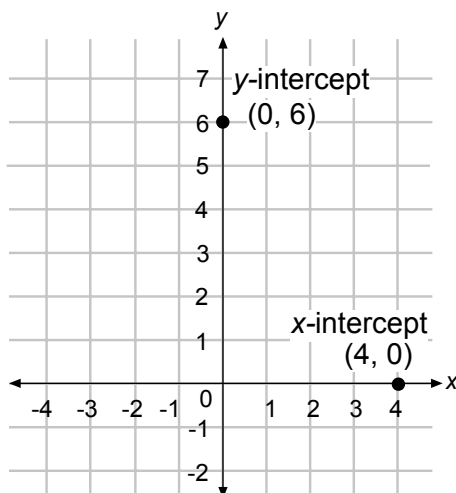
Remember that you must have two points to decide exactly where the line goes on the coordinate plane. So, we repeat the process, but this time replace  $y$  with 0.

$$\begin{aligned} 3x + 2y &= 12 \\ 3x + 2(0) &= 12 \\ 3x + 0 &= 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

This tells us that the point  $(4, 0)$  is also on the line. Did you guess that this is called the  **$x$ -intercept**?



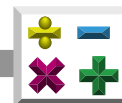
So, if we plot the two points  $(0, 6)$  and  $(4, 0)$ , we can draw a line connecting them.



Did you notice that we could find the slope of the line above either by using the slope formula with the  $x$ - and  $y$ -intercepts

$(m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{4 - 0} = \frac{-6}{4} = \frac{-3}{2})$  or by counting rise and run from the graph?





Let's try another example.

$$5x - y = 15$$

If  $x = 0$ ,

$$\begin{aligned} 5x - y &= 15 \\ 5(0) - y &= 15 \\ 0 - y &= 15 \\ -y &= 15 \\ y &= -15 \end{aligned}$$

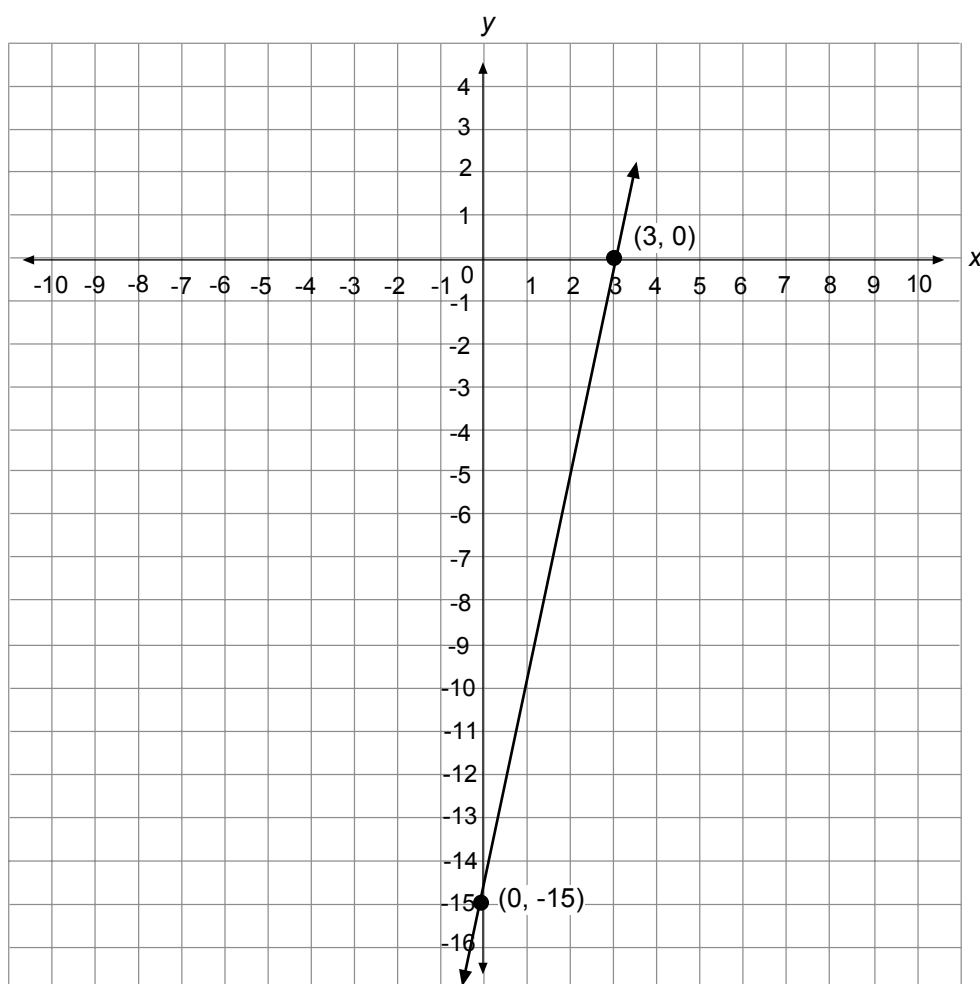
$(0, -15)$   $y$ -intercept

If  $y = 0$ ,

$$\begin{aligned} 5x - y &= 15 \\ 5x - y(0) &= 15 \\ 5x - 0 &= 15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

$(3, 0)$   $x$ -intercept

**Graph of  $5x - y = 15$**



Your turn.



## Practice

Use the equations in **standard form** to find the ***y*-intercepts**, find the ***x*-intercepts**, and **graph the lines** of the following.

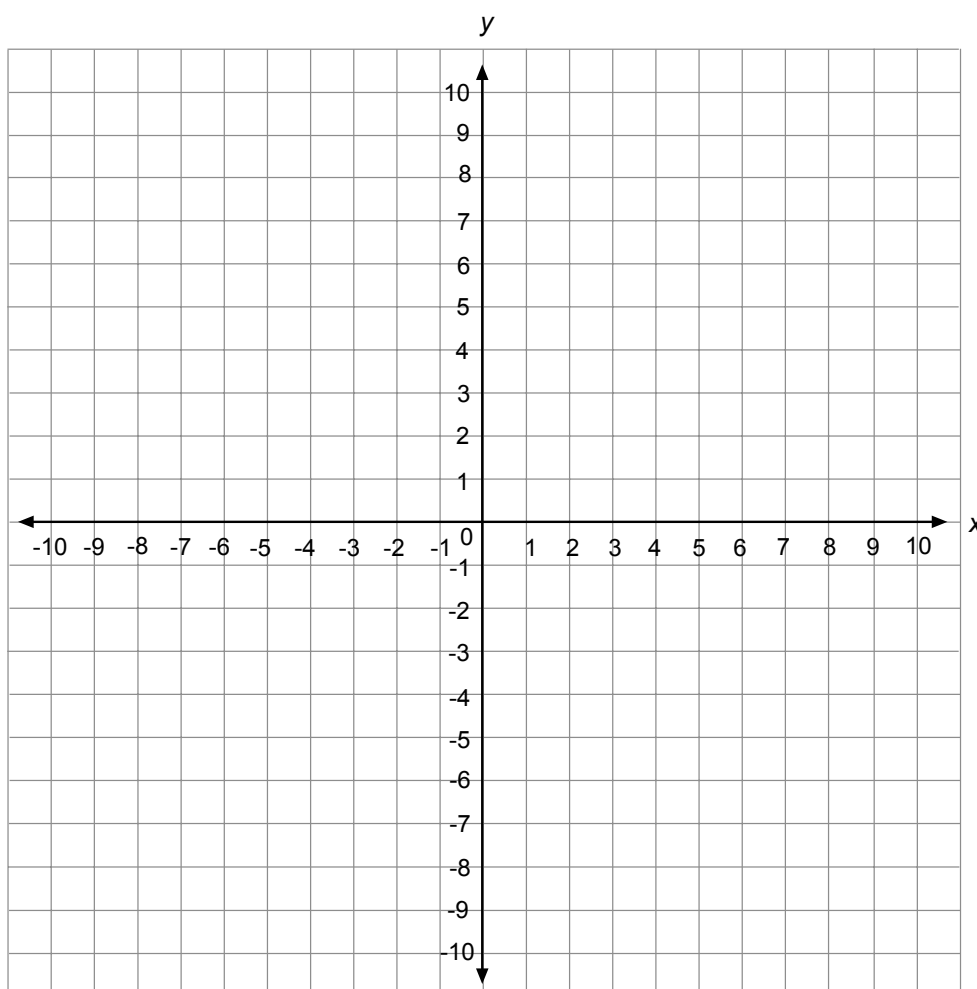
1.  $2x + 5y = 10$

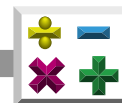
a. *y*-intercept = \_\_\_\_\_

b. *x*-intercept = \_\_\_\_\_

c. graph

**Graph of  $2x + 5y = 10$**





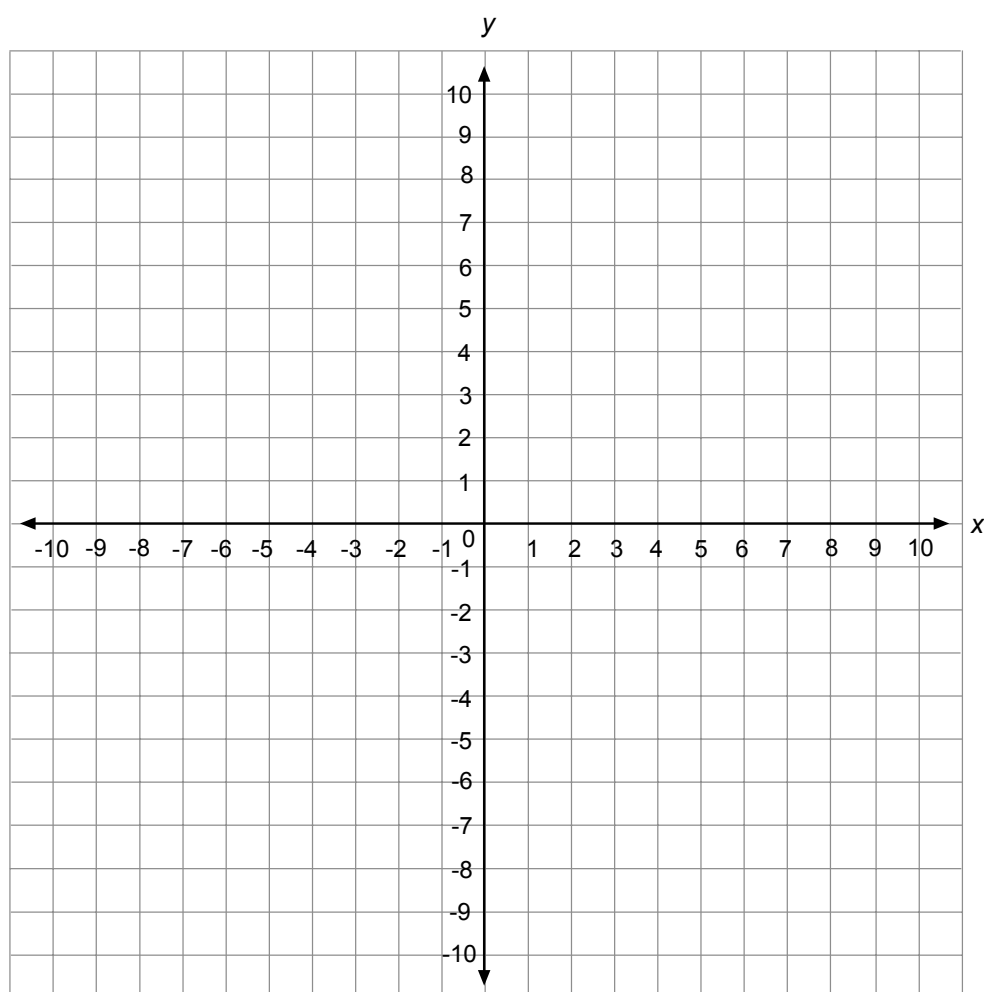
2.  $8x - 3y = 24$

a.  $y$ -intercept = \_\_\_\_\_

b.  $x$ -intercept = \_\_\_\_\_

c. graph

**Graph of  $8x - 3y = 24$**





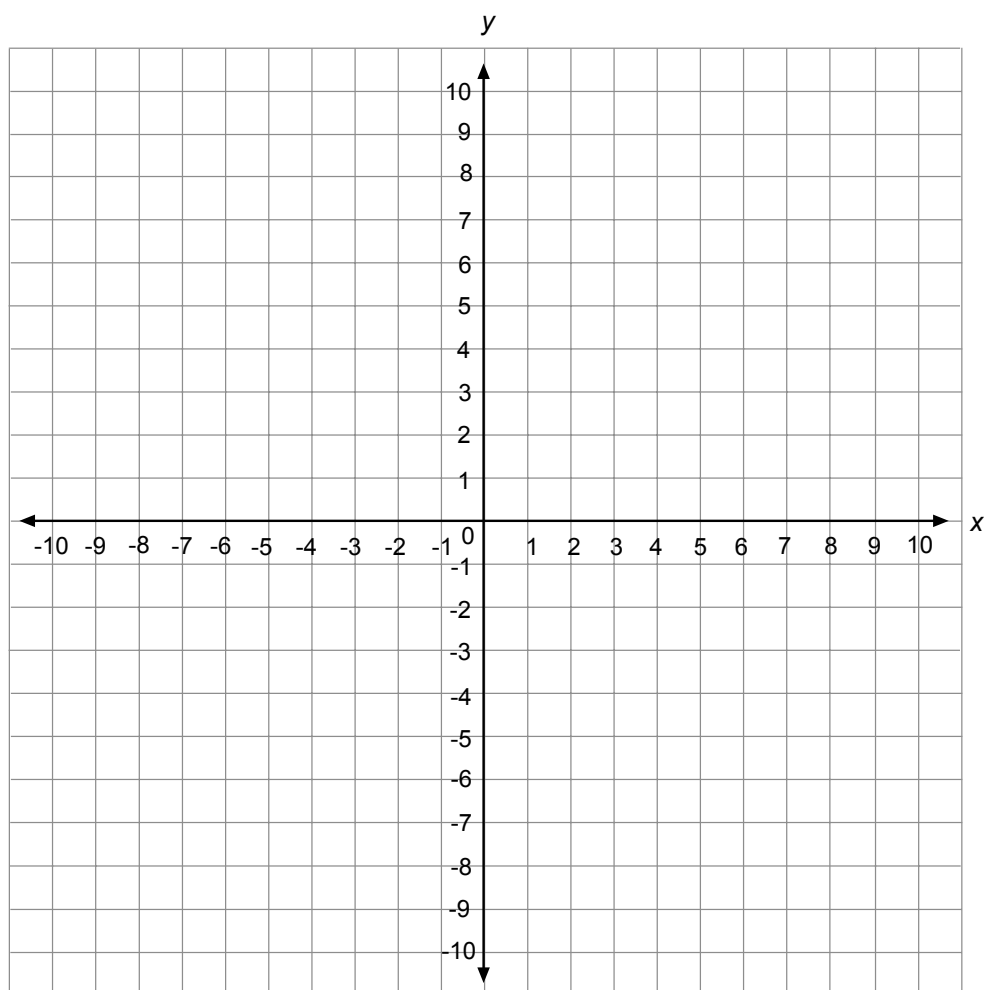
3.  $3x - 8y = 24$

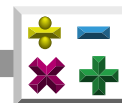
a.  $y$ -intercept = \_\_\_\_\_

b.  $x$ -intercept = \_\_\_\_\_

c. graph

**Graph of  $3x - 8y = 24$**





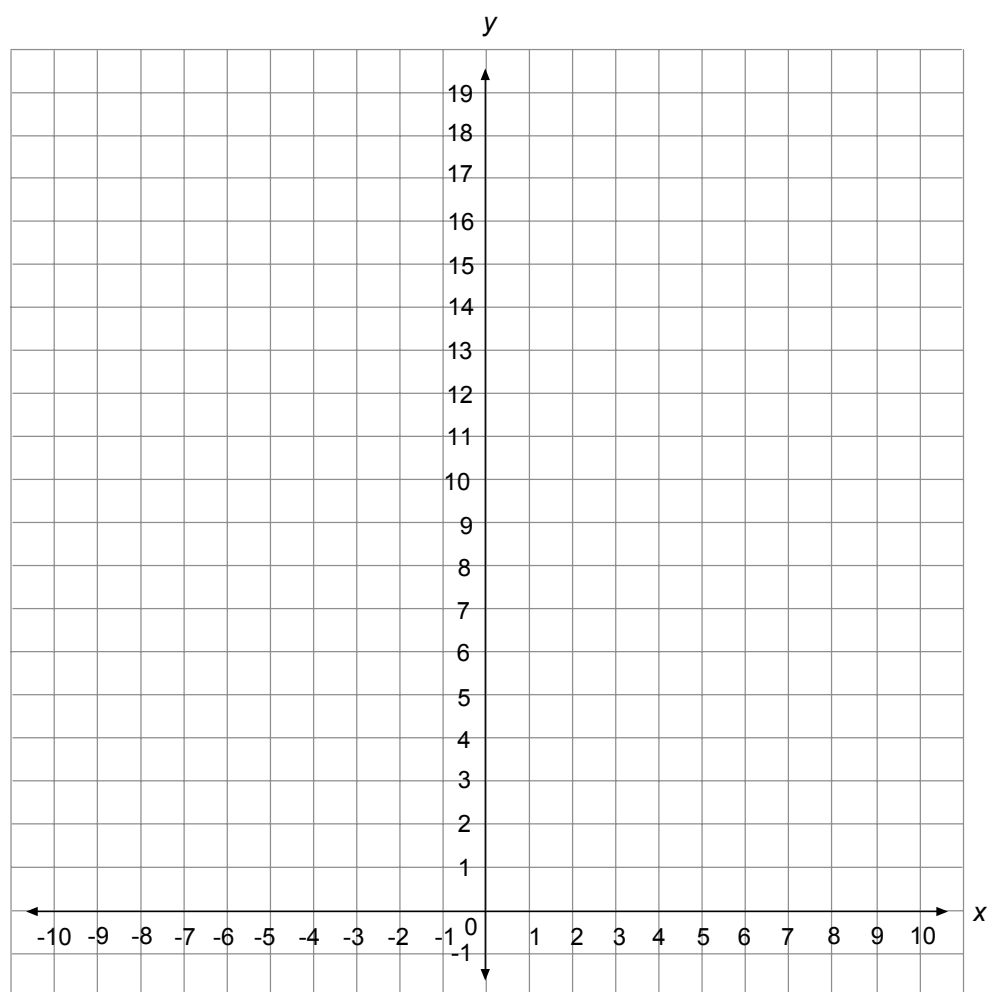
4.  $6x + y = 18$

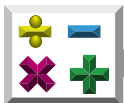
a.  $y$ -intercept = \_\_\_\_\_

b.  $x$ -intercept = \_\_\_\_\_

c. graph

**Graph of  $6x + y = 18$**





5.  $4x = 8$

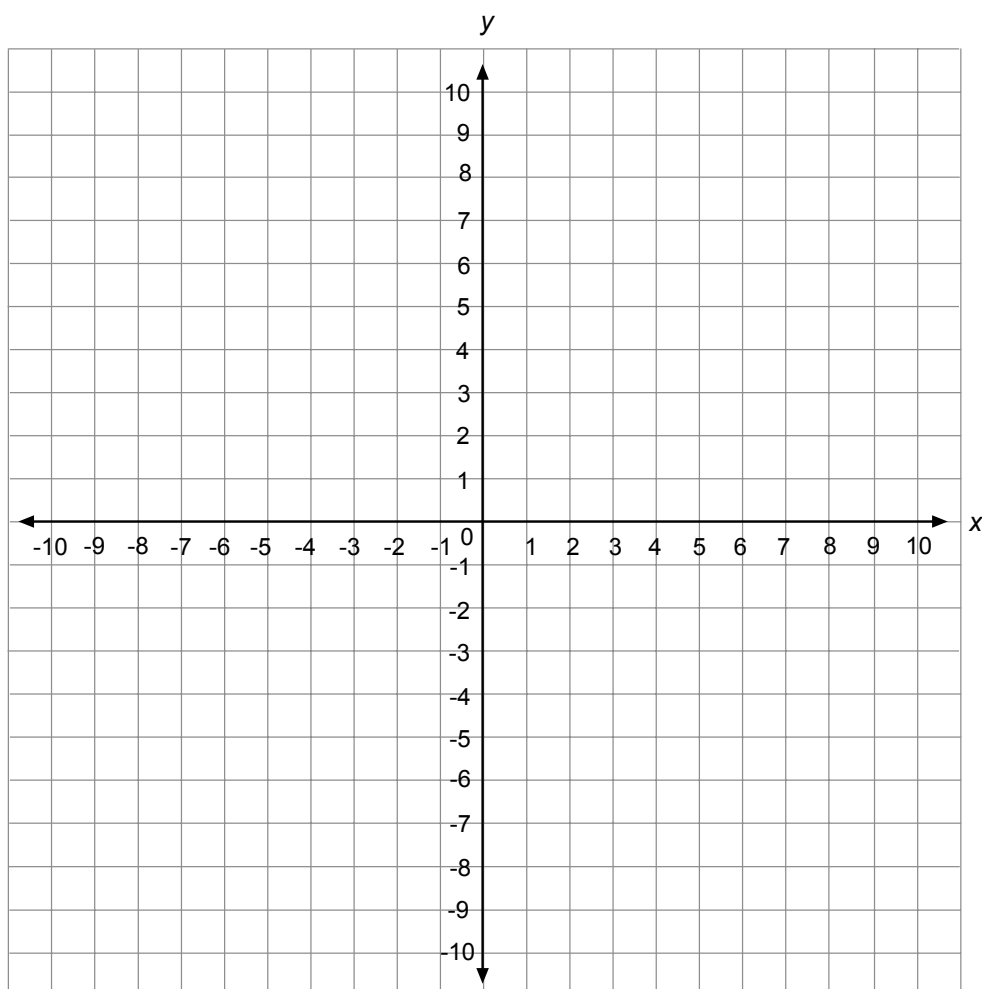
**Hint:** If there is *no*  $y$ -intercept, the line is *vertical*.  
If there is *no*  $x$ -intercept, the line is *horizontal*.

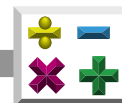
a.  $y$ -intercept = \_\_\_\_\_

b.  $x$ -intercept = \_\_\_\_\_

c. graph

**Graph of  $4x = 8$**





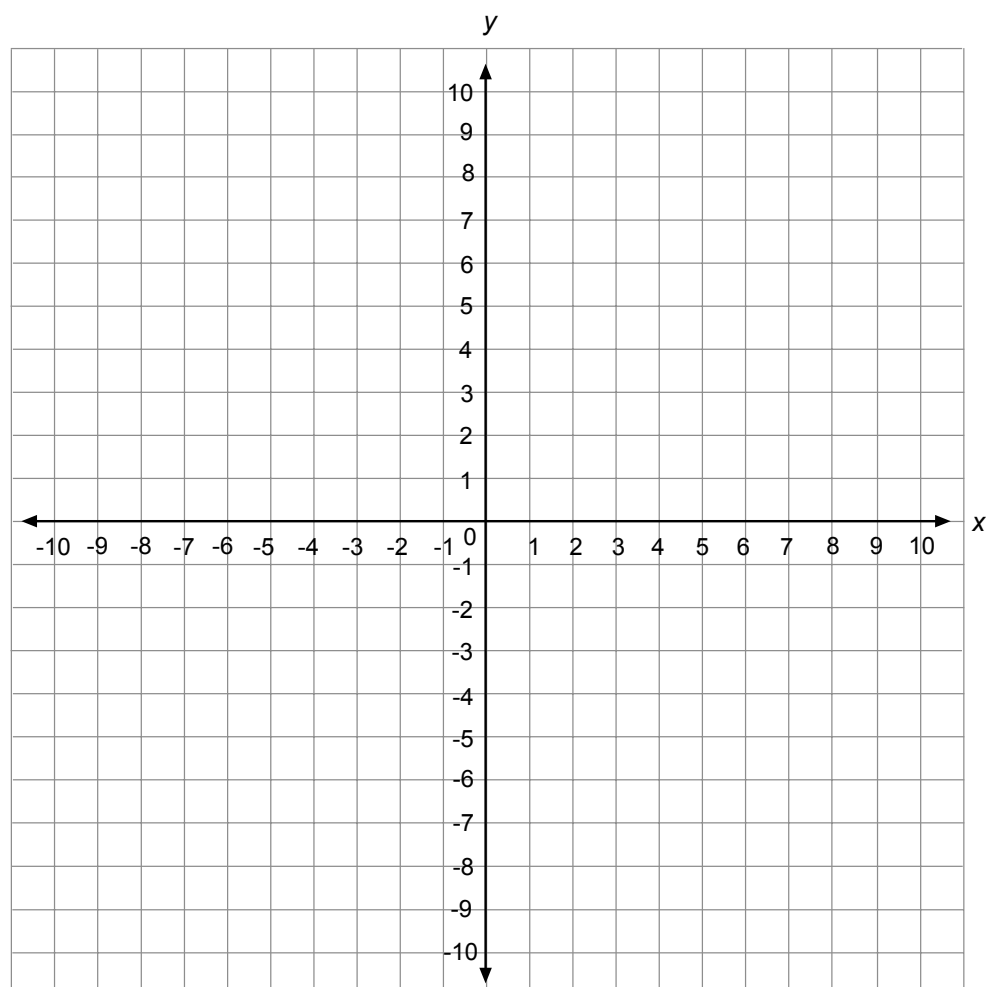
6.  $4y = 8$

a.  $y$ -intercept = \_\_\_\_\_

b.  $x$ -intercept = \_\_\_\_\_

c. graph

**Graph of  $4y = 8$**





## Slope-Intercept Form

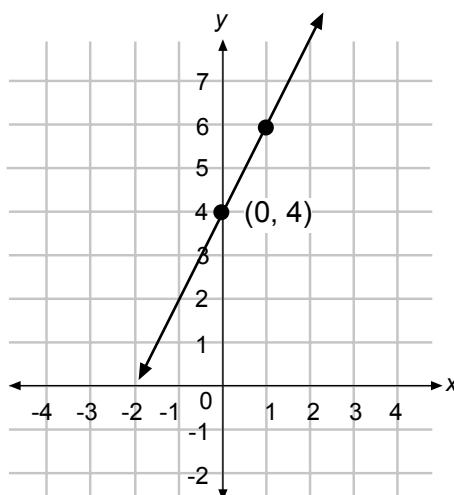
Many students prefer to use the **slope-intercept form** for the equation of a line. An equation in this form tells you the slope of a line and where it crosses the  $y$ -axis. The generic format looks like the following.

$$y = mx + b$$

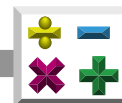
$m$  is the slope       $b$  is the  $y$ -intercept

So if  $y = 2x + 4$ , this line crosses the  $y$ -axis at 4 and has a slope of 2.

To graph this line, plot a point at  $(0, 4)$  and count the rise and run of the slope  $(\frac{2}{1})$  from that point and draw a line.



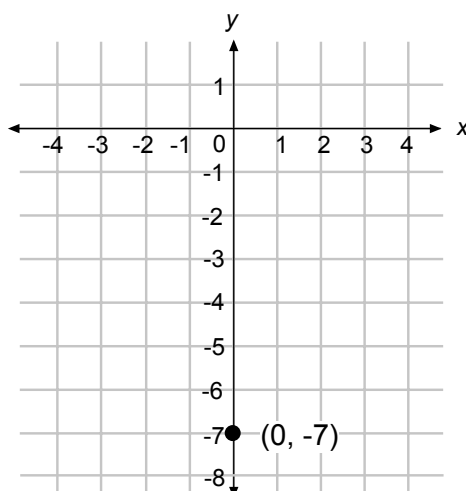




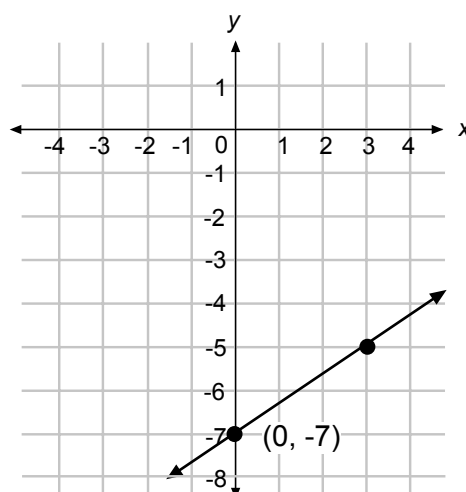
For the equation of the line

$$y = \frac{2}{3}x - 7,$$

the  $y$ -intercept is  $-7$ .

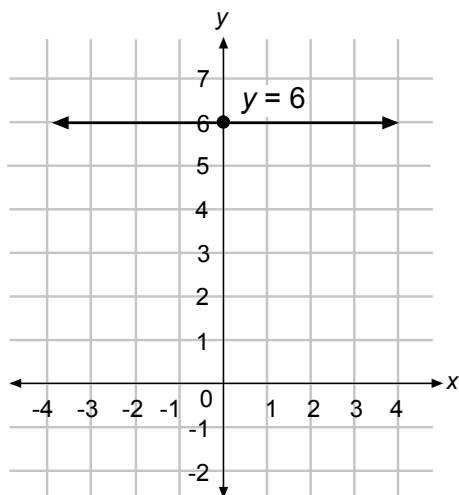


The slope is  $\frac{2}{3}$ , so the graph looks like the following.

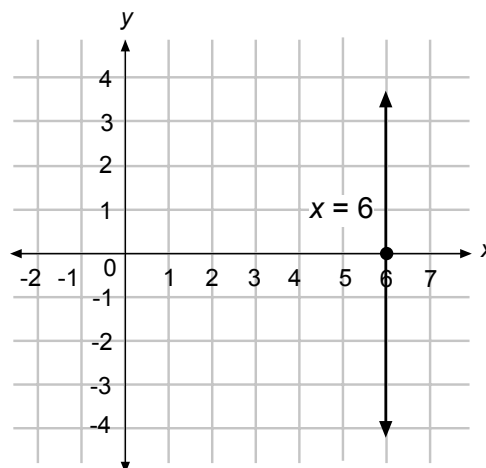




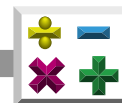
**Remember:** If the equation looks like  $y = 6$ , the line is *horizontal* and has *zero* slope. If the equation looks like  $x = 6$ , the line is *vertical* and has *no* slope. Look at the graphs below.



$y = 6$   
line is horizontal  
zero slope



$x = 6$   
line is vertical  
no slope  
(sometimes referred to as undefined)



## Practice

Use the equations to find the **y-intercepts**, find the **slopes**, and **graph the lines** of the following.

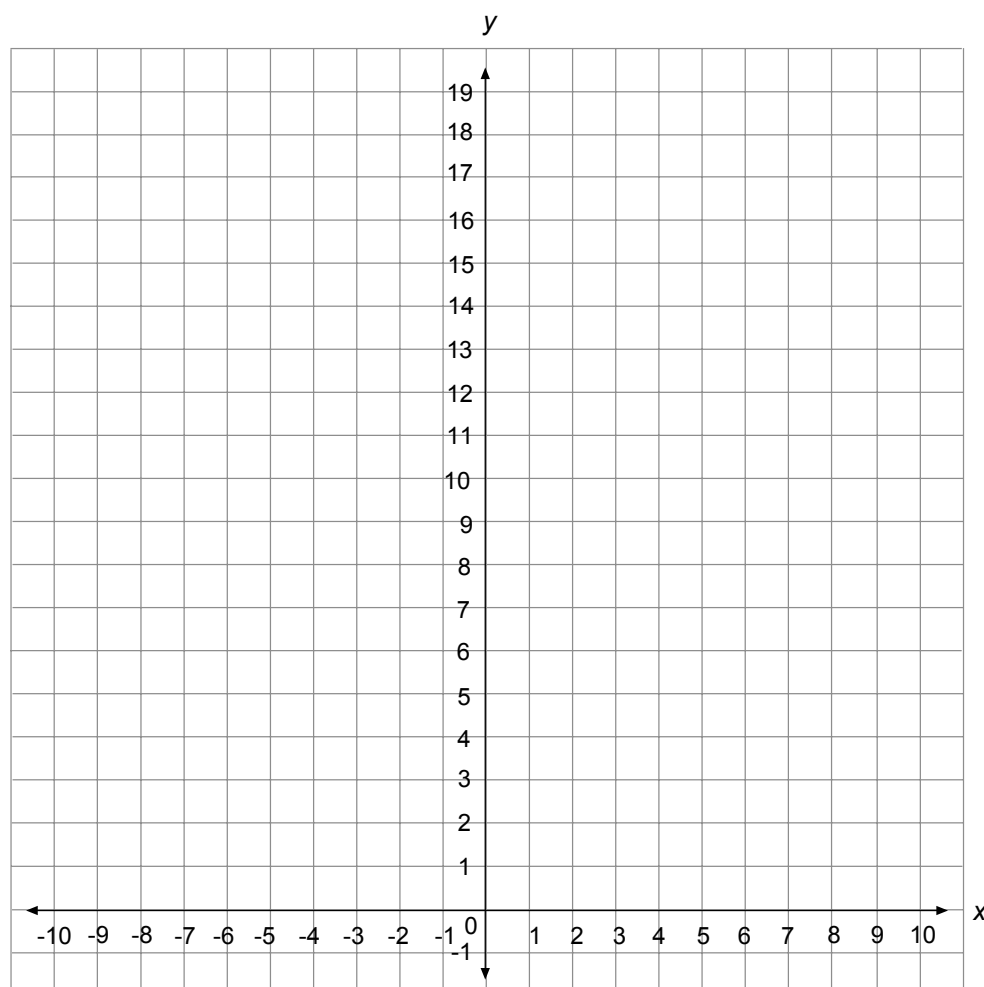
1.  $y = 5x + 7$

a. y-intercept = \_\_\_\_\_

b. slope = \_\_\_\_\_

c. graph

**Graph of  $y = 5x + 7$**





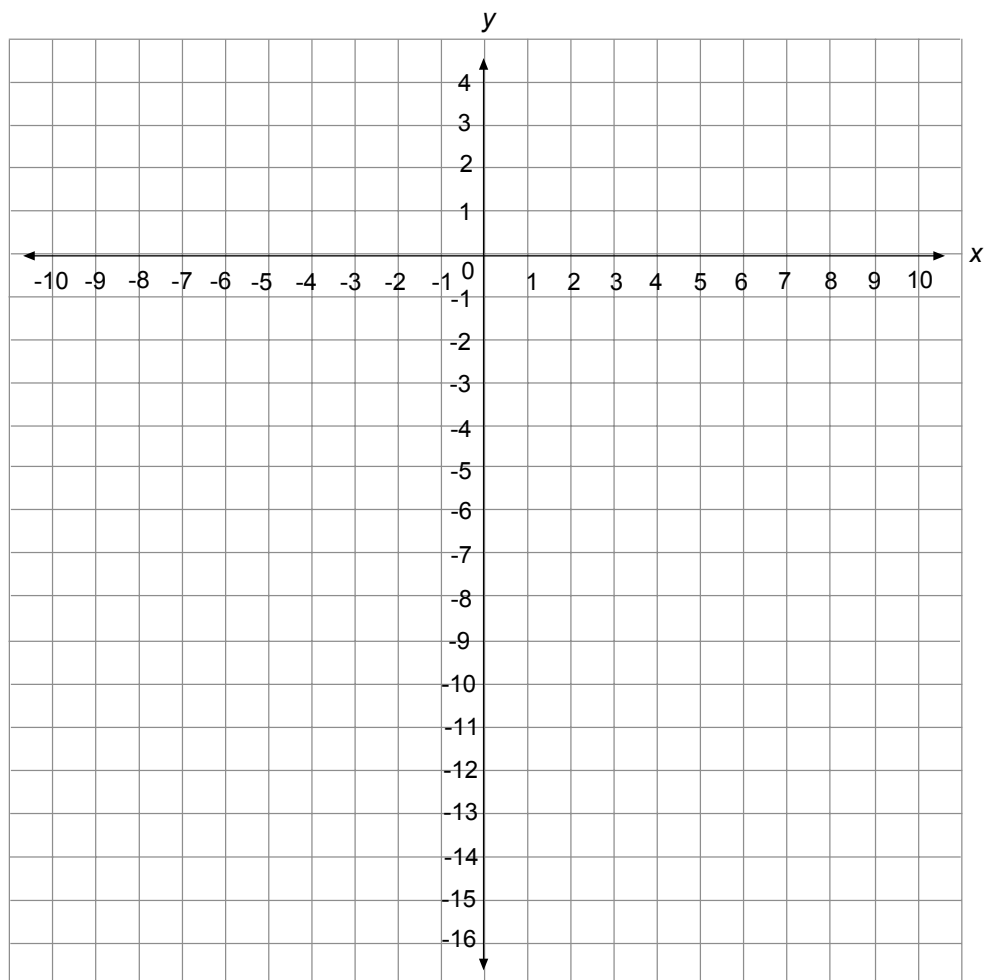
2.  $y = -3x - 9$

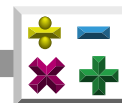
a.  $y$ -intercept = \_\_\_\_\_

b. slope = \_\_\_\_\_

c. graph

**Graph of  $y = -3x - 9$**





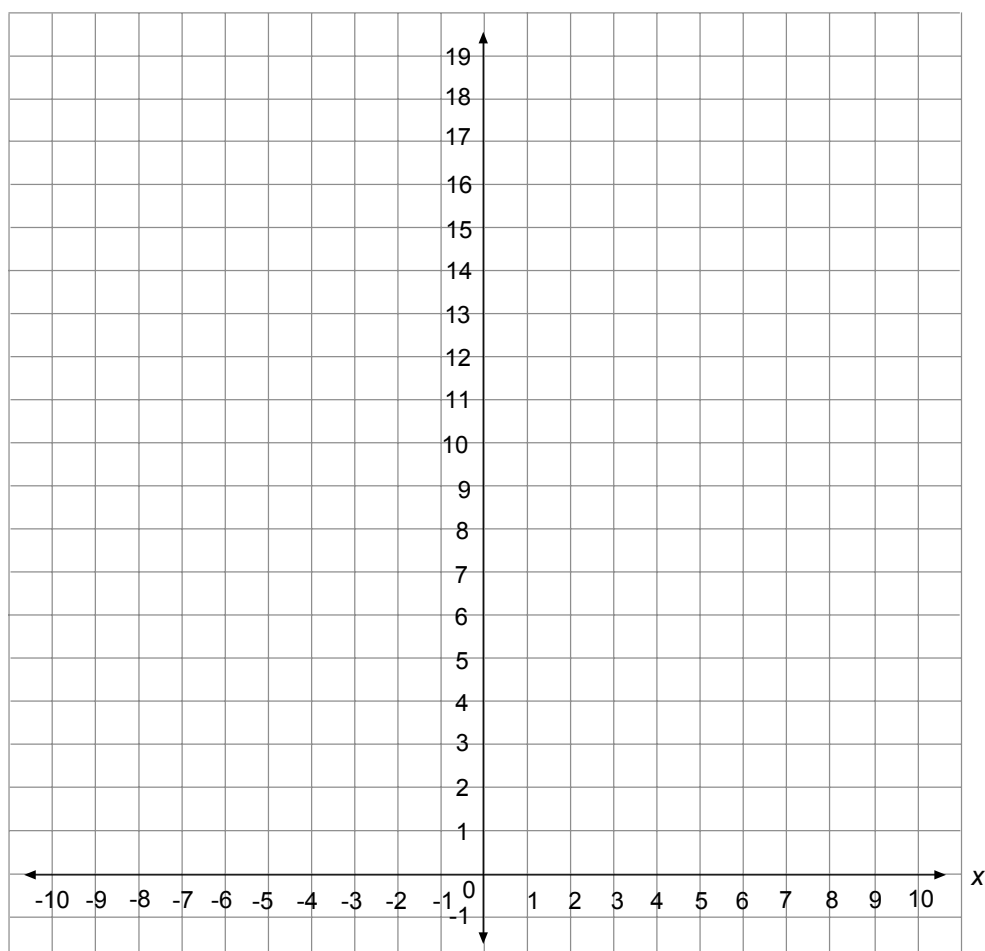
3.  $y = \frac{5}{7}x + 11$

a.  $y$ -intercept = \_\_\_\_\_

b. slope = \_\_\_\_\_

c. graph

**Graph of  $y = \frac{5}{7}x + 11$**





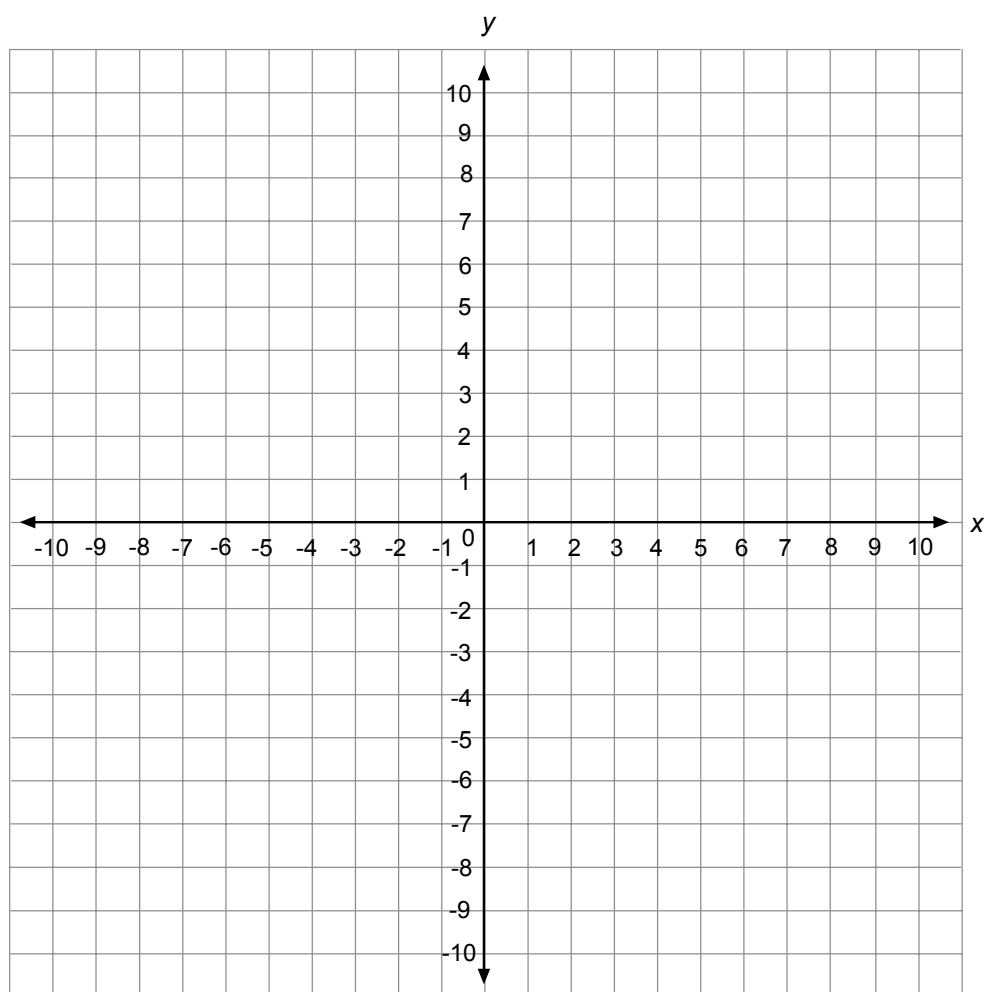
4.  $x = -7$

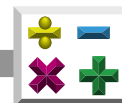
a.  $y$ -intercept = \_\_\_\_\_

b. slope = \_\_\_\_\_

c. graph

**Graph of  $x = -7$**





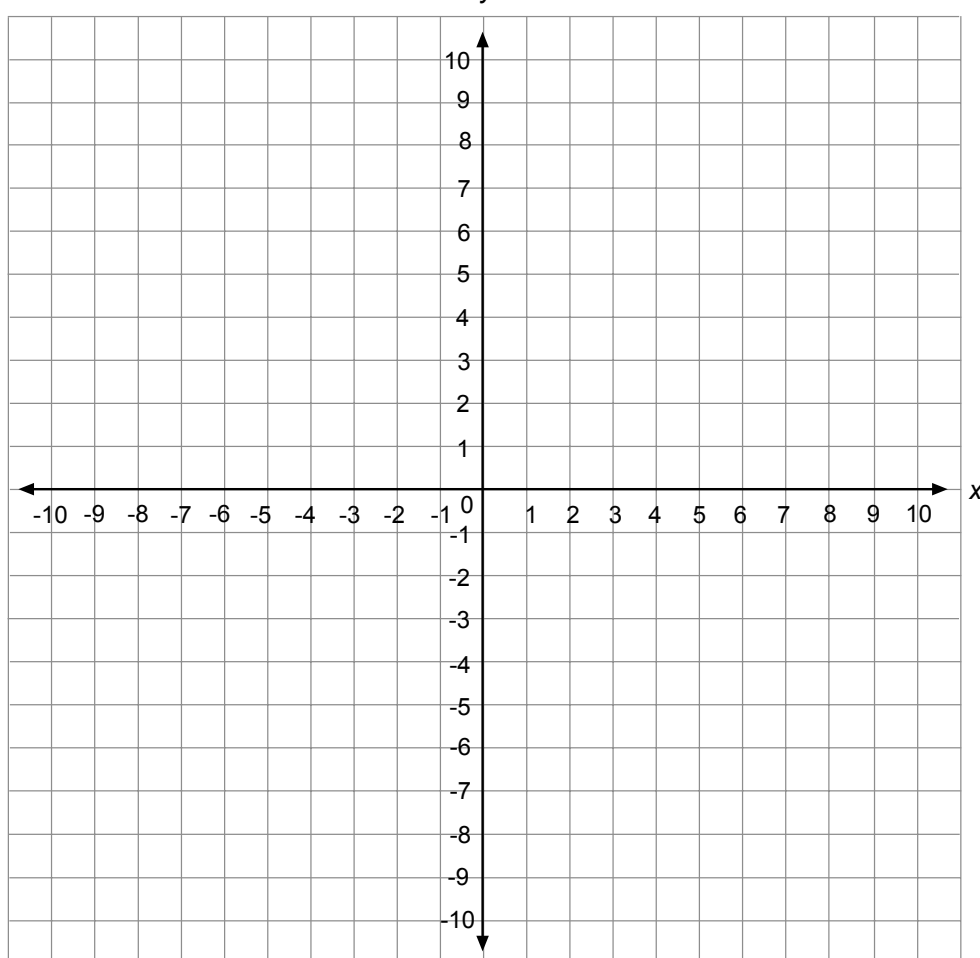
5.  $y = \frac{2}{3}x - 4$

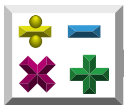
a.  $y$ -intercept = \_\_\_\_\_

b. slope = \_\_\_\_\_

c. graph

**Graph of  $y = \frac{2}{3}x - 4$**





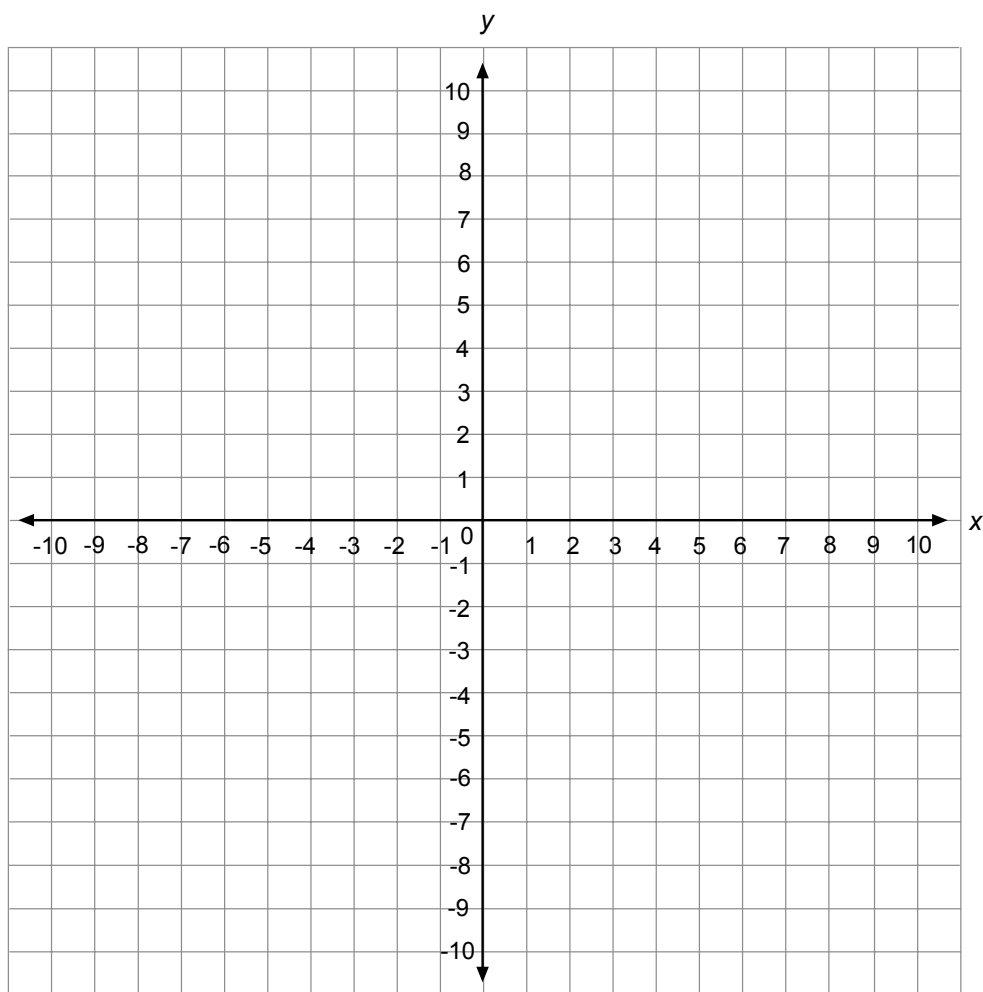
6.  $y = -4x + 2$

a.  $y$ -intercept = \_\_\_\_\_

b. slope = \_\_\_\_\_

c. graph

**Graph of  $y = -4x + 2$**







## Transforming Equations into Slope-Intercept Form

Sometimes it is necessary to transform an equation into the *slope-intercept form* so that we can readily identify the slope or the  $y$ -intercept or both.

Follow these examples. Remember, we want it to be in the  $y = mx + b$  format.

### Example 1

$$\begin{aligned} 6x - 3y &= 12 \\ -3y &= -6x + 12 && \longleftarrow \text{subtract } 6x \text{ from both sides} \\ y &= 2x - 4 && \longleftarrow \text{divide both sides by } -3 \end{aligned}$$

Now we can easily see that the slope is 2 and the  $y$ -intercept is -4.

### Example 2

$$\begin{aligned} x + \frac{2}{3}y &= 8 \\ \frac{2}{3}y &= -x + 8 && \longleftarrow \text{subtract } x \text{ from each side} \\ \left(\frac{3}{2}\right)\frac{2}{3}y &= -\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)8 && \longleftarrow \text{multiply both sides by } \frac{3}{2} \\ y &= -\frac{3}{2}x + 12 && \longleftarrow \text{simplify} \end{aligned}$$

$$\text{slope} = -\frac{3}{2} \qquad y\text{-intercept} = 12$$



## Practice

Express in **slope-intercept form**, find the **y-intercepts**, find the **slopes**, and **graph the lines** of the following.

1.  $5x + 3y = -18$

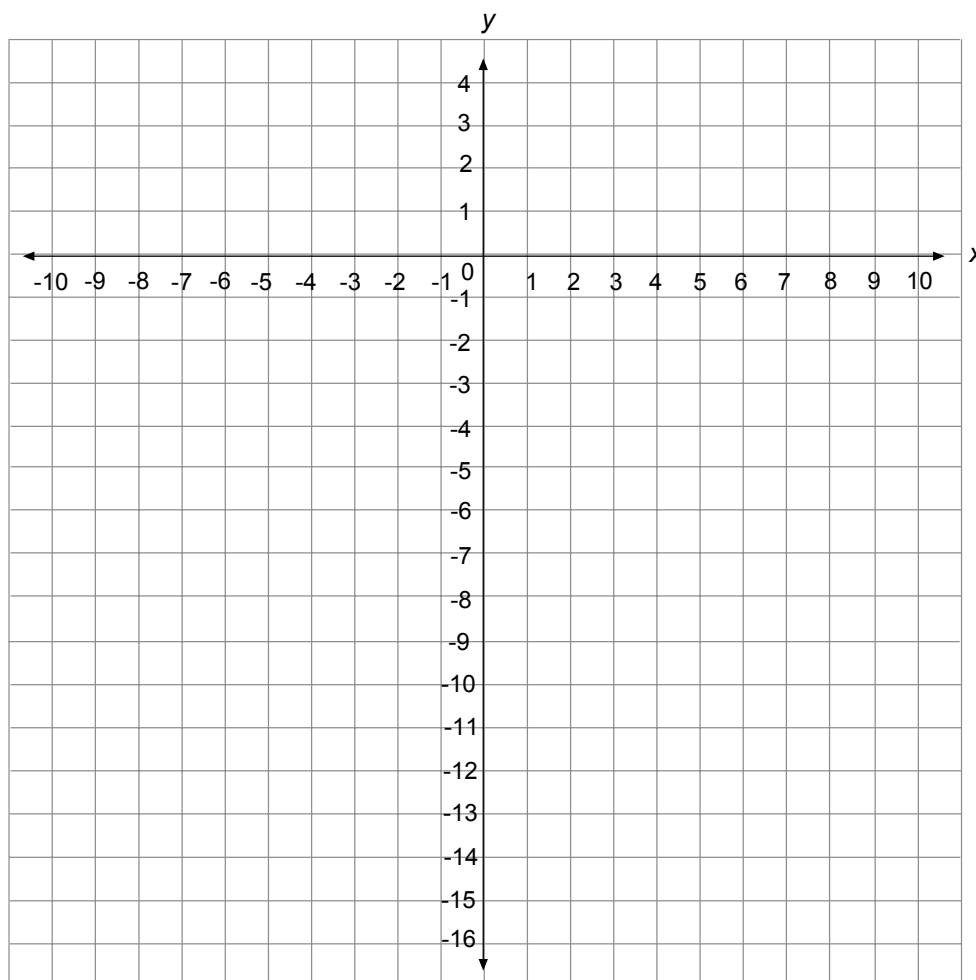
a. slope-intercept form = \_\_\_\_\_

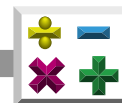
b. y-intercept = \_\_\_\_\_

c. slope = \_\_\_\_\_

d. graph

**Graph of  $5x + 3y = -18$**





2.  $2x + y = 8$

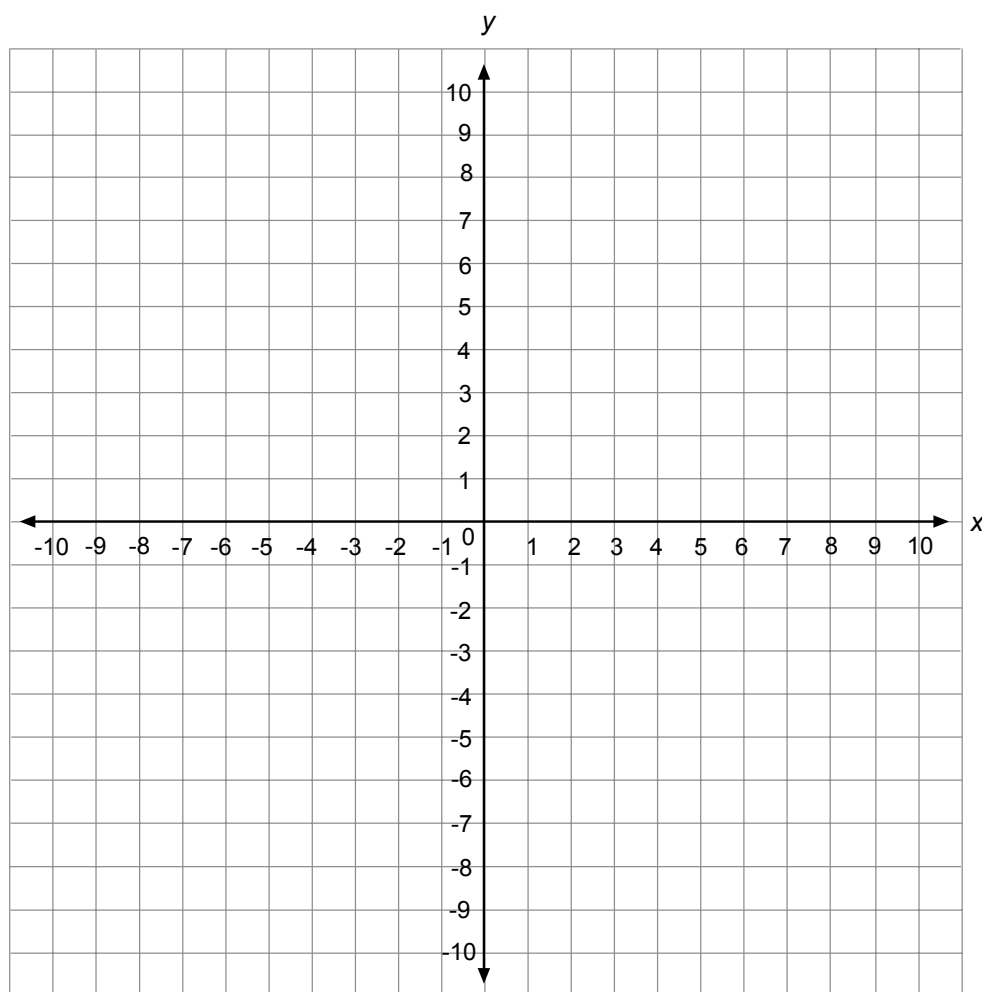
a. slope-intercept form = \_\_\_\_\_

b.  $y$ -intercept = \_\_\_\_\_

c. slope = \_\_\_\_\_

d. graph

**Graph of  $2x + y = 8$**





3.  $3x + 3y = 6$

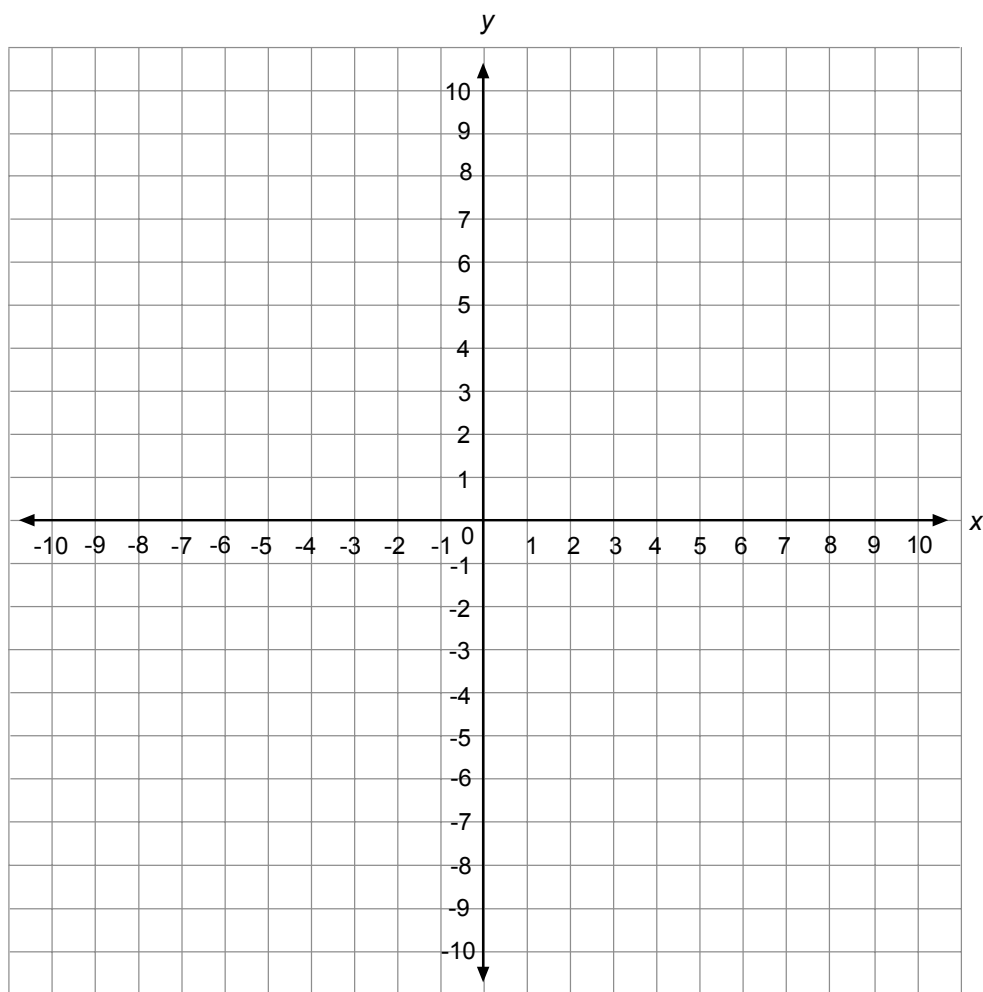
a. slope-intercept form = \_\_\_\_\_

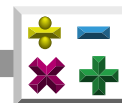
b.  $y$ -intercept = \_\_\_\_\_

c. slope = \_\_\_\_\_

d. graph

**Graph of  $3x + 3y = 6$**





4.  $5x + y = 0$

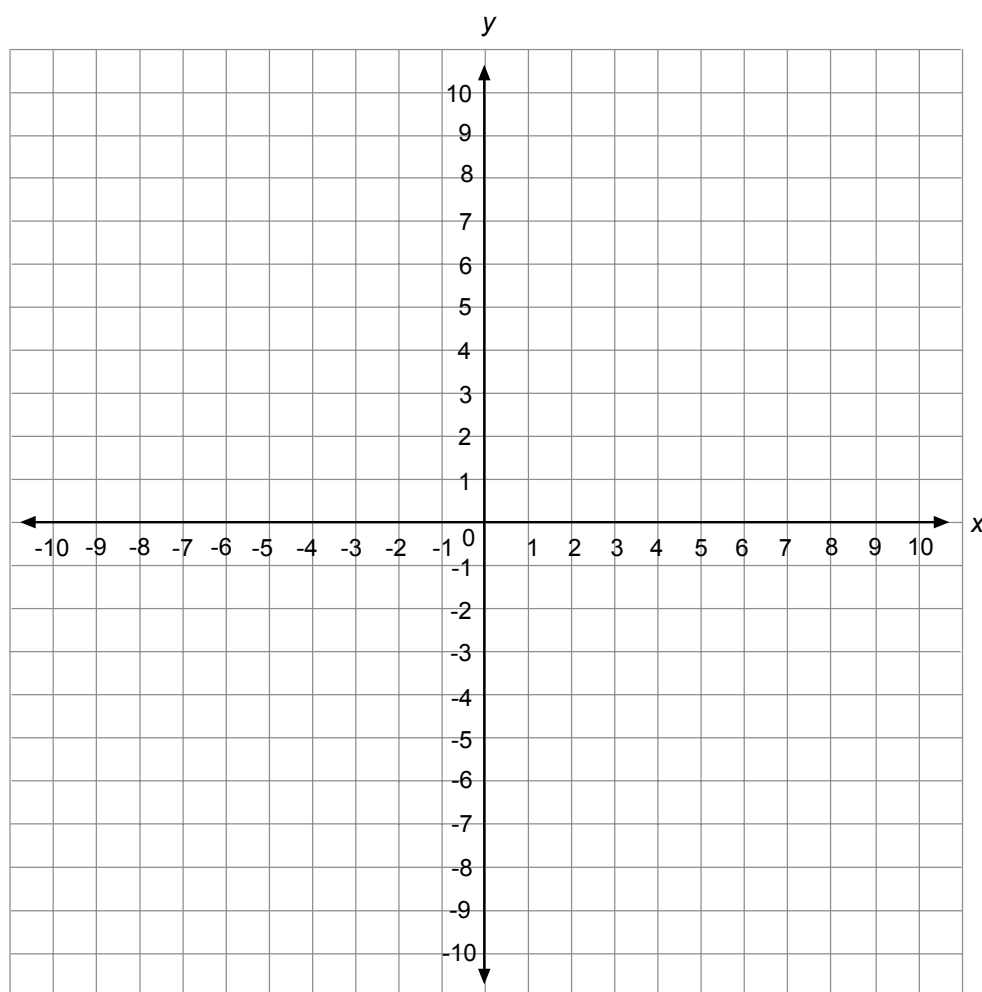
a. slope-intercept form = \_\_\_\_\_

b.  $y$ -intercept = \_\_\_\_\_

c. slope = \_\_\_\_\_

d. graph

**Graph of  $5x + y = 0$**





5.  $2x - y = -2$

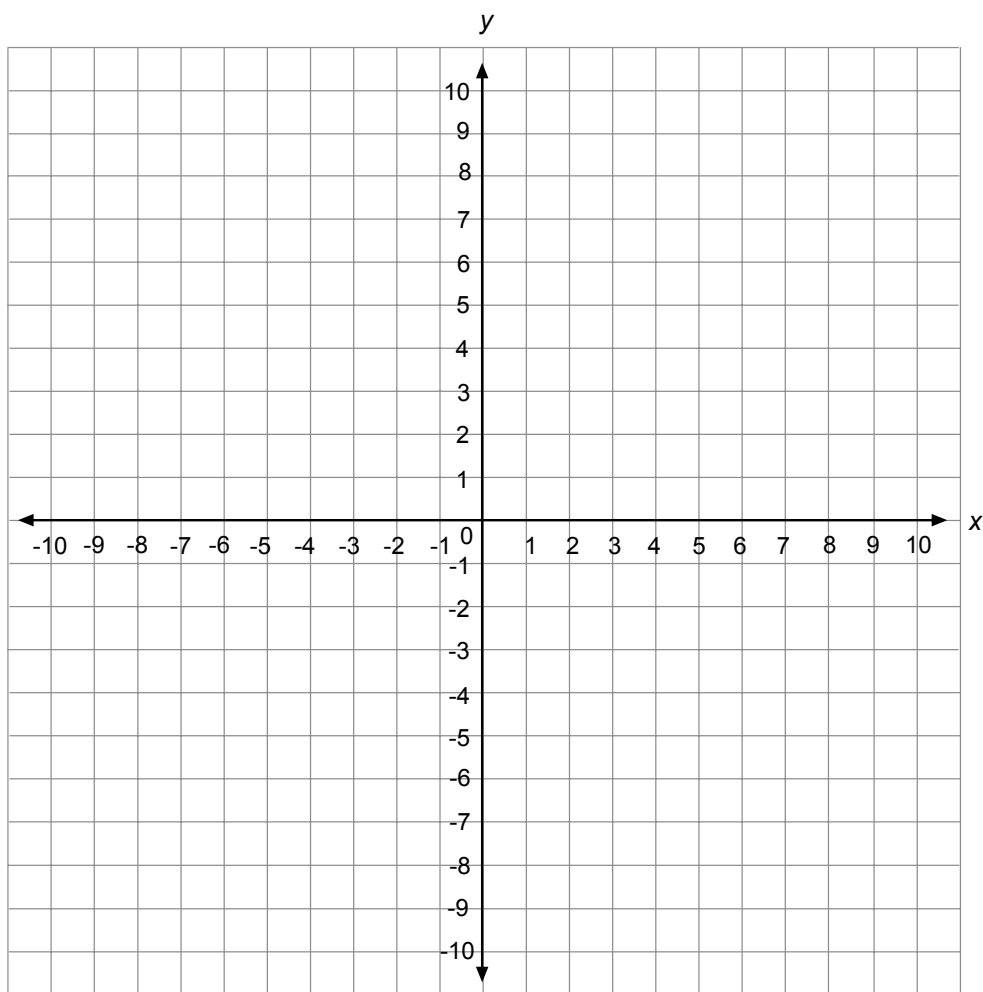
a. slope-intercept form = \_\_\_\_\_

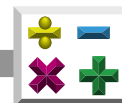
b.  $y$ -intercept = \_\_\_\_\_

c. slope = \_\_\_\_\_

d. graph

**Graph of  $2x - y = -2$**





6.  $x - y = -8$

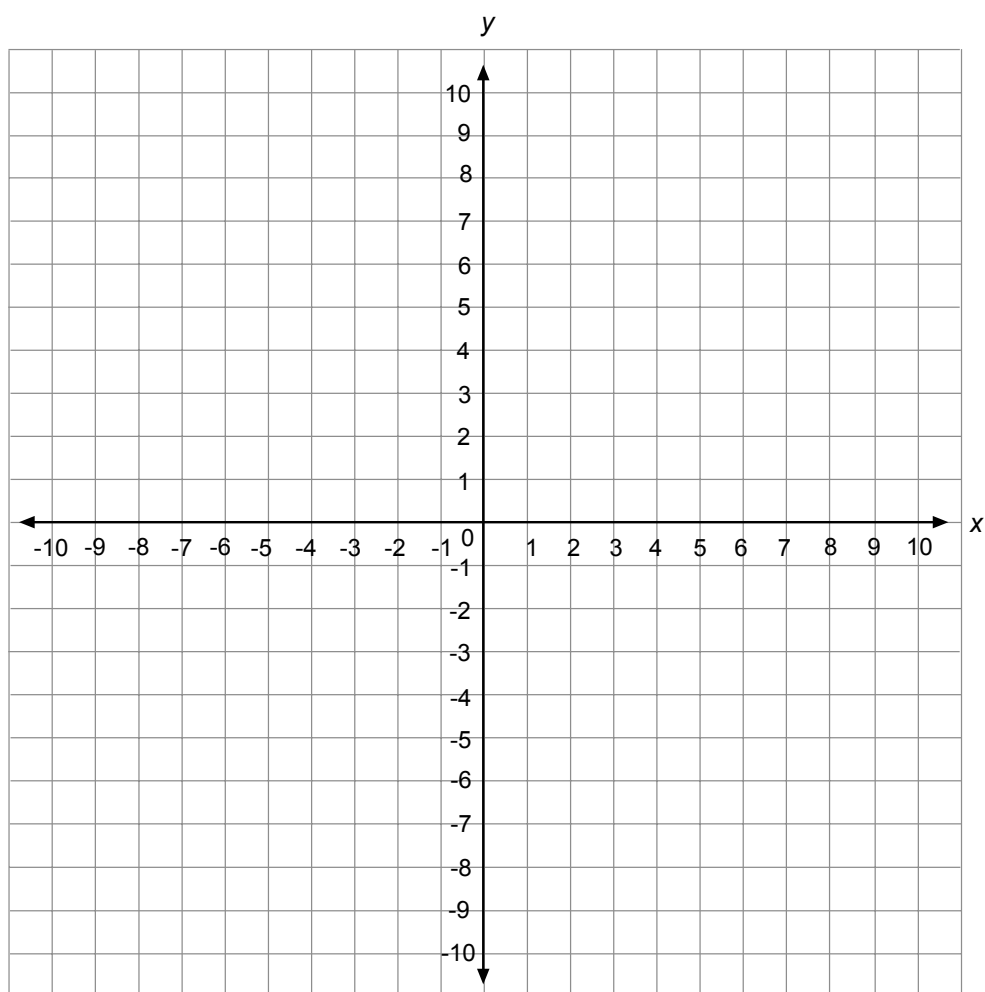
a. slope-intercept form = \_\_\_\_\_

b.  $y$ -intercept = \_\_\_\_\_

c. slope = \_\_\_\_\_

d. graph

**Graph of  $x - y = -8$**





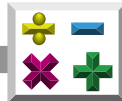
## Practice

Use the list below to write the correct term for each definition on the line provided.

<b>denominator</b>	<b>rise</b>	<b>slope</b>
<b>linear equation</b>	<b>run</b>	<b>slope-intercept form</b>
<b>numerator</b>		

- \_\_\_\_\_ 1. the vertical change on the graph between two points
- \_\_\_\_\_ 2. a form of a linear equation,  $y = mx + b$ , where  $m$  is the slope of the line and  $b$  is the  $y$ -intercept
- \_\_\_\_\_ 3. the ratio of change in the vertical axis ( $y$ -axis) to each unit change in the horizontal axis ( $x$ -axis) in the form  $\frac{\text{rise}}{\text{run}}$ ; the constant,  $m$ , in the linear equation for the slope-intercept form  $y = mx + b$
- \_\_\_\_\_ 4. the top number of a fraction, indicating the number of equal parts being considered
- \_\_\_\_\_ 5. the bottom number of a fraction, indicating the number of equal parts a whole was divided into
- \_\_\_\_\_ 6. the horizontal change on a graph between two points
- \_\_\_\_\_ 7. an equation whose graph in a coordinate plane is a straight line; an algebraic equation in which the variable quantity or quantities are raised to the zero or first power only and the graph is a straight line





## **Lesson Five Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

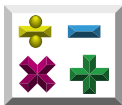
#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

### **Algebra Body of Knowledge**

#### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.10  
Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line.



## Geometry Body of Knowledge

### Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4  
Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

### Parallel and Perpendicular Lines

When two lines are on the same coordinate plane, there are two possibilities. Either the two lines are **parallel** ( $\parallel$ ) to each other or they **intersect** each other.

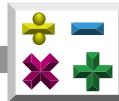
If two lines are *parallel* to each other, we can say that the lines are always the same distance apart and will never *intersect*. This happens when the two lines have the same *slant*. In other words, two **parallel lines** have *equal* slopes.

For example, the two lines,  $y = 5x + 13$  and  $y = 5x - 6$  are parallel because in each line,  $m$  has a value of 5.

If two lines intersect, they cross each other at some point. You may not see that point where they cross on the particular picture, but remember that lines extend forever and their slopes may be such that they will eventually cross. If the two lines intersect at a **right angle** or at 90 **degrees** ( $^\circ$ ), they are **perpendicular** ( $\perp$ ). Keep in mind that when this happens, their slopes will be negative **reciprocals** of each other.

A line whose equation is  $y = \frac{3}{2}x - 5$  is *perpendicular* to a line whose equation is  $y = -\frac{2}{3}x + 6$ . Notice that their slopes are  $\frac{3}{2}$  and  $-\frac{2}{3}$ .

**Note:** If you multiply the slopes of two **perpendicular lines**, the **product** will be -1, unless one of the lines was vertical.



## Practice

Use the slope formula below to find the **slopes** of  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$ . Then **multiply the slopes** to determine if they are **parallel**, **perpendicular**, or **neither**. Show **all your work**. Write the answer on the line provided. The first one has been done for you.

slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Remember:

- If slopes are **equal**, the lines are **parallel**.
- If slopes are **negative reciprocals**, the lines are **perpendicular**.

parallel

1. A (3, 2), B (-5, 6), C (-4, 1), D (-2, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

$$\frac{6 - 2}{-5 - 3} =$$

$$\frac{6 + -2}{-5 + -3} =$$

$$\frac{4}{-8} =$$

$$-\frac{1}{2} =$$

$$m = y_2 - y_1 =$$

$$\frac{0 - 1}{-2 - -4} =$$

$$\frac{0 + -1}{-2 + +4} =$$

$$-\frac{1}{2}$$

The slopes are equal; therefore, the lines are *parallel*.



\_\_\_\_\_ 2.  $A(5, 7), B(0, 4), C(2, -6), D(-3, 7)$

\_\_\_\_\_ 3.  $A(2, 4), B(-6, -6), C(3, -3), D(-1, -8)$

\_\_\_\_\_ 4.  $A(0, 5), B(3, 5), C(6, 7), D(6, 3)$



**Remember:**

$\frac{0}{x}$  = a line that is horizontal with a zero (0) slope

$\frac{y}{0}$  = a line that is vertical with no slope



\_\_\_\_\_ 5.  $A(3, 8), B(4, 5), C(0, 0), D(6, -4)$

\_\_\_\_\_ 6.  $A(4, 4), B(-4, -4), C(-4, 4), D(4, -4)$

\_\_\_\_\_ 7.  $A(-2, -2), B(2, 4), C(1, 6), D(-1, 3)$

\_\_\_\_\_ 8.  $A(8, -8), B(0, -6), C(3, 13), D(-3, -11)$



## Practice

Put equations in **slope-intercept form**. Show all your work. Determine if the following **lines** are **parallel**, **perpendicular**, or **neither**. Write the answer on the line provided.

**slope-intercept form**

$$y = mx + b$$

\_\_\_\_\_ 1.  $3x + y = 7$   
 $y + 6 = 3x$

\_\_\_\_\_ 2.  $x - y = -6$   
 $x + y = 6$

\_\_\_\_\_ 3.  $x - 3y = -21$   
 $x + 3y = 21$



\_\_\_\_\_ 4.  $5x - y = 9$   
 $5x - y = 4$

\_\_\_\_\_ 5.  $3x - y = 4$   
 $4x - y = -3$

\_\_\_\_\_ 6.  $x = 6$   
 $y = -1$

\_\_\_\_\_ 7.  $2x + 3y = 5$   
 $3x - 2y = 7$



## Practice

Use the list below to complete the following statements.

distance	line segment	perpendicular
horizontal	midpoint	slope
hypotenuse	parallel	vertical

1. The slant or \_\_\_\_\_ of a line is defined as  $\frac{\text{rise}}{\text{run}}$ .
2. A line that has no slope is called a \_\_\_\_\_ line.
3. The \_\_\_\_\_ between two points is the length of the segment that connects the two points.
4. The \_\_\_\_\_ is the segment in a right triangle that is opposite the right angle.
5. Lines in the same plane that do not intersect are called \_\_\_\_\_ lines.
6. A line that has zero slope is a \_\_\_\_\_ line.
7. The point located exactly halfway between two endpoints of a line segment is called the \_\_\_\_\_.
8. If two lines intersect to form right angles, they are \_\_\_\_\_ lines.
9. The figure that contains two defined endpoints and all the points in between is called a \_\_\_\_\_.

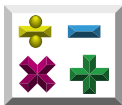




## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |   |                              |
|---|------------------------------|
| _____ 1. the square of the hypotenuse ( $c$ ) of a right triangle is equal to the sum of the square of the legs ( $a$ and $b$ ), as shown in the equation $c^2 = a^2 + b^2$ | A. formula                   |
| _____ 2. two lines, two line segments, or two planes that intersect to form a right angle   | B. intersect                 |
| _____ 3. an angle whose measure is exactly $90^\circ$   | C. parallel lines            |
| _____ 4. two lines in the same plane that are a constant distance apart; lines with equal slopes  | D. perpendicular ( $\perp$ ) |
| _____ 5. two numbers whose product is 1; also called <i>multiplicative inverses</i>   | E. product                   |
| _____ 6. to meet or cross at one point  | F. Pythagorean theorem       |
| _____ 7. a way of expressing a relationship using variables or symbols that represent numbers   | G. reciprocals               |
| _____ 8. the result of multiplying numbers together   | H. right angle               |



## **Lesson Six Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

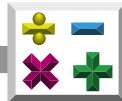
## **Algebra Body of Knowledge**

### **Standard 1: Real and Complex Number Systems**

- MA.912.A.1.8  
Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.7  
Rewrite equations of a line into slope-intercept form and standard form.



- MA.912.A.3.10  
Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line.

## Geometry Body of Knowledge

### Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4  
Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

## Point-Slope Form

We know that if we have two points we can draw a line that connects them. But did you know we can also produce the equation of that line using those points?

To do this, we will use yet another format for the equation of a line. It is called the point-slope form. Notice that it looks a bit like the slope-intercept format, but it has a little extra.

$$(y - y_1) = m(x - x_1) \quad \text{point-slope form}$$

$(x_1, y_1)$  is one of the coordinates given

$m$  = slope



Let's see how this works.

### Example 1

Find the equation of the line which passes through points (3, 5) and (-2, 1).

- Start with the following equation.

$$y - y_1 = m(x - x_1)$$

- Find the slope using the two points.

$$m = \frac{1-5}{-2-3} = \frac{-4}{-5} = \frac{4}{5} \quad \leftarrow \text{the slope is } \frac{4}{5}$$

- Select one of the given points (3, 5).
- Replace  $x_1$  and  $y_1$  with the coordinates from the point you selected, and then replace  $m$  with the slope ( $\frac{4}{5}$ ) that you found.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= \frac{4}{5}(x - 3) \end{aligned}$$

- Simplify.

$$\begin{aligned} y - 5 &= \frac{4}{5}(x - 3) \\ y - 5 &= \frac{4}{5}x - \frac{12}{5} && \leftarrow \text{distribute } \frac{4}{5} \\ y &= \frac{4}{5}x - \frac{12}{5} + 5 && \leftarrow \text{add 5 to both sides} \\ y &= \frac{4}{5}x - \frac{12}{5} + \frac{25}{5} && \leftarrow \text{get a **common denominator**} \\ y &= \frac{4}{5}x + \frac{13}{5} && \leftarrow \text{simplify} \end{aligned}$$

This is the equation of the line in slope-intercept form:  $y = mx + b$  with  $m = \frac{4}{5}$ ,  $b = \frac{13}{5}$ .

We could also transform this equation to standard form of  $ax + by = c$  using a bit of algebra.

$$\begin{aligned} y &= \frac{4}{5}x + \frac{13}{5} \\ -\frac{4}{5}x + y &= \frac{13}{5} && \leftarrow \text{subtract } \frac{4}{5}x \text{ from both sides} \\ 4x - 5y &= -13 && \leftarrow \text{multiply both sides by -5} \end{aligned}$$



How about another example before you try this yourself?

### Example 2

Find the equation of the line in both  $y$ -intercept and standard form that passes through the points  $(-4, 0)$  and  $(-2, 2)$ .

- Start with the following equation.

$$y - y_1 = m(x - x_1)$$

- Find the slope.

$$m = \frac{2-0}{-2-(-4)} = \frac{2}{2} = \frac{1}{1} = 1 \quad \longleftarrow \text{the slope is 1}$$

- Select a point.

$$(-2, 2)$$

- Replace  $x_1$ ,  $y_1$ , and  $m$ .

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x - (-2))$$

- Simplify.

$$y - 2 = 1(x + 2)$$

$$y - 2 = x + 2$$

$$y = x + 4$$

$\longleftarrow$  equation in slope-intercept form

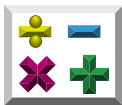
- Transform  $y = x + 4$  into standard form.

$$-x + y = 4$$

$$x - y = -4$$

$\longleftarrow$  multiply both sides by -1

$\longleftarrow$  equation in standard form



Look at some other situations when using the point-slope format is helpful.

### Example 3

Write an equation in point-slope form of the line that passes through (2, -3) and has a slope of  $-\frac{3}{8}$ .

$$y - y_1 = m(x - x_1)$$

- We can skip finding the slope—it is already done for us!

$$m = -\frac{3}{8}$$

- There is no need to select a point because we only have one to choose.

$$(2, -3)$$

- Replace  $x_1$ ,  $y_1$ , and  $m$ , and simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= -\frac{3}{8}(x - 2) \\ y + 3 &= -\frac{3}{8}(x - 2) \end{aligned}$$

Ta-da! We are finished! We have written the equation in point-slope form.



#### Example 4

Write an equation in point-slope form for a horizontal line passing through the point  $(-4, 2)$ .

$$y - y_1 = m(x - x_1)$$

- Slope = 0 (horizontal lines have zero slope)

$$m = 0$$

- Use the point  $(-4, 2)$ , replace  $x_1$ ,  $y_1$ , and  $m$ , and simplify.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 0(x - -4)$$

$$y - 2 = 0(x + 4)$$

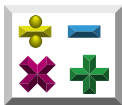
$$y - 2 = 0x + 0$$

$$y - 2 = 0$$

$$y = 2$$

All *horizontal* lines have equations that look like  $y = \text{"a number."}$  That number will *always* be the  $y$ -coordinate from any point on the line.

Time for some practice...here we go!



## Practice

Write an equation in **point-slope form** for the **line** that passes through the **given point** with the **given slope**.

1.  $(7, 2), m = -\frac{3}{4}$

2.  $(-1, -3), m = 8$

3.  $(4, -5), m = -\frac{3}{8}$

4.  $(2, 2), m = 0$





Put the following **equations of lines** into **slope-intercept form** and **standard form**.

**slope-intercept form**

$$y = mx + b$$

**standard form**

$$ax + by = c$$

5.  $y - 2 = \frac{3}{2}(x - 8)$

a. slope-intercept form = \_\_\_\_\_

b. standard form = \_\_\_\_\_

6.  $y + 3 = -5(x + 1)$

a. slope-intercept form = \_\_\_\_\_

b. standard form = \_\_\_\_\_



7.  $y + 5 = 2(x - 4)$

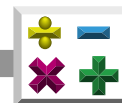
a. slope-intercept form = \_\_\_\_\_

b. standard form = \_\_\_\_\_

8.  $y - 4 = -6(x + 1)$

a. slope-intercept form = \_\_\_\_\_

b. standard form = \_\_\_\_\_

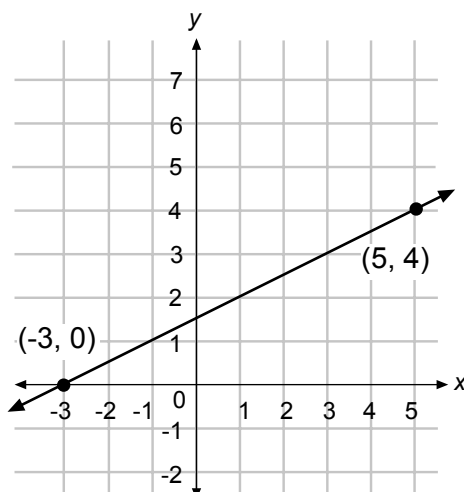


Write an equation in **slope-intercept form** for each line below.

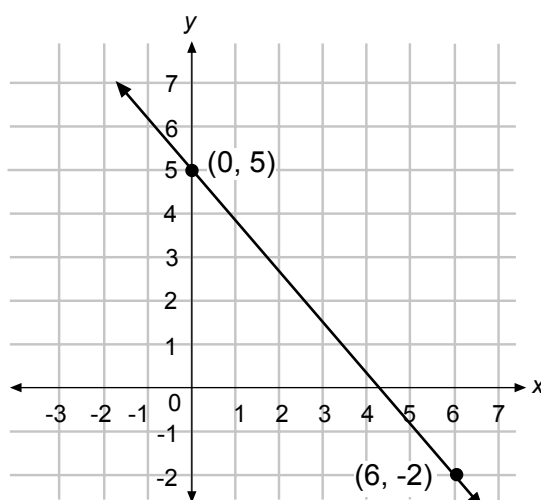
**Hint:** Use point-slope form and convert.

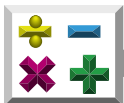
**point-slope form**  
 $(y - y_1) = m(x - x_1)$

9.

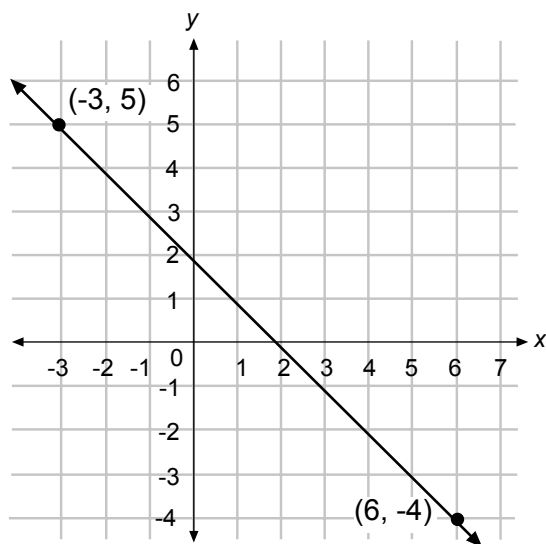


10.

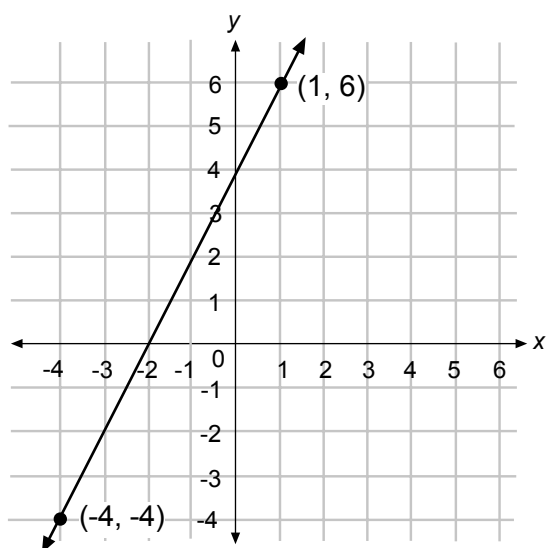




11.



12.





Write an equation in **slope-intercept form** for the **line** which passes through each **pair of points** below.

13.  $(-2, 4), (4, 5)$

14.  $(1, 0), (-3, 8)$

15.  $(0, 0), (7, -7)$

16.  $(4, -2), (-2, 8)$



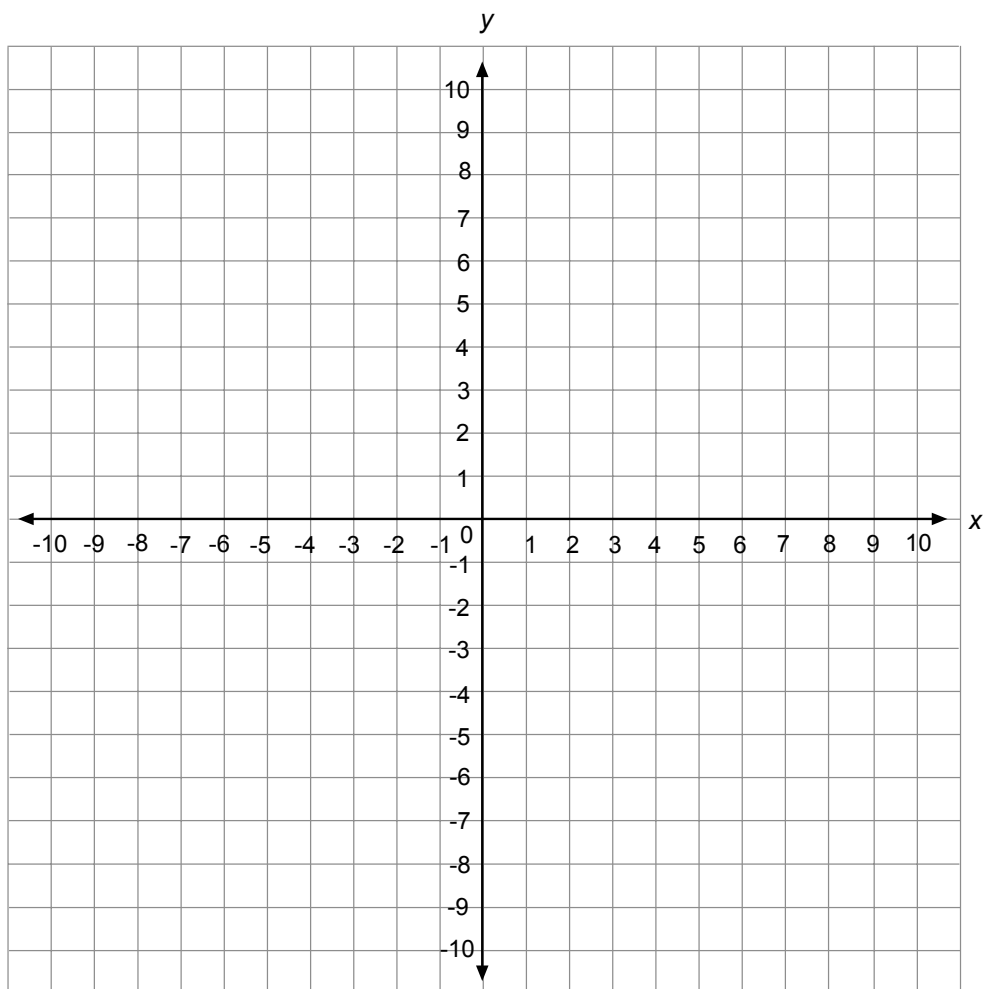
## Unit Review

*Solve the following.*

1. Plot points (3, -2) and (-6, 4). Draw a triangle and use the Pythagorean theorem below to find the distance between the two points.

**Pythagorean theorem**

$$a^2 + b^2 = c^2$$





2. Use the distance formula below to find the distance between  $(-2, 4)$  and  $(7, -3)$ .

**distance formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3. Use either the method above or the Pythagorean theorem below to find the distance between  $(5, 1)$  and  $(-1, 9)$ .

**Pythagorean theorem**

$$a^2 + b^2 = c^2$$

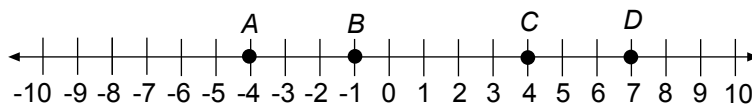


4. On the number line below, find the **midpoint** between  $A$  and  $B$ . Use either of the methods below.

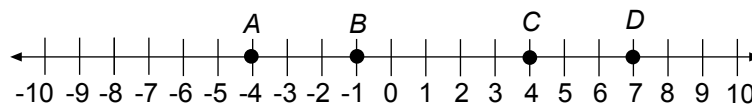
- Use the **number line and count in from both endpoints** of a line segment until you reach the middle to determine the midpoint.
- Use the **Method One midpoint formula** and add the two endpoints together, then divide by two. **Show all your work.**

**Method One midpoint formula**

$$\frac{a + b}{2}$$



5. On the number line below, find the **midpoint** of  $\overline{BD}$ . Use either method above.








Use the list below to correctly describe the following lines. Write the answer on the line provided.

falling  
horizontal  
rising  
vertical

\_\_\_\_\_ 6. 

\_\_\_\_\_ 7. 

\_\_\_\_\_ 8. 

\_\_\_\_\_ 9. 



Use the **slope formula** below to find the **slopes** through each **pair of points**.

**slope formula**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

10.  $(3, -8), (5, 7)$

11.  $(-2, 0), (6, -3)$

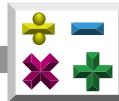
Use the **slope-intercept form** below to find the **slope** for each **line**.

**slope-intercept form**

$$y = mx + b$$

12.  $y = \frac{1}{2}x - 7$

13.  $y = -2x + 6$



Use the **slope formula** below to find the **slopes** of  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$ . Then **multiply the slopes** to determine if they are **parallel**, **perpendicular**, or **neither**. **Show all your work**. Write the answer on the line provided. .

**slope formula**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

\_\_\_\_\_ 14. A (2, -5), B (4, 5), C (-3, 8), D (2, 7)

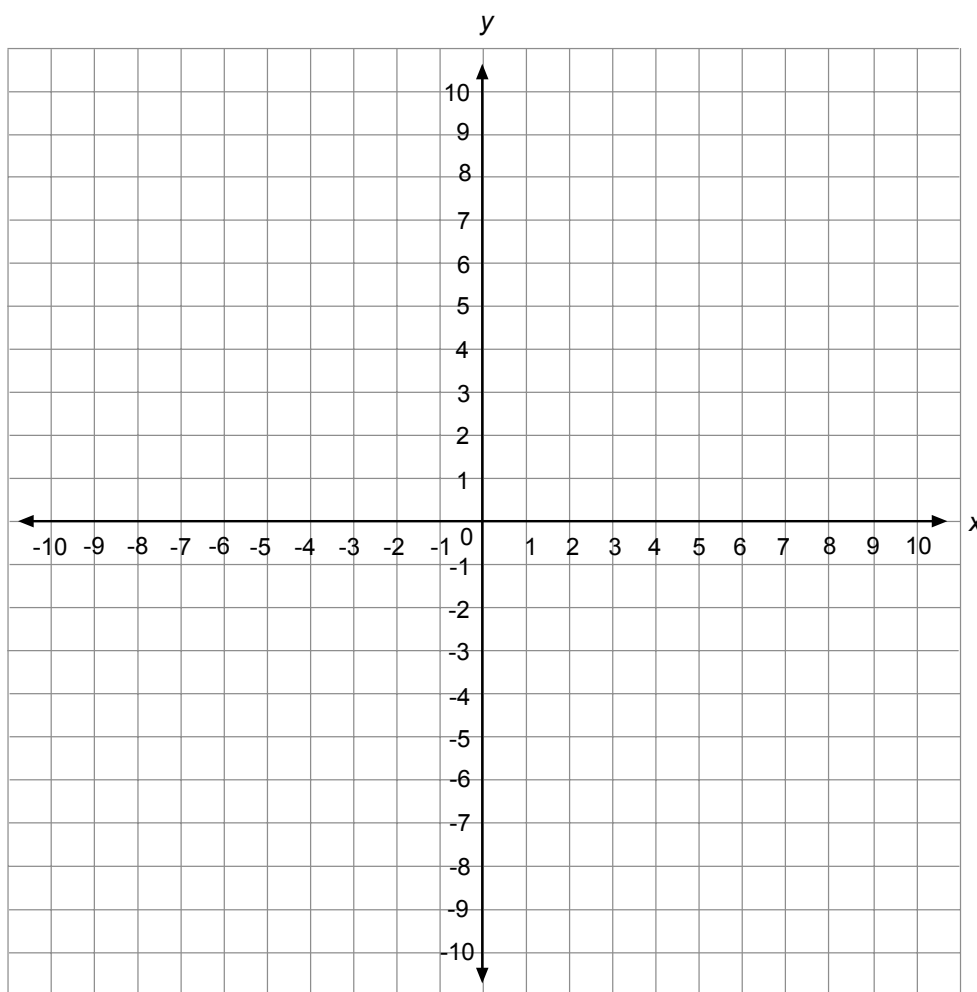
\_\_\_\_\_ 15. A (-4, 0), B (6, 1), C (4, 3), D (-6, 2)



Use the equation  $y = -\frac{3}{2}x + 4$  to answer each of the following.

16. Give the  $y$ -intercept. \_\_\_\_\_
17. Give the slope. \_\_\_\_\_
18. Graph the line.

**Graph of  $y = -\frac{3}{2}x + 4$**



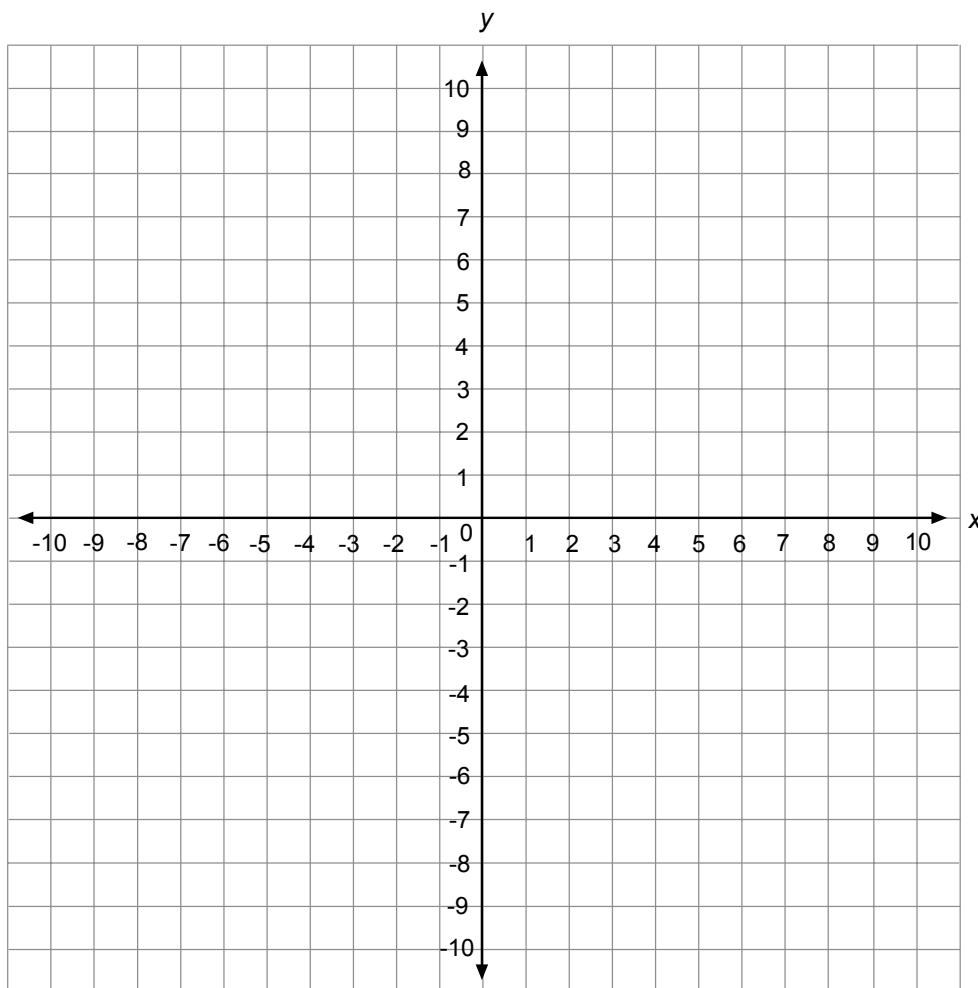
19. Write the equation in standard form.



Use the equation  $2x - 4y = -12$  to answer the following.

20. Find the  $y$ -intercept. \_\_\_\_\_
21. Find the  $x$ -intercept. \_\_\_\_\_
22. Graph the line.

**Graph of  $2x - 4y = -12$**





Express these lines in **slope-intercept form**.

23.  $2x - 3y = 4$

24.  $2x + y = 16$

Put equations in **slope-intercept form**. **Show all your work**. Determine if the following **lines** are **parallel**, **perpendicular**, or **neither**. Write the answer on the line provided.

\_\_\_\_\_ 25.  $x - 2y = 12$   
 $2x - y = -6$

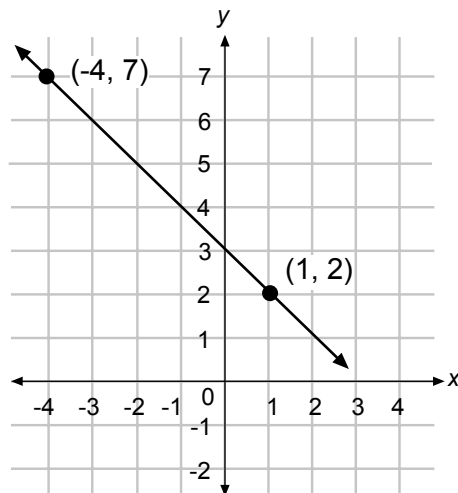
\_\_\_\_\_ 26.  $5x - 8y = -8$   
 $8x + 5y = -15$



27. Write an equation in point-slope form for the line that passes through point  $(-2, 5)$  with a slope of  $\frac{2}{3}$ .

28. Put  $y - 4 = \frac{3}{8}(x + 2)$  into slope-intercept form.

29. Write an equation in slope-intercept form for the line given.



30. Write an equation in slope-intercept form for the line which passes through points  $(-3, 8)$  and  $(5, 7)$ .





## **Unit 9: Having Fun with Functions**

Students will learn and use the terminology and symbolism associated with functions.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Writing Process Strand**

##### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

### **Algebra Body of Knowledge**

##### **Standard 1: Real and Complex Number Systems**

- MA.912.A.1.8  
Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

**Standard 2: Relations and Functions**

- MA.912.A.2.3  
Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions.
- MA.912.A.2.4  
Determine the domain and range of a relation.
- MA.912.A.2.13  
Solve real-world problems involving relations and functions.

**Standard 3: Linear Equations and Inequalities**

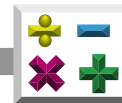
- MA.912.A.3.11  
Write an equation of a line that models a data set and use the equation or the graph to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

**Standard 7: Quadratic Equations**

- MA.912.A.7.1  
Graph quadratic equations with and without graphing technology.
- MA.912.A.7.10  
Use graphing technology to find approximate solutions of quadratic equations.

**Standard 10: Mathematical Reasoning and Problem Solving**

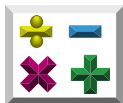
- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).



## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

- axis of symmetry** .....vertical line passing through the maximum or minimum point of a parabola
- coefficient** .....the number that multiplies the variable(s) in an algebraic expression  
*Example:* In  $4xy$ , the coefficient of  $xy$  is 4.  
If no number is specified, the coefficient is 1.
- coordinates** .....numbers that correspond to points on a coordinate plane in the form  $(x, y)$ , or a number that corresponds to a point on a number line
- data** .....information in the form of numbers gathered for statistical purposes
- domain** .....set of  $x$ -values of a relation
- element** .....one of the objects in a set
- equation** .....a mathematical sentence stating that the two expressions have the same value  
*Example:*  $2x = 10$
- estimation** .....the use of rounding and/or other strategies to determine a reasonably accurate approximation, without calculating an exact answer  
*Examples:* clustering, front-end estimating, grouping, etc.



**expression** ..... a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables

*Examples:*  $4r^2$ ;  $3x + 2y$ ;  $\sqrt{25}$

An expression does *not* contain equal (=) or inequality (<, >, ≤, ≥, or ≠) signs.

**factor** ..... a number or expression that divides evenly into another number; one of the numbers multiplied to get a product

*Examples:* 1, 2, 4, 5, 10, and 20 are factors of 20 and  $(x + 1)$  is one of the factors of  $(x^2 - 1)$ .

**factoring** ..... expressing a polynomial expression as the product of monomials and polynomials

*Example:*  $x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

**FOIL method** ..... a pattern used to multiply two binomials; multiply the first, outside, inside, and last terms:

**F** First terms

**O** Outside terms

**I** Inside terms

**L** Last terms.

*Example:*

$$\begin{array}{c}
 \begin{array}{ccc}
 & 2 \text{ Outside} & \\
 \swarrow & & \searrow \\
 (a + b)(x - y) & & \\
 \nwarrow & & \nearrow \\
 & 4 \text{ Last} & 
 \end{array}
 \end{array}
 \begin{array}{c}
 1 \text{ First} \\
 \swarrow \quad \searrow \\
 \nwarrow \quad \nearrow \\
 3 \text{ Inside}
 \end{array}
 = \overset{\text{F}}{ax} - \overset{\text{O}}{ay} + \overset{\text{I}}{bx} - \overset{\text{L}}{by}$$

**function notation** ..... a way to name a function that is defined by an equation

*Example:* In function notation, the equation  $x = 5x + 4$  is written as  $f(x) = 5x + 4$ .

**function (of  $x$ )** ..... a relation in which each value of  $x$  is paired with a unique value of  $y$

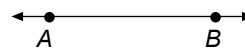


**horizontal** .....parallel to or in the same plane of the horizon



**intersect** .....to meet or cross at one point

**line ( $\leftrightarrow$ )** .....a collection of an infinite number of points forming a straight path extending in opposite directions having unlimited length and no width



**linear function** .....an equation whose graph is a nonvertical line

**maximum** .....the highest point on the vertex of a parabola, which opens downward

**mean (or average)** .....the arithmetic average of a set of numbers; a measure of central tendency

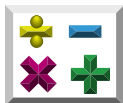
**minimum** .....the lowest point on the vertex of a parabola, which opens upward

**ordered pair**.....the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the  $x$ -axis and  $y$ -axis, respectively  
*Examples:  $(x, y)$  or  $(3, -4)$*

**origin** .....the point of intersection of the  $x$ - and  $y$ -axes in a rectangular coordinate system, where the  $x$ -coordinate and  $y$ -coordinate are both zero (0)

**parabola** .....the graph of a quadratic equation

**point** .....a specific location in space that has no discernable length or width



**quadratic equation** .....an equation in the form of  
 $ax^2 + bx + c = 0$

**quadratic function** .....an equation in the form  
 $y = ax^2 + bx + c$ , where  $a \neq 0$

**range** .....set of  $y$ -values of a relation

**relation** .....a set of ordered pairs  $(x, y)$

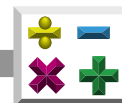
**roots** .....the solutions to a quadratic equation

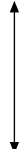
**rounded number** .....a number approximated to a specified place  
*Example:* A commonly used rule to round a number is as follows.

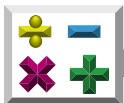
- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place (461 rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place (441 rounded to the nearest hundred is 400).

**set** .....a collection of distinct objects or numbers

**slope** .....the ratio of change in the vertical axis ( $y$ -axis) to each unit change in the horizontal axis ( $x$ -axis) in the form  $\frac{\text{rise}}{\text{run}}$  or  $\frac{\Delta y}{\Delta x}$ ; the constant,  $m$ , in the linear equation for the slope-intercept form  $y = mx + b$



- solution** .....any value for a variable that makes an equation or inequality a true statement  
*Example:* In  $y = 8 + 9$   
 $y = 17$       17 is the solution.
- solve** .....to find all numbers that make an equation or inequality true
- value (of a variable)** .....any of the numbers represented by the variable
- variable** .....any symbol, usually a letter, which could represent a number
- vertex** .....the maximum or minimum point of a parabola
- vertical** .....at right angles to the horizon; straight up and down 
- vertical line test** .....if any vertical line passes through no more than one point of the graph of a relation, then the relation is a function
- $x$ -axis** .....the horizontal number line on a rectangular coordinate system
- $x$ -intercept** .....the value of  $x$  at the point where a line or graph intersects the  $x$ -axis; the value of  $y$  is zero (0) at this point
- $y$ -axis** .....the vertical number line on a rectangular coordinate system



**$y$ -intercept** .....the value of  $y$  at the point where a line or graph intersects the  $y$ -axis; the value of  $x$  is zero (0) at this point

**zero product property** .....for all numbers  $a$  and  $b$ , if  $ab = 0$ , then  $a = 0$  and/or  $b = 0$

**zeros** .....the points where a graph crosses the  $x$ -axis; the roots, or  $x$ -intercepts, of a quadratic function





## Unit 9: Having Fun with Functions

### Introduction

We will explore a number of relations through the use of tables and graphs. We will create tables and graphs for specific problems. We will also link equations to functions when given a function.

### Lesson One Purpose

#### Reading Process Strand

##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### Writing Process Strand

##### Standard 3: Prewriting

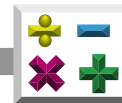
- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.



## **Algebra Body of Knowledge**

### **Standard 2: Relations and Functions**

- MA.912.A.2.3  
Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions.
- MA.912.A.2.4  
Determine the domain and range of a relation.



## Functions

In the unit on Venn diagrams we learned that a **set of ordered pairs**, such as  $\{(2, 4), (3, 8), (5, 7), (-2, 1)\}$ , is called a **relation**. Each **element** in a *relation* has a **value**—an  $x$ -value and a  $y$ -value  $(x, y)$ . The **ordered pairs** are called **coordinates**  $(x, y)$  of a point on a graph.

The *set* containing all of the  $x$ -values is called the **domain**, while the set of all  $y$ -values is called the **range**.

From the example  $\{(2, 4), (3, 8), (5, 7), (-2, 1)\}$  the *domain* would be  $\{2, 3, 5, -2\}$  and the *range* would be  $\{4, 8, 7, 1\}$ .

A *relation* in which no  $x$ -value is repeated is called a **function**. Another way to say that is each element of the domain is paired with only one element of the range.

### Set of Ordered Pairs—Relation

$\{(2, 4), (3, 8), (5, 7), (-2, 1)\}$

$x$ -values, or first numbers of the ordered pairs—domain =  $\{2, 3, 5, -2\}$



$(2, 4)$   
 $(3, 8)$   
 $(5, 7)$   
 $(-2, 1)$

This relation is a function because of the following.

- no  $x$ -value is repeated
- each element of the domain, or  $x$ -values, can be paired with only one element of the range, or  $y$ -values



$y$ -values, or second numbers of the ordered pairs—range =  $\{4, 8, 7, 1\}$

**Note:** Usually *values* are listed in numerical order. However, for giving the domain ( $x$ -values) and the range ( $y$ -values) for relations, numerical order is *not* required. If a value in a domain or in a range is repeated, list the value *one* time.



## Practice

Decide if the **relations** below are **functions**. Write **yes** if it is a function, write **no** if it is not a function.

- \_\_\_\_\_ 1.  $\{(2, 3), (5, 6), (4, 9), (3, 8)\}$
- \_\_\_\_\_ 2.  $\{(3, 6), (4, 7), (3, -9), (8, 2)\}$
- \_\_\_\_\_ 3.  $\{(4, 2), (2, 4), (3, 6), (6, 3)\}$
- \_\_\_\_\_ 4.  $\{(4, -1), (5, 8), (4, 6), (3, 0)\}$
- \_\_\_\_\_ 5.  $\{(10, 4), (8, -6), (0, 0), (10, 3)\}$
- \_\_\_\_\_ 6.  $\{(6, 3), (6, 2), (6, 0), (6, -2)\}$
- \_\_\_\_\_ 7.  $\{(4, 1), (5, 1), (6, 1)\}$
- \_\_\_\_\_ 8.  $\{(3, 4), (4, 5), (5, 6), (6, 7), (7, 8)\}$



## Practice

Give the **domain** and the **range** for each **relation**.



**Remember:** The domains and ranges do *not* have to be listed in numerical order. If a value in a domain or in a range is repeated, list the value *one* time.

1.  $\{(2, 3), (5, 6), (4, 9), (3, 8)\}$

a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_

2.  $\{(3, 6), (4, 7), (3, -9), (8, 2)\}$

a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_

3.  $\{(4, 2), (2, 4), (3, 6), (6, 3)\}$

a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_

4.  $\{(4, -1), (5, 8), (4, 6), (3, 0)\}$

a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_



5.  $\{(10, 4), (8, -6), (0, 0), (10, 3)\}$

a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_

6.  $\{(6, 3), (6, 2), (6, 0), (6, -2)\}$

a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_

7.  $\{(4, 1), (5, 1), (6, 1)\}$

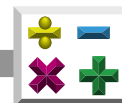
a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_

8.  $\{(3, 4), (4, 5), (5, 6), (6, 7), (7, 8)\}$

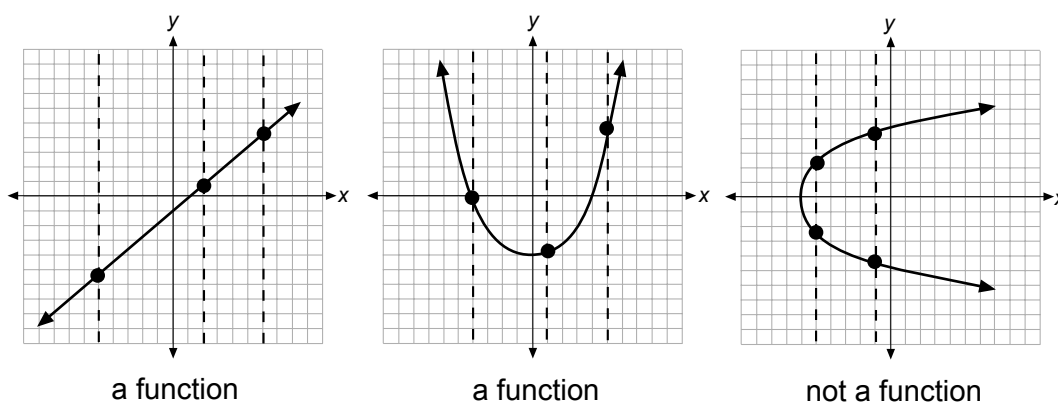
a. domain = \_\_\_\_\_

b. range = \_\_\_\_\_

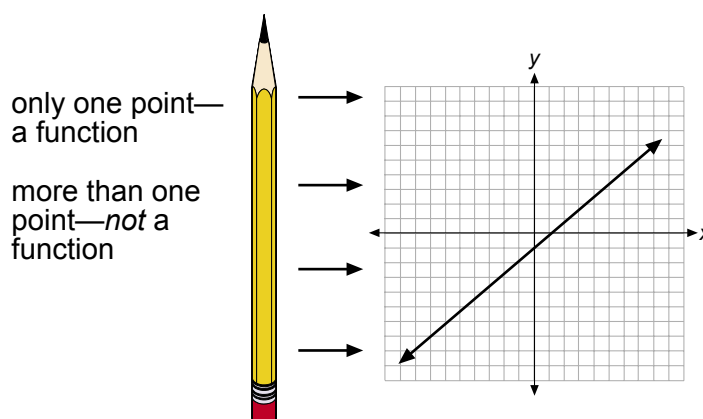


## Graphs of Functions

Using the **vertical line test**, it is possible to tell from a graph whether a relation is a *function* or not. If any **vertical** line (line that is straight up and down) can be drawn that touches the graph at *no more than one point* of the graph, then the relation is a function. However, if the *vertical* line touches the graph at *more than one point*, the relation is *not* a function.



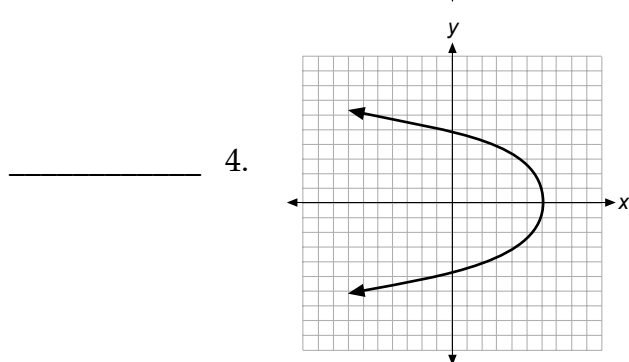
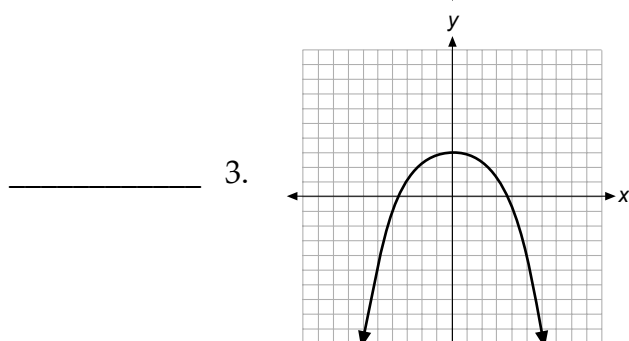
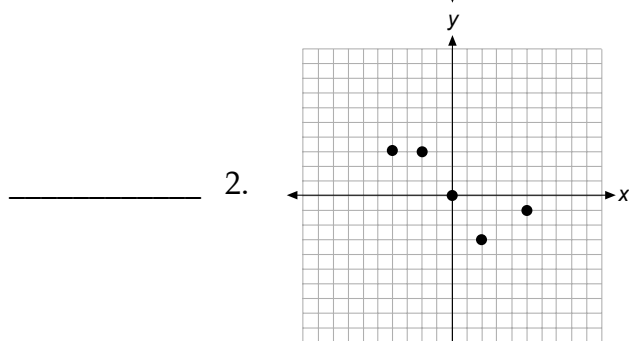
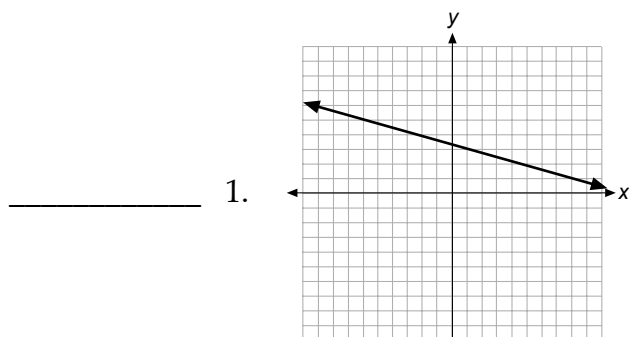
**Tip:** A vertical test line can use any straight-edged object, such as a pencil or pen, to perform the test. Place your pencil next to the graph. Line the pencil up vertically with the graph and move it slowly across the graph.



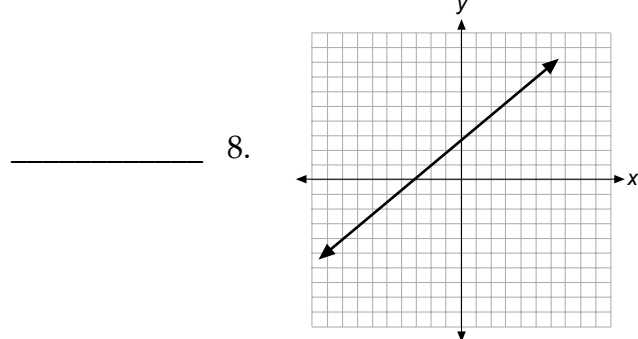
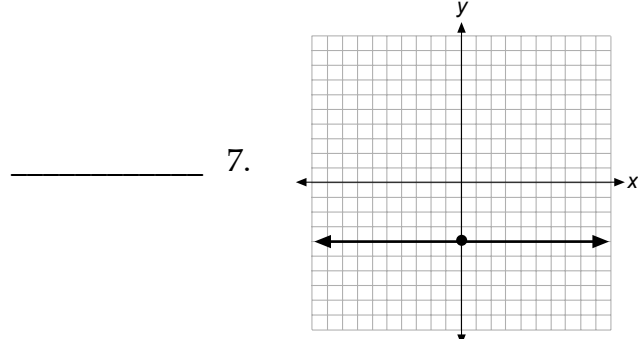
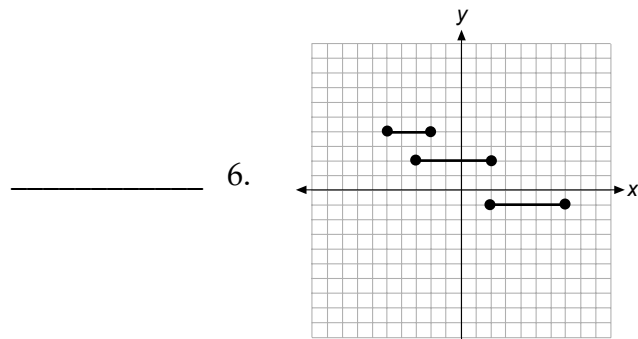
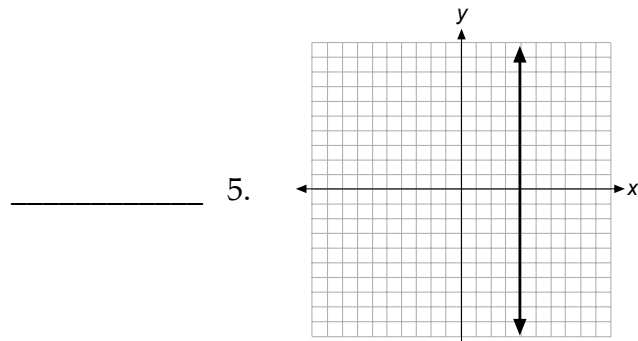


## Practice

Determine if these **graphs** represent **functions**. Write **yes** if it is a function. Write **no** if it is not a function.







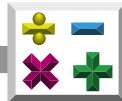


## Practice

Use the list below to complete the following statements.

<b>domain</b>	<b>range</b>
<b>function</b>	<b>relation</b>
<b>ordered pair</b>	

1.  $(x, y)$  represents a(n) \_\_\_\_\_ .
2. A set of ordered pairs is called a(n) \_\_\_\_\_ .
3. A relation in which no  $x$ -value is repeated is called a(n)  
\_\_\_\_\_ .
4. The set of  $x$ -values from a relation is the \_\_\_\_\_ .
5. The set containing the  $y$ -values from a set of ordered pairs is the  
\_\_\_\_\_ .



## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Algebra Body of Knowledge**

#### **Standard 2: Relations and Functions**

- MA.912.A.2.3  
Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions.



## The Function of X

Functions are so important that they have their own notation called a **function notation**. A *function notation* is a way to name a function defined by an **equation**. An *equation* is a mathematical sentence stating that the two **expressions** have the same value, connected by an equal sign (=). Think of a function as a math machine that will work problems the way you instruct it.

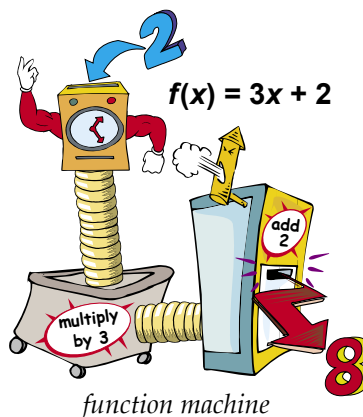
### Function Machine

$$\boxed{f(x) = 3x + 2}$$

Notice the notation on the function machine above— $f(x) = 3x + 2$ . The  $f(x)$  is read “the function of  $x$ .” We sometimes shorten that and read the entire sentence as  $f$  of  $x$  equals  $3x + 2$ .

The machine works when you put in numbers from a domain (set of  $x$ -values). So if our domain is  $\{2, 4, 5, 9\}$  and we use the function machine, we get the following.

2	$\boxed{f(x) = 3x + 2}$	8	because $3(2) + 2 = 8$
4	$\boxed{f(x) = 3x + 2}$	14	$3(4) + 2 = 14$
5	$\boxed{f(x) = 3x + 2}$	17	$3(5) + 2 = 17$
9	$\boxed{f(x) = 3x + 2}$	29	$3(9) + 2 = 29$



So now we see our range ( $y$ -values) is  $\{8, 14, 17, 29\}$ .

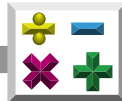
Together the domain and range give us the relation.

$$\{(2, 8), (4, 14), (5, 17), (9, 29)\}$$

This relation is a function because no  $y$ -value is repeated.

Although  $f(x)$  is most commonly used, it is not unusual to see a function expressed as  $g(x)$  or  $h(x)$  and occasionally other letters as well. Did you notice we work these the same as if the problem had read  $y = 3x + 2$ ?

Let's practice a bit, shall we?



## Practice

Use the **domain** below to give the **range** for the following **functions**.

$\{-4, -2, 0, 1, 3\}$

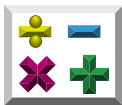


**Remember:** The range does *not* have to be listed in numerical order. If a value in a range is repeated, list the value *one* time.

1.  $f(x) = 2x - 9$

2.  $f(x) = 5x + 4$

3.  $g(x) = -2x - 3$



Use the **domain** below to give the **range** for the following **functions**.

$\{-4, -2, 0, 1, 3\}$

4.  $f(x) = x^2$

5.  $f(x) = 3x^2$

6.  $h(x) = x^2 + 2$



Use the **domain** below to give the **range** for the following **functions**.

$\{-4, -2, 0, 1, 3\}$

7.  $f(x) = (5x)^2$

8.  $g(x) = (x + 3)^2$



## **Lesson Three Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Algebra Body of Knowledge**

#### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.11  
Write an equation of a line that models a data set and use the equation or the graph to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

#### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).





## Graphing Functions

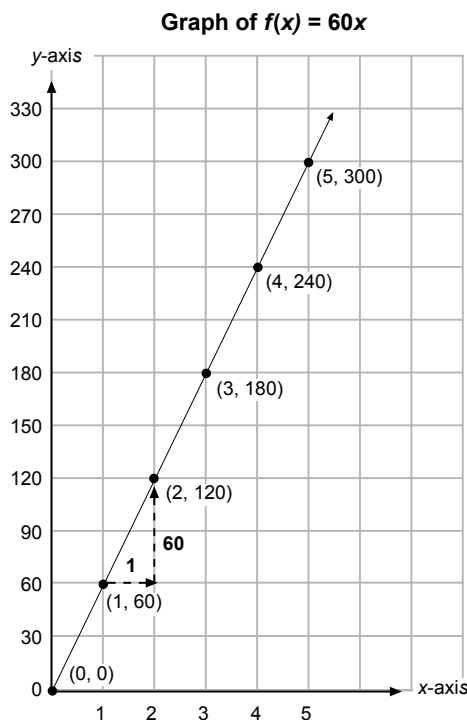
We graph functions in the same way we do equations. We can still identify **slopes** and **y-intercepts** from functions whose graph is a **line**. Remember the slope-intercept form  $y = mx + b$ ? Well, if a function is expressed as  $f(x) = mx + b$ , it is a **linear function**. A *linear function* is an equation whose graph is a nonvertical *line*.

Sometimes a graph will pass through the **origin**. That happens when  $f(0) = 0$  or when the point  $(0, 0)$  is in the relation.

As we know, the set

$$\{(0, 0), (1, 60), (2, 120), (3, 180), (4, 240), (5, 300)\}$$

can be called a relation (which is any set of ordered pairs). These ordered pairs could be graphed by hand on a coordinate grid or on a graphing calculator. A function is a relation in which each value of  $x$  is paired with a unique value of  $y$ . This relation is also a function because its graph is a *nonvertical line*.



equation:  $y = 60x$

slope:  $60$  or  $\frac{60}{1}$



### Think about This!

- As the first *coordinate* increases by 1, the second coordinate increases by 60.
- If these points were plotted, they would lie in a line.
- An equation for the line would be  $y = 60x$ .
- The line will pass through the origin so the ***x*-intercept** is  $(0, 0)$  and the ***y*-intercept** is  $(0, 0)$ .

$$\begin{array}{c} \text{60} \\ \text{(1, 60), (2, 120)} \\ \text{1} \end{array}$$

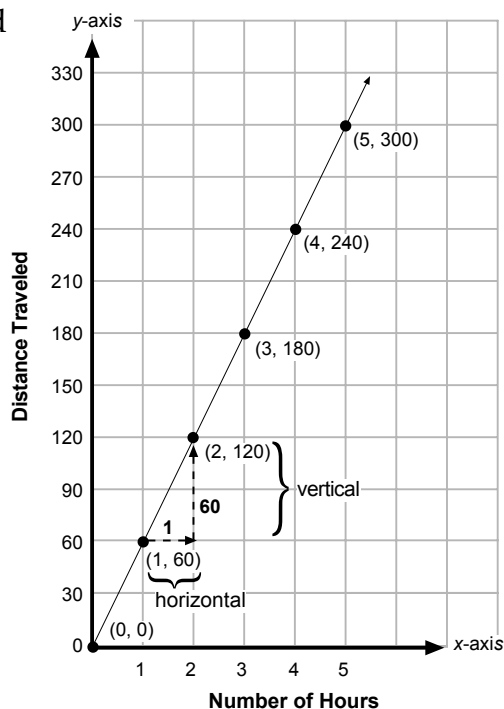


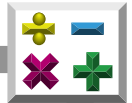
**Remember:** The *x*-intercept is the value of *x* on a graph when *y* is zero. The line passes through the ***x*-axis** at this point. The *y*-intercept is the value of *y* on a graph when *x* is zero. The line passes through the ***y*-axis** at this point.

- The slope of the line will be 60 or  $\frac{60}{1}$  because for each increase of 1 in *x*, there is an increase of 60 in *y*. From any given point on the line, a **horizontal** ( $\leftrightarrow$ ) movement of 1 unit followed by a vertical ( $\updownarrow$ ) movement of 60 units will produce another point on the line.
- If the ordered pairs are describing the distance traveled at a rate of 60 miles per hour, then *x* could represent the number of hours and *y* would represent the distance traveled.

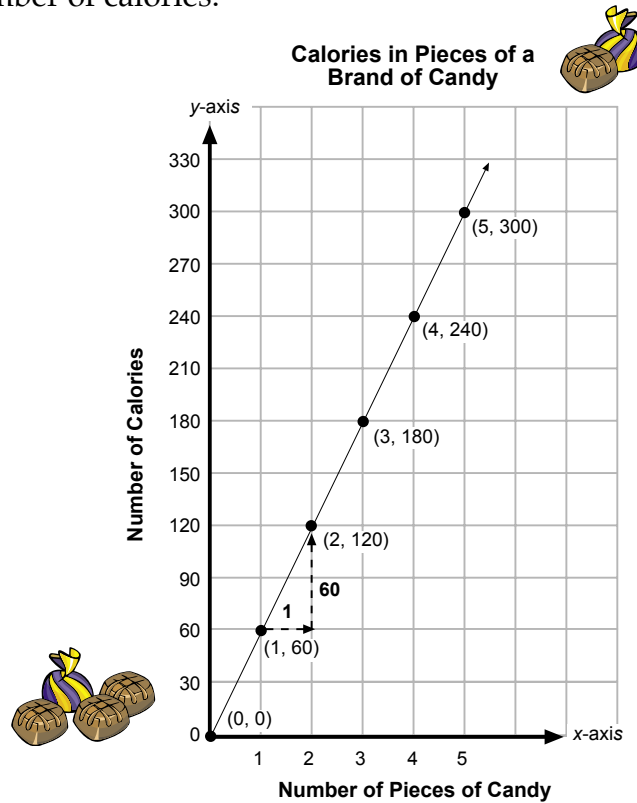
From the function  $f(x) = 60x$  or its graph, we can predict how far we could travel in 8 hours at 60 mph. If  $f(x) = 60x$  and  $x = 8$ ,  $f(8) = 60(8)$ . We could travel 480 miles in 8 hours.

Distance Traveled at a Rate of 60 Miles per Hour





- If the ordered pairs are describing the number of calories in a certain brand of candy, then  $x$  could represent the number of pieces of candy and  $y$  would represent the number of calories.



What function could be written to describe the graph above?

The function would be

$$f(x) = 60x$$

because the relationship between  $x$  and  $y$  in each ordered pair indicates that  $x$  times 60 =  $y$ .



## Practice

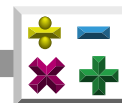
Complete the following for the **set of ordered pairs** below.

$\{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$

1. As the first coordinate increases by 1, the second coordinate increases by \_\_\_\_\_ .
2. If these points are plotted, they \_\_\_\_\_ (always, sometimes, never) lie in a line.
3. This line \_\_\_\_\_ (always, sometimes, never) passes through the origin.
4. The slope of the line will be \_\_\_\_\_ because for each increase of 1 in  $x$ , there is an increase of \_\_\_\_\_ in  $y$ . From any given point on the line, a *horizontal* ( $\leftrightarrow$ ) movement of 1 unit followed by a vertical ( $\updownarrow$ ) movement of \_\_\_\_\_ unit(s) will produce another point on the line.
5. The function for the line would be  $f(x) =$  \_\_\_\_\_ .
6. Create a situation the set of order pairs might describe.

---

---



## Practice

Graph each function **two** times. Use a **table of values** in one of your graphs and the **slope-intercept method** in the other. Use **x-values** of -2, 0, and 2 in your table.

1.  $f(x) = 3x$

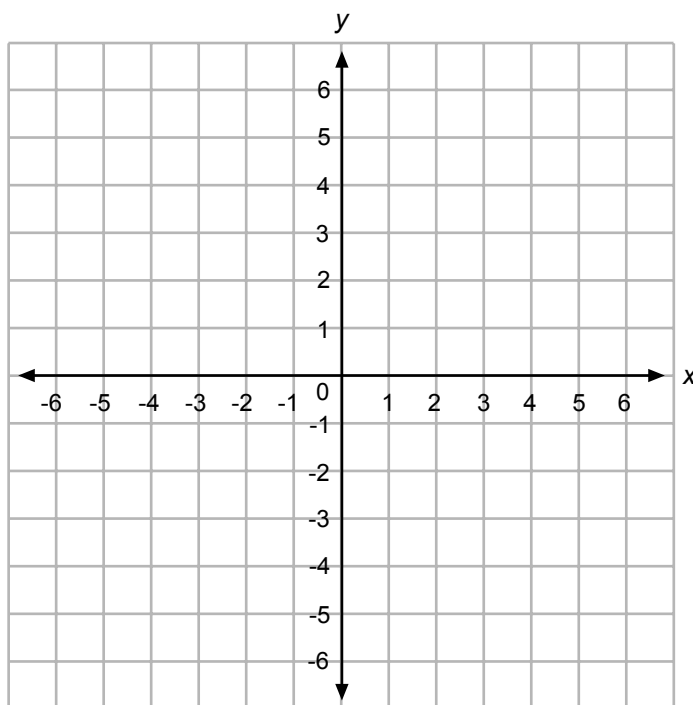
a. Table of Values Method

**Table of Values**

$f(x) = 3x$	
$x$	$f(x)$
-2	
0	
2	

b.

**Graph of  $f(x) = 3x$**





c. Slope-Intercept Method

$$f(x) = mx + b$$

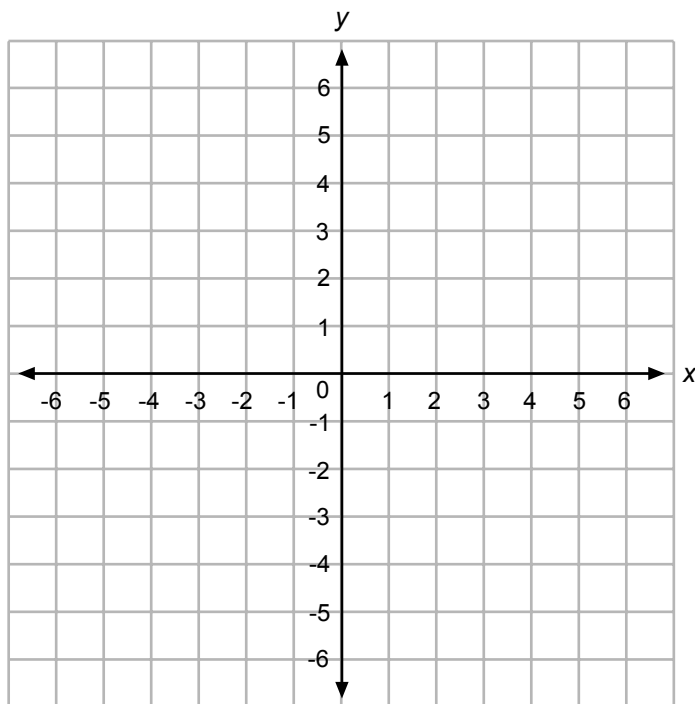
$$f(x) = 3x$$

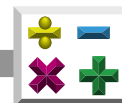


**Remember:** In the function  $f(x) = 3x$ , the **variable**  $b$ , which is the  $y$ -intercept, is zero. A *variable* is any symbol, usually a letter, which could represent a number.

Slope is \_\_\_\_\_ or  $\frac{1}{1}$

d. **Graph of  $f(x) = 3x$**





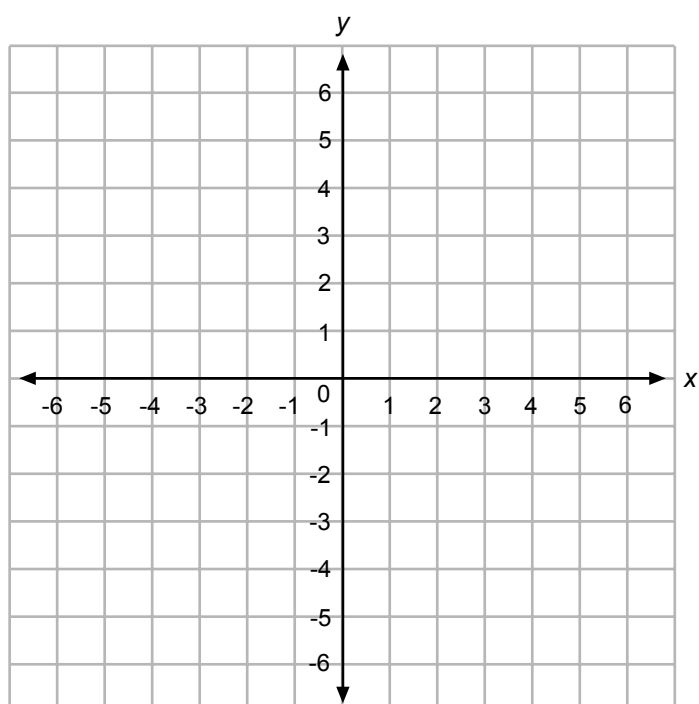
2.  $f(x) = -2x + 1$

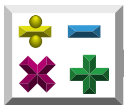
a. Table of Values Method

**Table of Values**

$f(x) = -2x + 1$	
$x$	$f(x)$
-2	
0	
2	

b. **Graph of  $f(x) = -2x + 1$**





c. Slope-Intercept Method

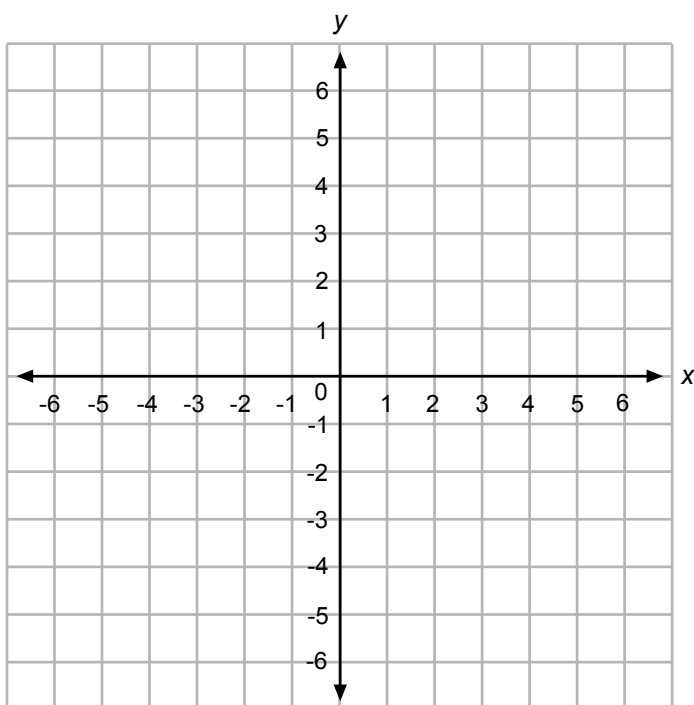
$$f(x) = mx + b$$

$$f(x) = -2x + 1$$

Slope is \_\_\_\_\_ or  $\frac{1}{1}$

d.

**Graph of  $f(x) = -2x + 1$**







## Linear Relations in the Real World

As you look ahead and consider the cost of higher education, you will find that *tuition costs* tend to represent a *linear relationship*. Universities tend to have a *fixed price for each semester hour of credit*. There is often a difference in the fixed price for a semester hour of undergraduate credit and a semester hour of graduate credit. Special areas of study may have increased costs. The following set of practices deals with costs involved in higher education. You will likely find technology, such as computer programs and some calculators support the making of tables and graphs, which are often used when considering **data** to be displayed when making comparisons.

HIGHER  
EDUCATION  
COST



## Practice

Answer the following.

1. The function used to determine the tuition for each semester hour of undergraduate credit for Florida residents at Florida State University was  $f(x) = 84.58x + 4.9x$  for all main-campus students. If a student was enrolled in a course at an extension site, the equation was  $g(x) = 84.58x$ . Most students take 12 to 15 hours each semester.

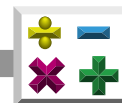
Complete the table below.

**Tuition for Florida State University—Florida Residents**

<b>Number of Semester Hours <math>x</math></b>	<b>Cost for Main-Campus Students (Florida Residents) <math>f(x) = 84.58x + 4.9x</math></b>	<b>Cost for Extension-Site Students (Florida Residents) <math>g(x) = 84.58x</math></b>
12		
13		
14		
15		

If these points were plotted on a coordinate grid, they would appear linear. If we make a graph of  $g(x) = 84.58x$  when  $x$  can be any number, we tend to show the line passing through the points.

The additional charge of \$4.90 per semester hour for main-campus students used to improve the overall campus infrastructure for all students is called the transportation access fee.



2. The function used to determine the tuition for each semester hour of undergraduate credit for non-Florida residents at Florida State University was  $f(x) = 402.71x + 4.9x$  for all main-campus students. If a student was enrolled in a course at an extension site, the equation was  $g(x) = 402.71x$ . Most students take 12 to 15 hours each semester.

Complete the table below.

**Tuition for Florida State University—Non-Florida Residents**

<b>Number of Semester Hours <math>x</math></b>	<b>Cost for Main-Campus Students (Non-Florida Residents) <math>f(x) = 402.71x + 4.9x</math></b>	<b>Cost for Extension-Site Students (Non-Florida Residents) <math>g(x) = 402.71x</math></b>
12		
13		
14		
15		

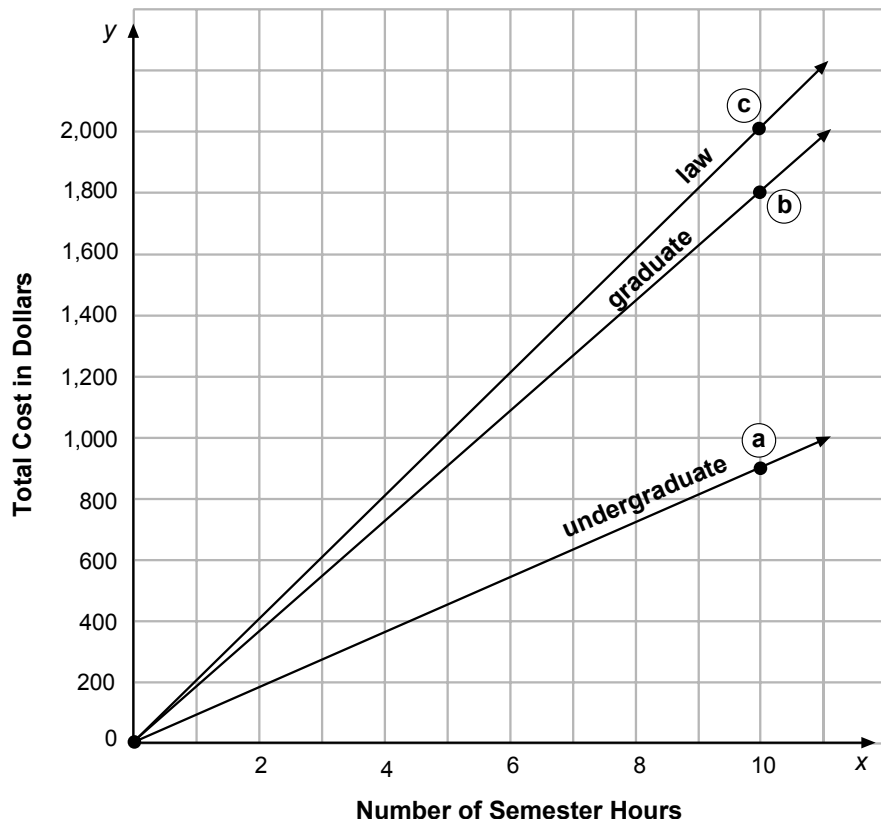


3. The graph below shows the three functions related to tuition costs for Florida residents who are undergraduate students, graduate students, and law students. The total cost per semester hour was rounded to the nearest \$10 and included the transportation fee. The points were connected to emphasize the slope of each line. (The slope directly relates to *per hour costs*.)

Use the graph to **estimate** the cost per semester hour for each of the three types for Florida residents.

- undergraduate students: approximately \_\_\_\_\_ per semester hour
- graduate students: approximately \_\_\_\_\_ per semester hour
- law students: approximately \_\_\_\_\_ per semester hour

**Tuition for Florida State University—Florida Residents**



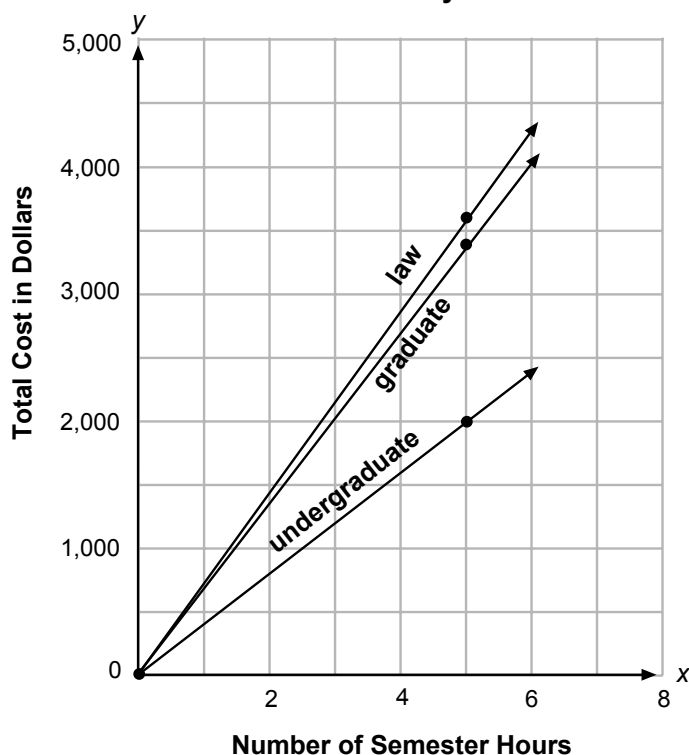


4. The cost per semester hour including the transportation access fee for each of the three types for non-Florida residents was as follows.

- undergraduate school:  $\$402.71 + 4.90$  per semester hour
- graduate school:  $\$670.92 + 4.90$  per semester hour
- law school:  $\$712.59 + 4.90$  per semester hour

A graph is provided below for this data.

**Tuition for Florida State University—Non-Florida Residents**





Write **three** statements comparing tuition for **each type** of student, based on being Florida residents *or* non-Florida residents. Explain whether you prefer to use equation, table, or graph models when writing such comparison statements.

Statement 1: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Statement 2: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Statement 3: \_\_\_\_\_

\_\_\_\_\_

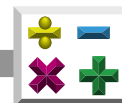
\_\_\_\_\_

Preference: \_\_\_\_\_

Explanation: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



5. The following functions would allow you to compute the tuition for a semester hour of credit in an undergraduate course at the University of Florida for Florida residents and non-Florida residents.

$f(x) = 92.68x$  where  $f(x)$  is the total tuition for a Florida resident for  $x$  number of hours of undergraduate-level courses.

$g(x) = 460.28x$  where  $g(x)$  is the total tuition for a non-Florida resident for  $x$  number of hours of undergraduate-level courses.

Complete the table below.

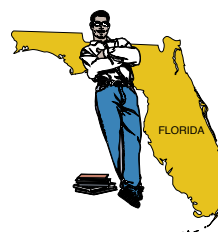
**Tuition for University of Florida—Florida Residents  
and Non-Florida Residents**

<b>Number of Semester Hours <math>x</math></b>	<b>Cost for Florida Resident in Undergraduate Courses <math>f(x) = 92.68x</math></b>	<b>Cost for Non-Florida Resident in Undergraduate Courses <math>g(x) = 460.28x</math></b>
6		
9		
12		
15		



6. Complete the following.

- a. A Florida resident paid \$2,873.64 in tuition for 14 hours of graduate-level courses. What was the cost per credit hour?

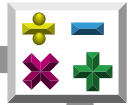


Answer: \_\_\_\_\_

- b. A non-Florida resident received \$27.96 in change from his payment of \$10,000 for 12 credit hours in law courses. What was the cost per credit hour? **Round to the nearest dollar.**

Answer: \_\_\_\_\_





- c. A non-Florida resident has a budget of \$25,000 for tuition for two semesters and is taking graduate-level courses with a cost per credit hour of \$774.53. What is the greatest number of hours she can take and not exceed her budget?

Answer: \_\_\_\_\_



## More about the Slope of a Line

You are a member of a private club that offers valet parking for its members. The club charges you \$3.00 to have the parking attendant park and retrieve your car and \$2 per hour for parking. A set of ordered pairs for this situation would include the following.

$$\{(1, 5), (2, 7), (3, 9), (4, 11), (5, 13)\}$$

If  $x$  represents the number of hours your car is parked, then  $y$  would represent the cost.

The equation for this line would be

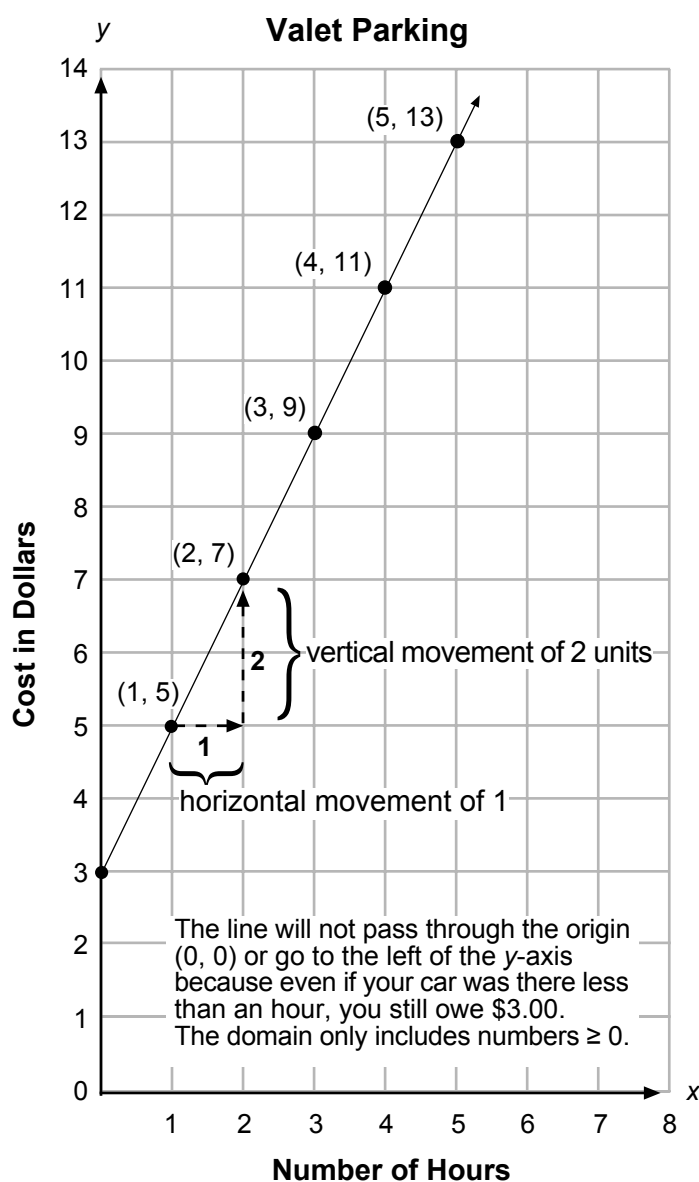
$$y = 2x + 3.$$

In function notation,

$$f(x) = 2x + 3.$$



*valet parking for members*





## Practice

Complete the following for the **set of ordered pairs** below.

$\{(0, 10), (1, 13), (2, 16), (3, 19), (4, 22), (5, 25)\}$
--

1. As the first coordinate increases by 1, the second coordinate increases by \_\_\_\_\_ .
2. If these points were plotted, they \_\_\_\_\_ would (always, sometimes, never) lie in a line.
3. The line \_\_\_\_\_ (will, will not) pass through the origin.
4. The slope of the line will be \_\_\_\_\_ or  $\frac{1}{1}$  because for each increase of 1 in  $x$ , there is an increase of \_\_\_\_\_ in  $y$ . From any given point on the line, a horizontal movement of 1 unit, followed by a vertical movement of \_\_\_\_\_ unit(s), will produce another point on the line.
5. The equation for the line would be  $y =$  \_\_\_\_\_ and in function notation it would be  $f(x) =$  \_\_\_\_\_ .



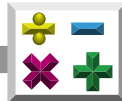
6. Create a situation the set of order pairs might describe.

Situation: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



## Practice

*Answer the following.*

1. The cost for Florida residents per credit hour at Tallahassee Community College was \$53 per credit hour, plus a \$10 student service fee and \$10 for the math lab.
  - a. Write a function that would permit tuition and fees to be calculated for  $x$  number of hours.  
  
Function: \_\_\_\_\_
  - b. If a graph were made for your function, the slope would be \_\_\_\_\_ and the  $y$ -intercept would be located at (0, \_\_\_\_\_). You know that an active student takes one or more credit hours. Therefore, the  $y$ -intercept has meaning for the general equation but not for its specific application when used for tuition and fee costs.
2. Tuition at the University of Miami was \$1,074 per credit hour for undergraduate students taking 1-11 hours. In addition, the university charged \$299.50 for a combination of four different student fees: activity, athletic, wellness center, and university.
  - a. Write a function that would permit tuition and fees to be calculated for  $x$  number of hours where  $x$  can be from 1 through 11.  
  
Function: \_\_\_\_\_



- b. Use your function to determine the cost of tuition and fees for a student taking 9 hours.

Answer: \_\_\_\_\_

3. The University of Miami charged a flat rate of \$12,919 for 12-20 credit hours plus \$226.50 for a combination of three fees for full-time students.

- a. Write a function that would represent the total cost of tuition and fees for a student taking 15 credit hours.

Function: \_\_\_\_\_

- b. Use your function to determine the total cost for 12 hours.

Answer: \_\_\_\_\_

- c. Based on your answer for **b**, what is the **mean (or average)** cost per credit hour when 12 credit hours are taken? **Round to the nearest hundredth.**

Answer: \_\_\_\_\_

- d. Use your function to determine the total cost for 20 hours.

Answer: \_\_\_\_\_



## Practice

Answer the following.

1. Harvard charged a flat rate of \$26,066 for tuition, plus fees of \$1,142 for health services, \$1,852 for student services, and \$35 for the undergraduate council. Determine the *mean* cost per credit hour for a student taking 15 credit hours. **Round to the nearest hundredth.**

Answer: \_\_\_\_\_

2. The cost per credit hour at the undergraduate level at Florida A & M University was \$90.09, plus fees of \$45 for transportation and access, \$59 for health if taking 6 or more hours, and a materials and supply fee ranging from \$15 to \$60. Assuming the materials and supply fee was \$37.50, write a function that would allow you to calculate cost for  $x$  number of hours where  $x$  represents 6 or more.

Function: \_\_\_\_\_

3. Consider the following.

### Public Undergraduate Tuition Rates and Fees

Institution of Higher Learning (Public)	Cost of Tuition and Fees for 15 Undergraduate Credit Hours for Florida Resident
Florida A & M University	\$1,492.85
Florida State University	\$1,342.20
Tallahassee Community College	\$815.00
University of Florida	\$1,390.20



### Private Undergraduate Tuition Rates and Fees

Institution of Higher Learning (Private)	Cost of Tuition and Fees for 15 Undergraduate Credit Hours
Harvard University	\$29,095.00
University of Miami	\$13,145.50

- a. Use the figures in the table above and on the previous page. Explain why you could not divide each of the costs by 15 and then multiply by 12 to get the costs for 12 credit hours for each institution.

Explanation: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- b. If you were considering these schools in your future, would you find information pertaining to tuition and fees more helpful if this were modeled by equations, tables, or graphs? Explain the basis for your choice.

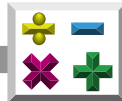
Explanation: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_





## **Lesson Four Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Algebra Body of Knowledge**

#### **Standard 1: Real and Complex Number Systems**

- MA.912.A.1.8  
Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

#### **Standard 7: Quadratic Equations**

- MA.912.A.7.1  
Graph quadratic equations with and without graphing technology.
- MA.912.A.7.10  
Use graphing technology to find approximate solutions of quadratic equations.



## Graphing Quadratics

Any function whose equation is in the format  $f(x) = ax^2 + bx + c$  (when  $a \neq 0$ ) is called a **quadratic function**. The presence of the  $ax^2$  term is a big hint that this is a quadratic expression. You'll also remember that the  $ax^2$  term is a hint that **factoring** is involved for solving  $x$ .

Graphs of *quadratic functions* are called **parabolas** and have a shape that looks like an airplane wing.

Let's look at two examples.

### Example 1

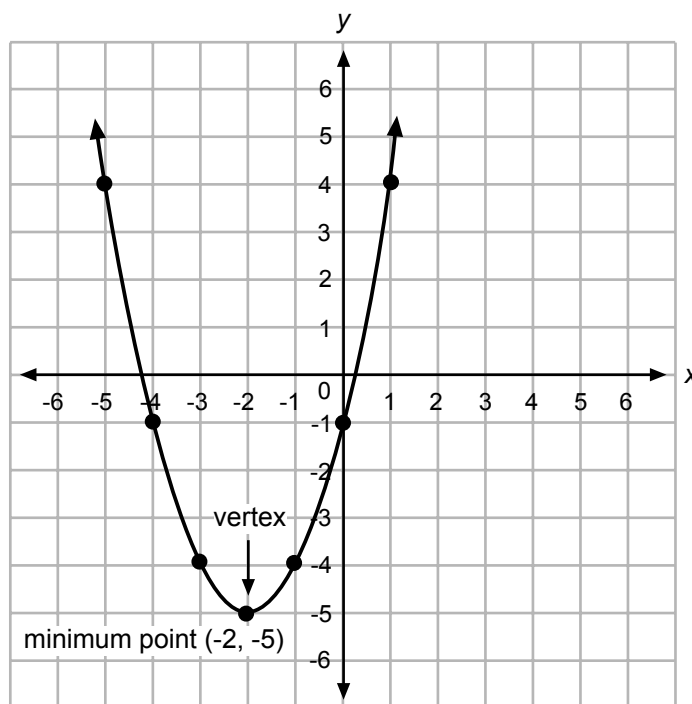
$$f(x) = x^2 + 4x - 1$$

We will use a table of values to graph this function.

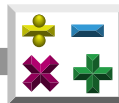
**Table of Values**

$f(x) = x^2 + 4x - 1$	
$x$	$f(x)$
-5	4
-4	-1
-3	-4
-2	-5
-1	-4
0	-1
1	4

**Graph of  $f(x) = x^2 + 4x - 1$**



We plot the points, and knowing that the graph will look like an airplane wing, we connect the dots with a smooth curve. A **coefficient** is the number part in front of an algebraic term. The *coefficient* in front of  $x^2$  in the function  $f(x) = x^2 + 4x - 1$  is understood to be a +1.



Because the  $x$  value of the coefficient is positive, the *parabola* will open upward and will have a *lowest point*, or **vertex**, called the **minimum** point.

To tell exactly where that *minimum* point will be on our graph, we use information from the equation. Remembering that  $f(x) = ax^2 + bx + c$ , we use  $x = \frac{-b}{2a}$  to tell us the  $x$ -value of the lowest point.

So from our function  $f(x) = x^2 + 4x - 1$ , where  $a = 1$ ,  $b = 4$ ,  $c = -1$ , we get the following.

$$\begin{aligned}x &= \frac{-b}{2a} \\x &= \frac{-4}{2(1)} \\x &= \frac{-4}{2} \\x &= -2\end{aligned}$$

So, the minimum point occurs when  $x = -2$ .

Using the function again,



**Remember:**  $f(x) = ax^2 + bx + c$   
 $f(x) = x^2 + 4x - 1$ , where  
 $a = 1$ ,  $b = 4$ ,  $c = -1$

$$\begin{aligned}f(-2) &= (-2)^2 + 4(-2) - 1 \\f(-2) &= 4 + -8 - 1 \\f(-2) &= -5\end{aligned}$$

Therefore, the minimum point is  $(-2, -5)$ .

Another thing we can tell from the equation  $x = -2$  in the box above is the **axis of symmetry**. Recall that the graph of  $x = -2$  is a vertical line through  $-2$  on the  $x$ -axis. This is the line that divides the parabola exactly in half. If you fold the graph along the *axis of symmetry*, each half of the parabola will match the other side exactly.

Note that the graph is a function because it passes the *vertical line test*. Any vertical line you draw will only **intersect** the graph (parabola) at one point.

Let's look at another example.



## Example 2

$$f(x) = -x^2 + 2x - 3$$

Notice that the coefficient of  $x^2$  is -1.

Because the value of the coefficient of  $x$  is negative, the parabola will open downward and have a highest point, or *vertex*, called a **maximum** point.

Find the axis of symmetry.

$$x = \frac{-b}{2a}$$

$$x = \frac{-2}{-2}$$

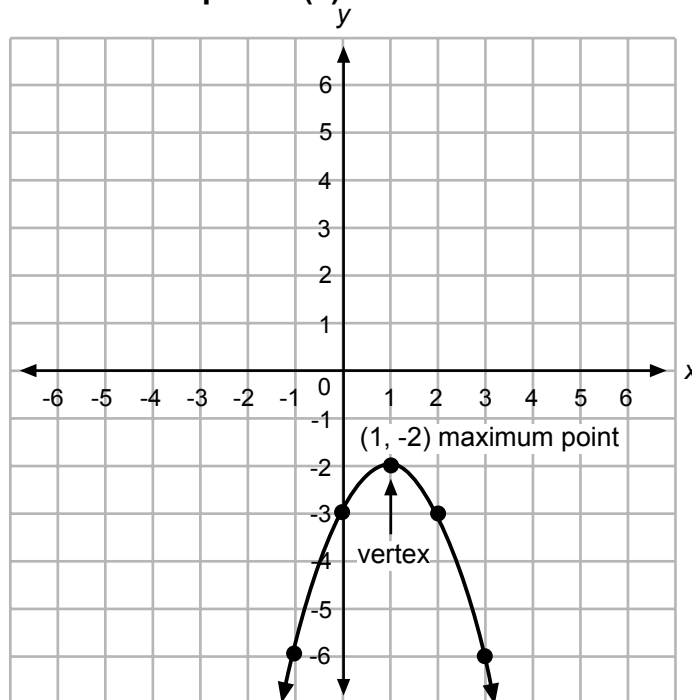
$$x = 1 \quad \leftarrow \text{axis of symmetry}$$

Our *maximum* point occurs when  $x = 1$ . Let's make a table of values (be sure to include 1 as a value for  $x$ ).

**Table of Values**

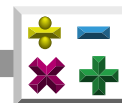
$f(x) = -x^2 + 2x - 3$	
$x$	$f(x)$
-1	-6
0	-3
1	-2
2	-3
3	-6

**Graph of  $f(x) = -x^2 + 2x - 3$**



Graph the ordered pairs and connect them with a smooth curve. Note that the vertex of the parabola has a maximum point at  $(1, -2)$  and the line of symmetry is at  $x = 1$ .

Refer to the examples above as you try the following.



## Practice

For each **function** do the following.

- Find the equation for the axis of symmetry.
- Find the coordinates of the vertex of the graph.
- Tell whether the vertex is a maximum or minimum vertex.
- Graph the function.

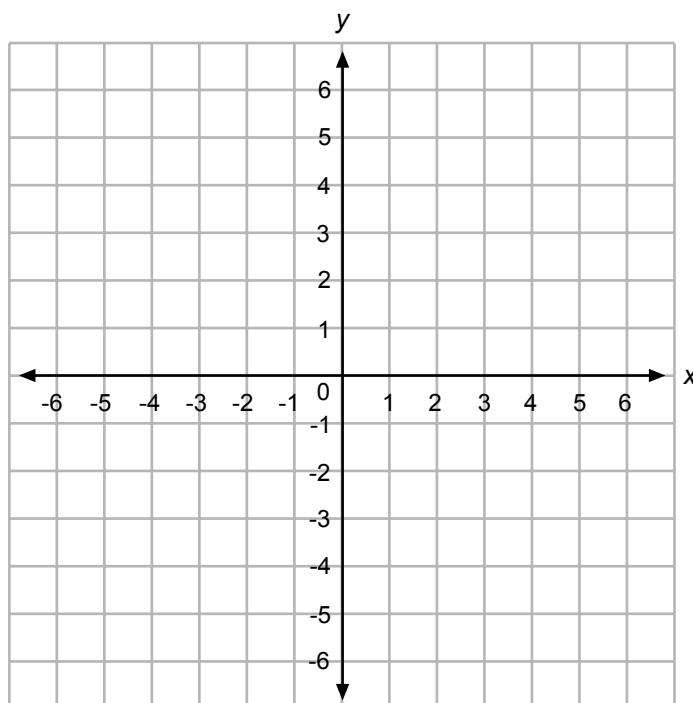
1.  $f(x) = x^2 + 2x - 3$

- axis of symmetry = \_\_\_\_\_
- coordinates of vertex = \_\_\_\_\_
- maximum or minimum = \_\_\_\_\_
- graph

**Table of Values**

$f(x) = x^2 + 2x - 3$	
$x$	$f(x)$

**Graph of  $f(x) = x^2 + 2x - 3$**





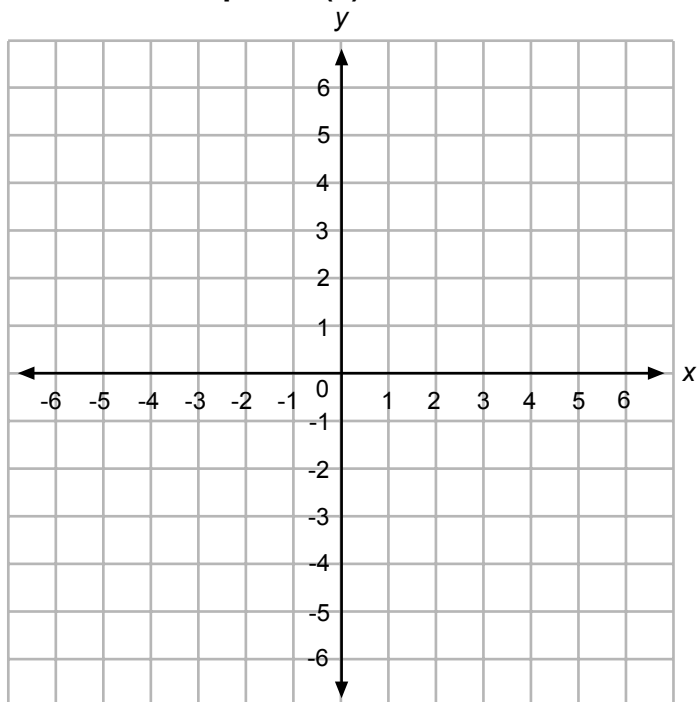
2.  $f(x) = x^2 - 2x - 3$

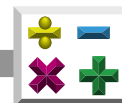
- a. axis of symmetry = \_\_\_\_\_
- b. coordinates of vertex = \_\_\_\_\_
- c. maximum or minimum = \_\_\_\_\_
- d. graph

**Table of Values**

$f(x) = x^2 - 2x - 3$	
$x$	$f(x)$

**Graph of  $f(x) = x^2 - 2x - 3$**





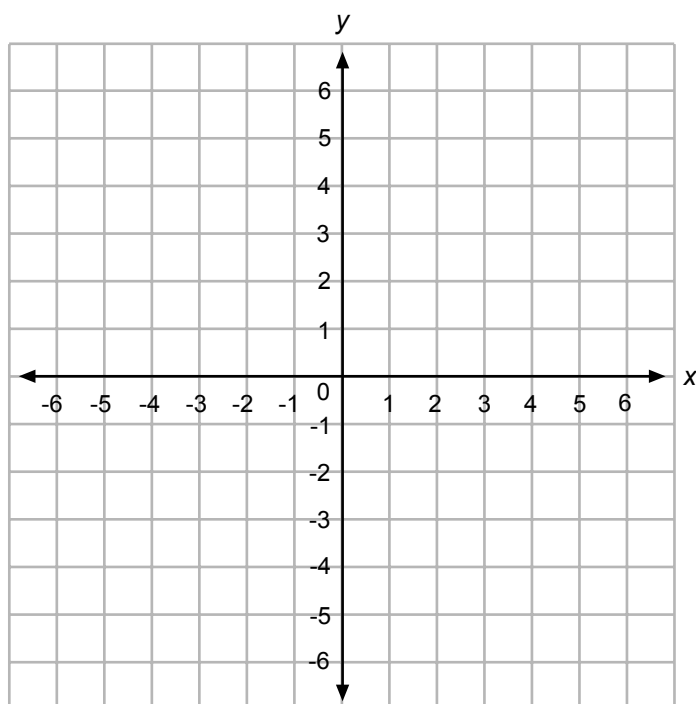
3.  $f(x) = -x^2 + 1$

- a. axis of symmetry = \_\_\_\_\_
- b. coordinates of vertex = \_\_\_\_\_
- c. maximum or minimum = \_\_\_\_\_
- d. graph

**Table of Values**

$f(x) = -x^2 + 1$	
$x$	$f(x)$

**Graph of  $f(x) = -x^2 + 1$**





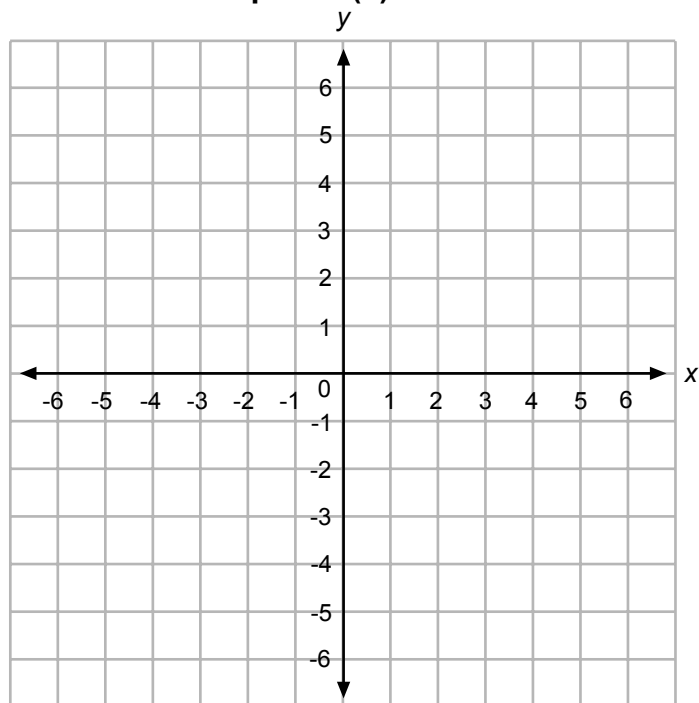
4.  $f(x) = x^2 + 2x$

- a. axis of symmetry = \_\_\_\_\_
- b. coordinates of vertex = \_\_\_\_\_
- c. maximum or minimum = \_\_\_\_\_
- d. graph

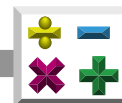
**Table of Values**

$f(x) = x^2 + 2x$	
$x$	$f(x)$

**Graph of  $f(x) = x^2 + 2x$**







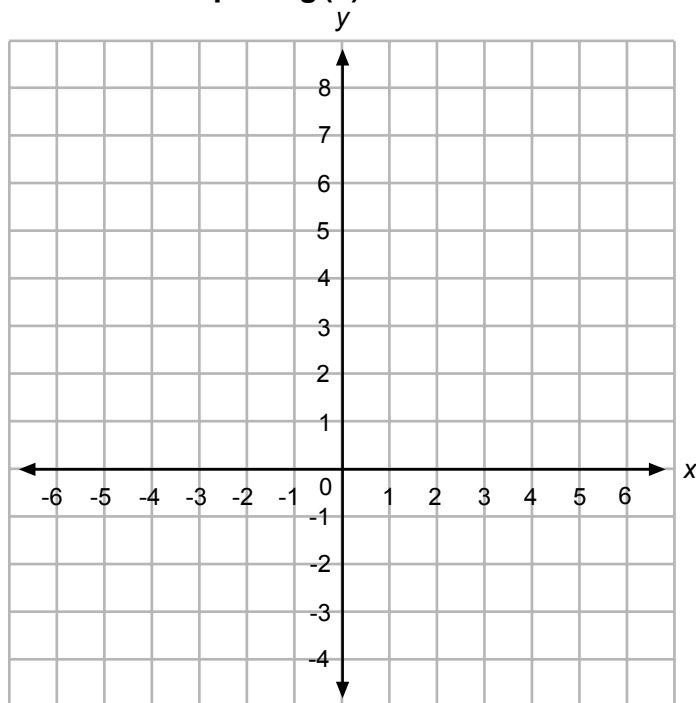
5.  $g(x) = -x^2 - 4x + 4$

- axis of symmetry = \_\_\_\_\_
- coordinates of vertex = \_\_\_\_\_
- maximum or minimum = \_\_\_\_\_
- graph

**Table of Values**

$g(x) = -x^2 - 4x + 4$	
$x$	$g(x)$

**Graph of  $g(x) = -x^2 - 4x + 4$**





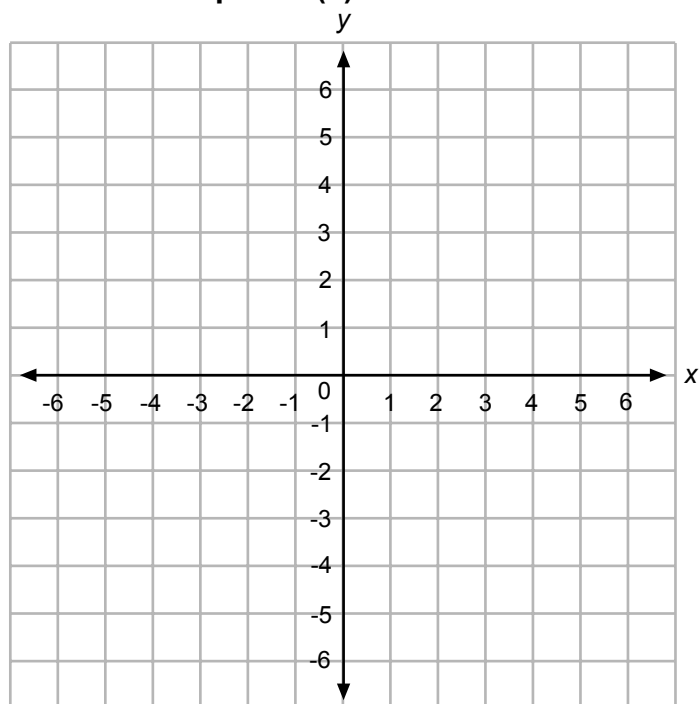
6.  $f(x) = 3x^2 + 6x - 2$

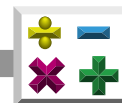
- a. axis of symmetry = \_\_\_\_\_
- b. coordinates of vertex = \_\_\_\_\_
- c. maximum or minimum = \_\_\_\_\_
- d. graph

**Table of Values**

$f(x) = 3x^2 + 6x - 2$	
$x$	$f(x)$

**Graph of  $f(x) = 3x^2 + 6x - 2$**





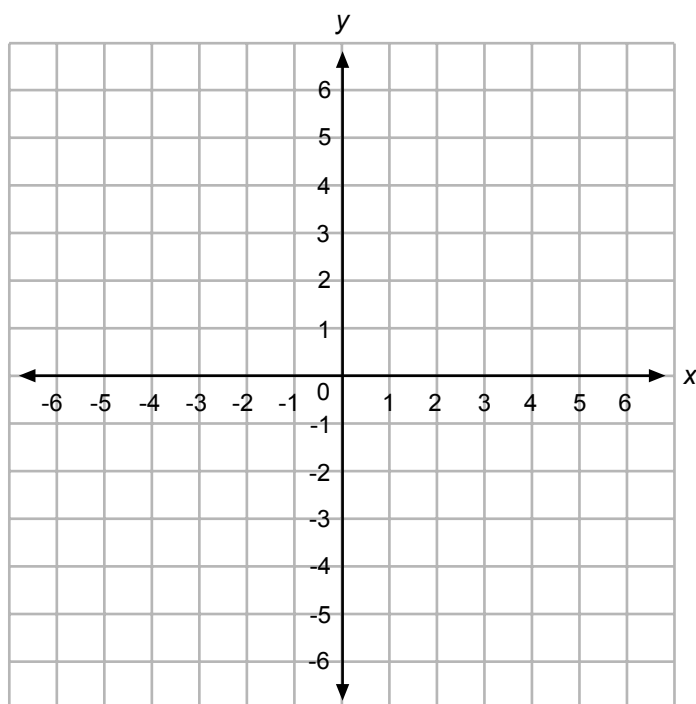
7.  $g(x) = -x^2 + 2x$

- a. axis of symmetry = \_\_\_\_\_
- b. coordinates of vertex = \_\_\_\_\_
- c. maximum or minimum = \_\_\_\_\_
- d. graph

**Table of Values**

$g(x) = -x^2 + 2x$	
$x$	$g(x)$

**Graph of  $g(x) = -x^2 + 2x$**





8.  $g(x) = (x - 1)^2$

**Note:** You must first use the **FOIL method**.



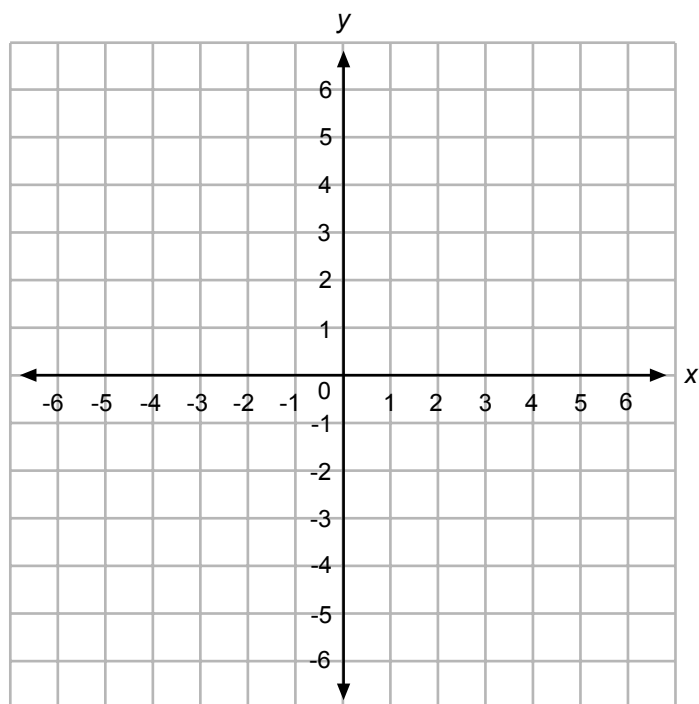
**Remember:** F First terms  
O Outside terms  
I Inside terms  
L Last terms.

- axis of symmetry = \_\_\_\_\_
- coordinates of vertex = \_\_\_\_\_
- maximum or minimum = \_\_\_\_\_
- graph

**Table of Values**

$g(x) = (x - 1)^2$	
$x$	$g(x)$

**Graph of  $g(x) = (x - 1)^2$**





## Solving Quadratic Equations

The **solutions** to **quadratic equations** are called the **roots** of the equations. In factoring *quadratic equations*, set each **factor** equal to 0 to **solve** for values of  $x$ . Those values of  $x$  are the *roots* of the equation.

### Example

#### Solve by factoring

$$\begin{aligned}x^2 + 10x + 9 &= 0 && \swarrow \text{factor} \\(x + 1)(x + 9) &= 0 && \swarrow\end{aligned}$$

#### Set each factor equal to 0

$$\begin{aligned}x + 1 &= 0 && \swarrow \text{zero product property} \\x &= -1 && \swarrow \text{add -1 to each side} \\x + 9 &= 0 && \swarrow \text{zero product property} \\x &= -9 && \swarrow \text{add -9 to each side}\end{aligned}$$

-1 and -9 are roots

We can also find these roots by graphing the related function  $f(x) = x^2 + 10x + 9$  and finding the  $x$ -intercepts. The  $x$ -intercepts are the points where the graph crosses the  $x$ -axis, which are also known as the **zeros** of the function.

Let's see how this works.

$$f(x) = x^2 + 10x + 9$$

The equation for the axis of symmetry is as follows.

$$\begin{aligned}x &= \frac{-10}{2(1)} \\x &= -5\end{aligned}$$

$$\begin{aligned}f(-5) &= (-5)^2 + 10(-5) + 9 \\f(-5) &= -16\end{aligned}$$

The vertex is at (-5, -16).

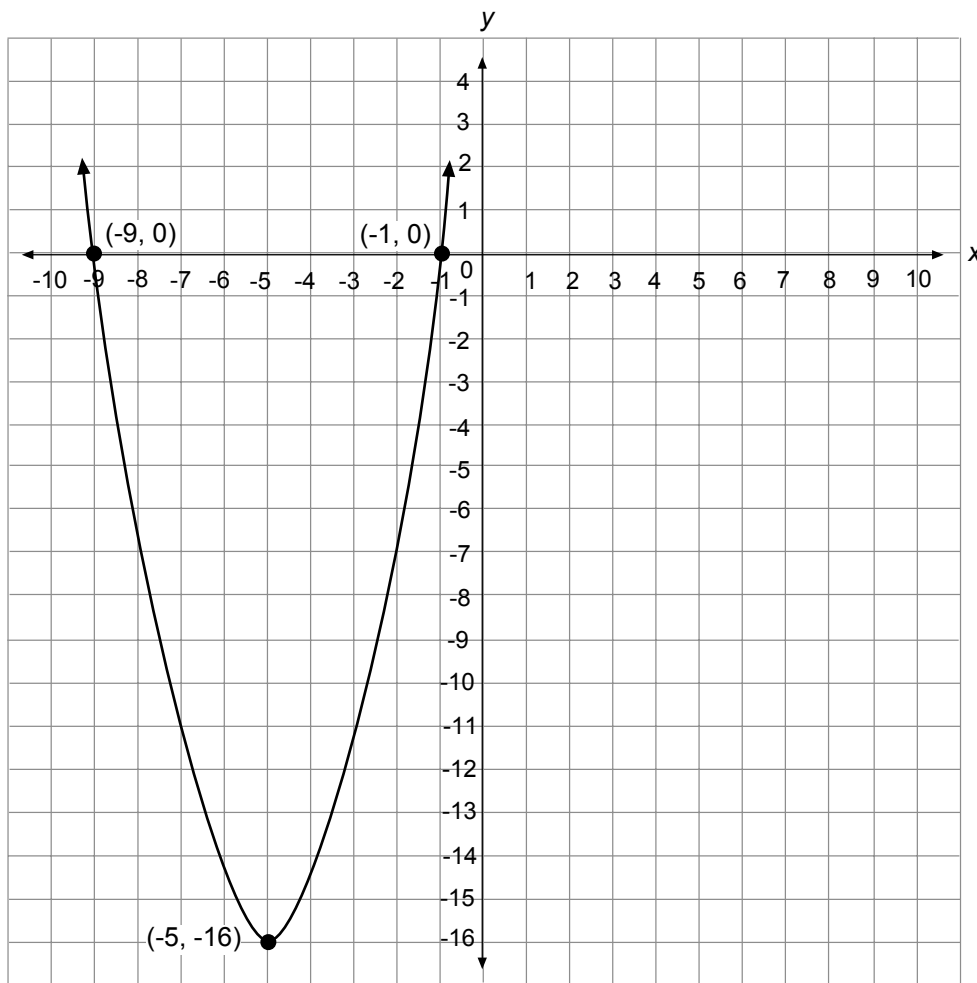


Find the  $x$ -intercepts by letting  $f(x) = 0$ .

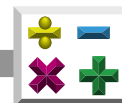
$$0 = x^2 + 10x + 9$$

The  $x$ -intercepts are at  $(-9, -1)$ . Thus the solutions are  $-9$  and  $-1$ .

**Graph of  $f(x) = x^2 + 10x + 9$**



You may find the solutions more efficiently by using your graphing calculator. When the  $x$ -intercepts are *not* integers, use your calculator to estimate them to the nearest integer.



## Practice

Solve the following by **graphing**. Show each step indicated.



Check your work  
using your graphing  
calculator.

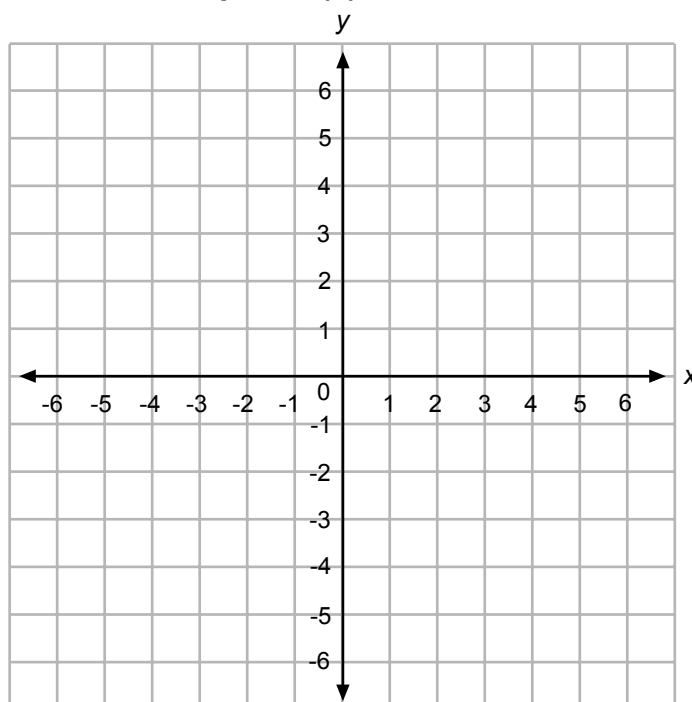
1.  $x^2 + 3x + 2 = 0$

a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = x^2 + 3x + 2$**



d. solutions = \_\_\_\_\_



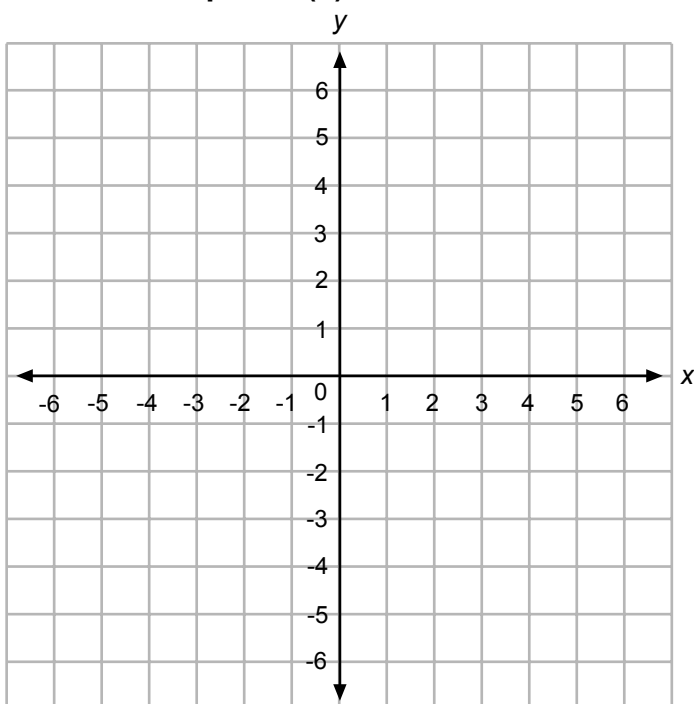
2.  $x^2 - 3x + 2 = 0$

a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

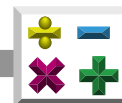
c. graph

**Graph of  $f(x) = x^2 - 3x + 2$**



d. solutions = \_\_\_\_\_





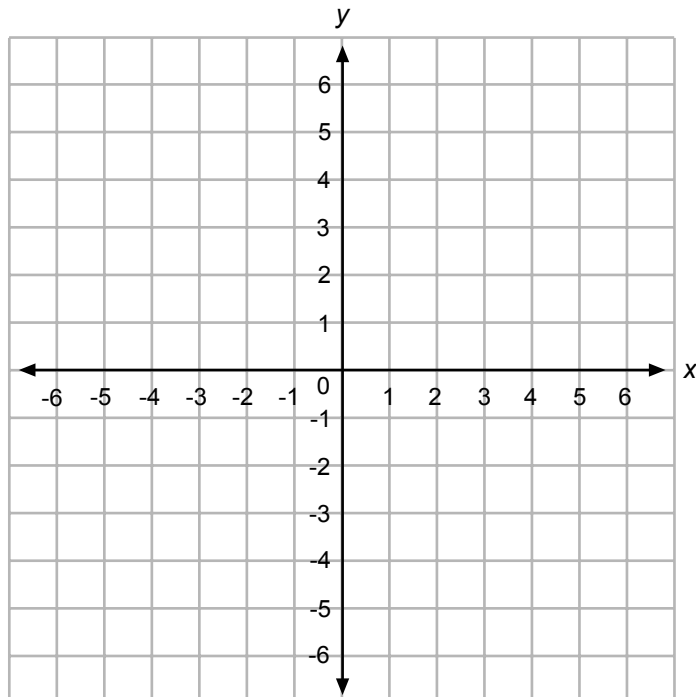
3.  $-x^2 - 3x - 2 = 0$

a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = -x^2 - 3x - 2$**



d. solutions = \_\_\_\_\_



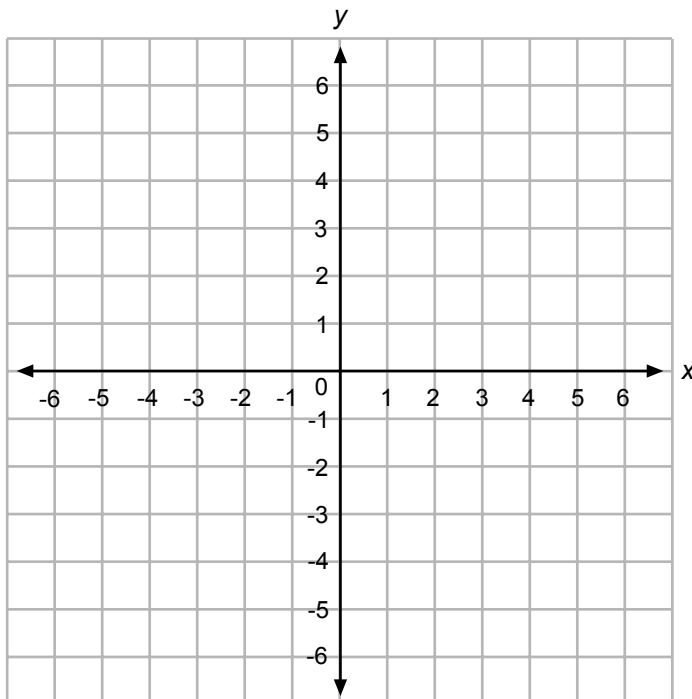
4.  $x^2 + 9x + 14 = 0$

a. axis of symmetry = \_\_\_\_\_

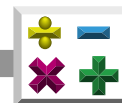
b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = x^2 + 9x + 14$**



d. solutions = \_\_\_\_\_



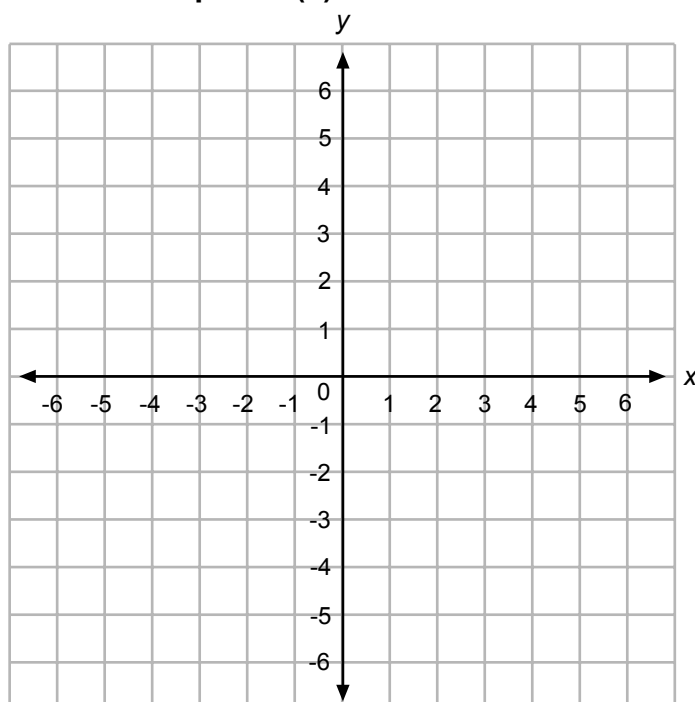
5.  $-x^2 + 9x - 14 = 0$

a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = -x^2 + 9x - 14$**



d. solutions = \_\_\_\_\_



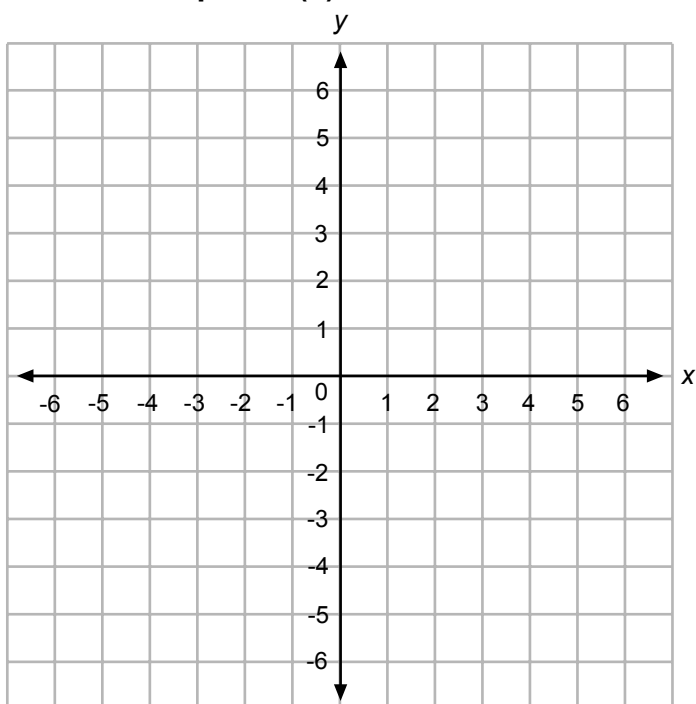
6.  $2x^2 + 3x + 1 = 0$

a. axis of symmetry = \_\_\_\_\_

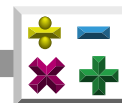
b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = 2x^2 + 3x + 1$**



d. solutions = \_\_\_\_\_



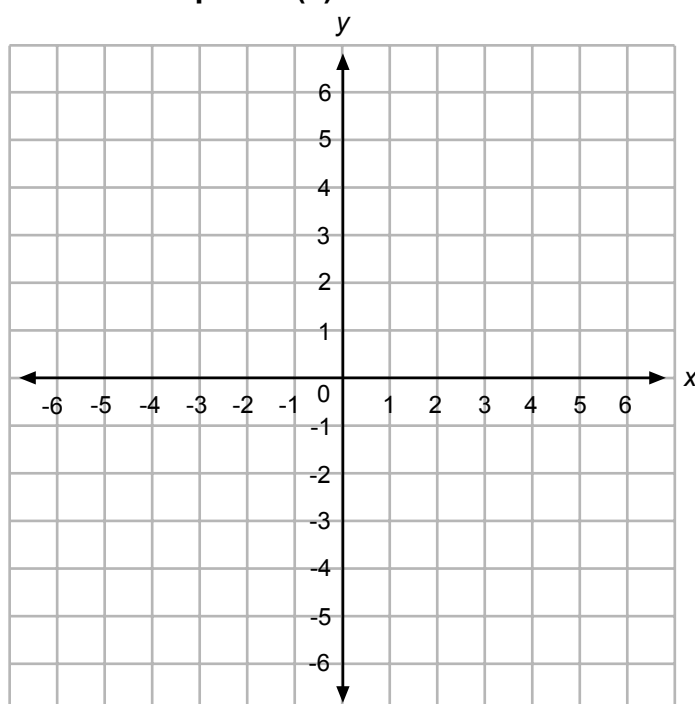
7.  $-2x^2 - 5x - 3 = 0$

a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = -2x^2 - 5x - 3$**



d. solutions = \_\_\_\_\_



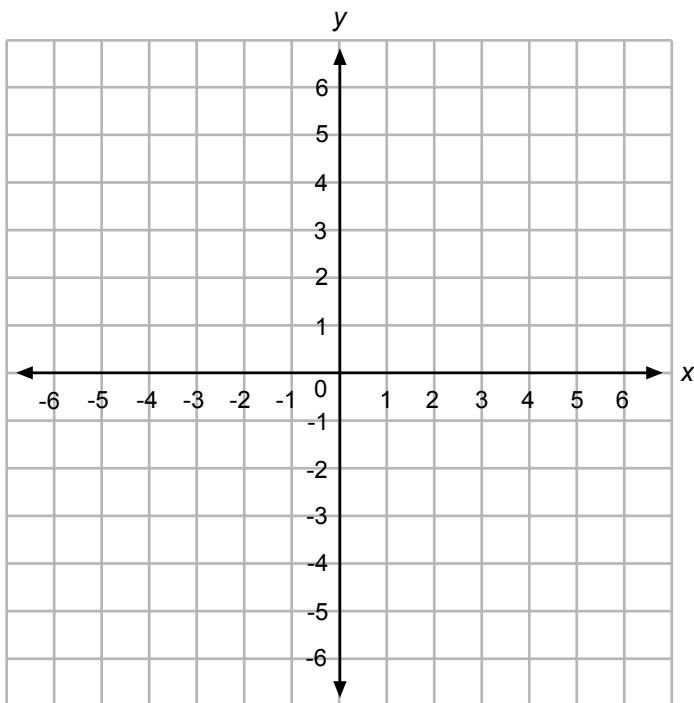
8.  $2x^2 - 5x = 0$

a. axis of symmetry = \_\_\_\_\_

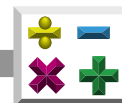
b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = 2x^2 - 5x$**



d. solutions = \_\_\_\_\_



9.  $x^2 + 6x + 6 = 0$

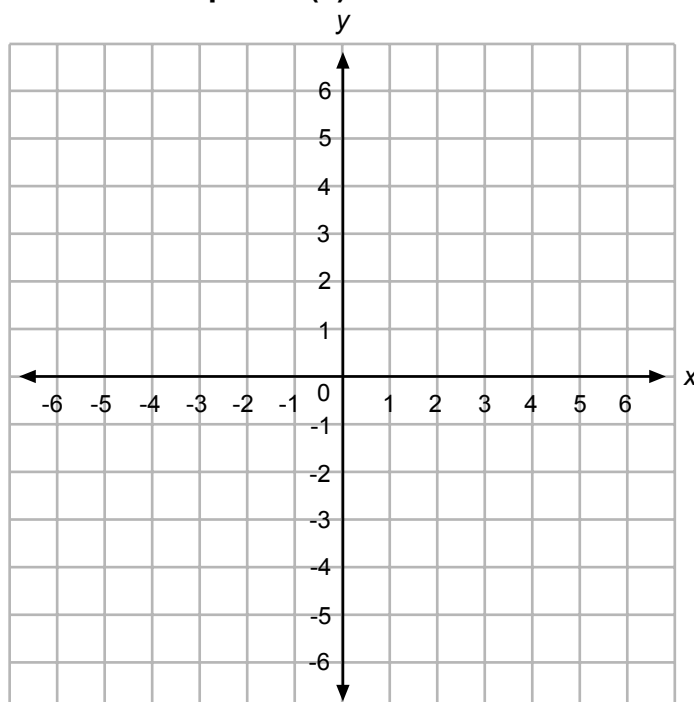
a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

**Hint:** Use your calculator to estimate  $x$ -intercepts.

c. graph

**Graph of  $f(x) = x^2 + 6x + 6$**



d. solutions = \_\_\_\_\_



10.  $x^2 - x - 4 = 0$

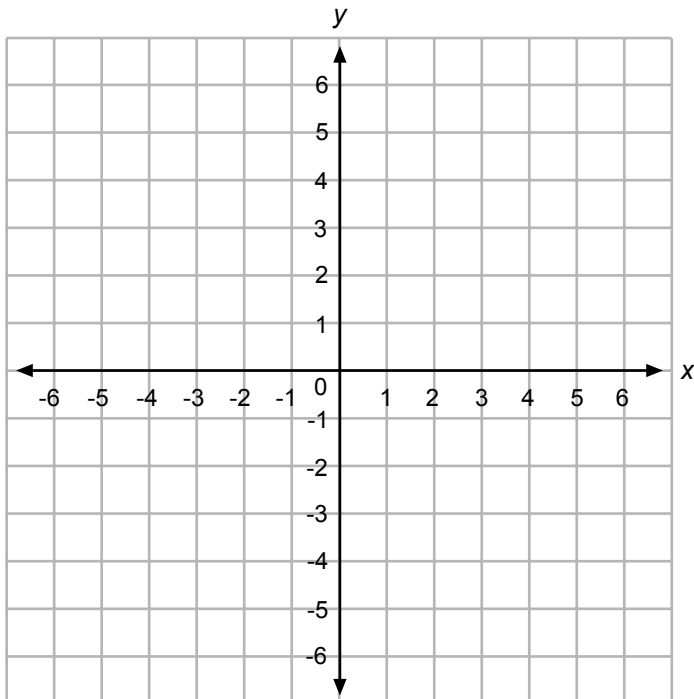
a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

**Hint:** Use your calculator to estimate  $x$ -intercepts.

c. graph

**Graph of  $f(x) = x^2 - x - 4$**



d. solutions = \_\_\_\_\_





## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |   |                       |
|---|-----------------------|
| _____ 1. the lowest point on the vertex of a parabola, which opens upward   | A. axis of symmetry   |
| _____ 2. set of $y$ -values of a relation   | B. domain             |
| _____ 3. the value of $x$ at the point where a line or graph intersects the $x$ -axis; the value of $y$ is zero (0) at this point | C. function (of $x$ ) |
| _____ 4. set of $x$ -values of a relation   | D. maximum            |
| _____ 5. a set of ordered pairs $(x, y)$  | E. minimum            |
| _____ 6. vertical line passing through the maximum or minimum point of a parabola   | F. range              |
| _____ 7. the highest point on the vertex of a parabola, which opens downward  | G. relation           |
| _____ 8. a relation in which each value of $x$ is paired with a unique value of $y$   | H. vertex             |
| _____ 9. the maximum or minimum point of a parabola   | I. $x$ -intercept     |



## Unit Review

Use the **set** below to answer the following.

$\{(2, 6), (3, -1), (7, 2), (-3, 5), (0, -2)\}$

1. Is the set a relation? \_\_\_\_\_
2. Give the domain. \_\_\_\_\_

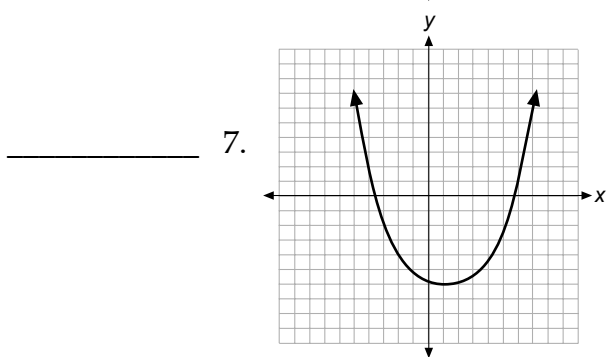
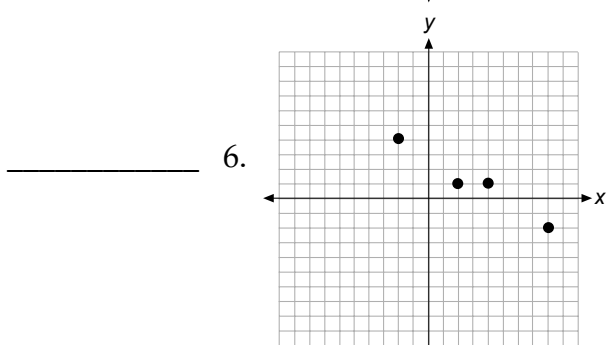
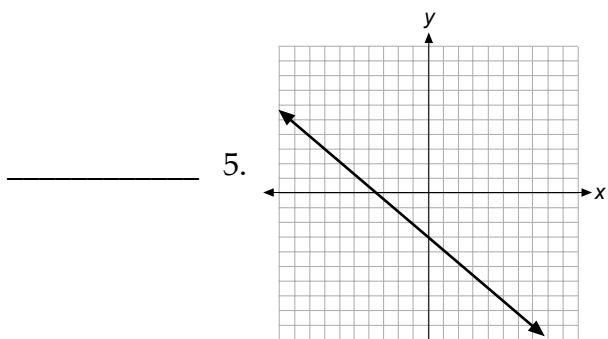


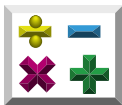
**Remember:** The domains and ranges do *not* have to be listed in numerical order. If a value in a domain or in a range is repeated, list the value *one* time.

3. Give the range. \_\_\_\_\_
4. Is the set a function? \_\_\_\_\_



Determine if the **graphs** represent **functions**.





Use the **domain** below to give the **range** for the following **functions**.

$\{-2, -1, 0, 1, 2\}$



**Remember:** The domains and ranges do *not* have to be listed in numerical order. If a value in a domain or in a range is repeated, list the value *one* time.

8.  $f(x) = 2x + 5$

9.  $g(x) = x^2 + 2$

10.  $h(x) = -3x + 2$



Use the **relation** below to answer the following.

$$\{(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)\}$$

11. If these points are plotted, they will \_\_\_\_\_  
(always, sometimes, never) lie in a line.
12. The function for the line will be  $f(x) =$  \_\_\_\_\_.
13. Create a situation the relation might describe.

---

---

---



For each **function**, fill in the **table of values** and then **graph the function**.

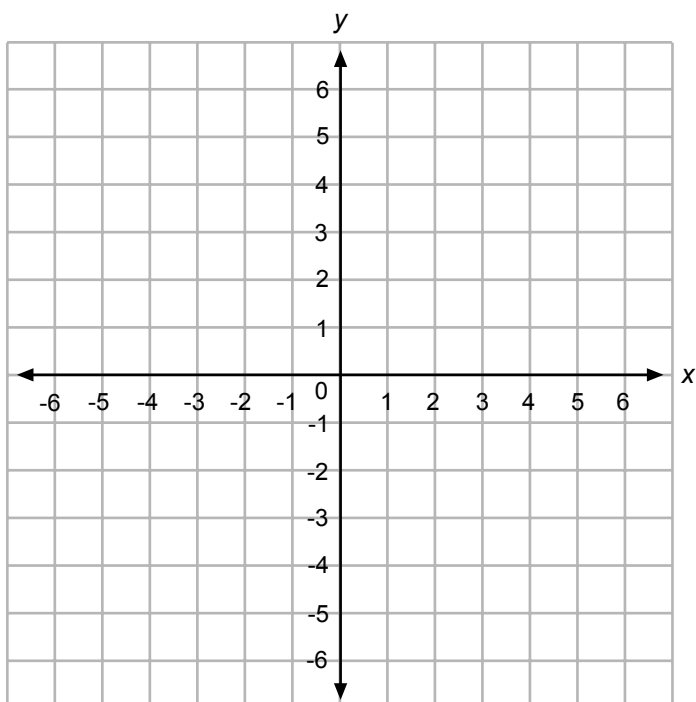
14.  $f(x) = 2x - 2$

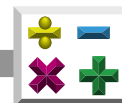
a. **Table of Values**

$f(x) = 2x - 2$	
$x$	$f(x)$
-2	
0	
2	

b. graph

**Graph of  $f(x) = 2x - 2$**





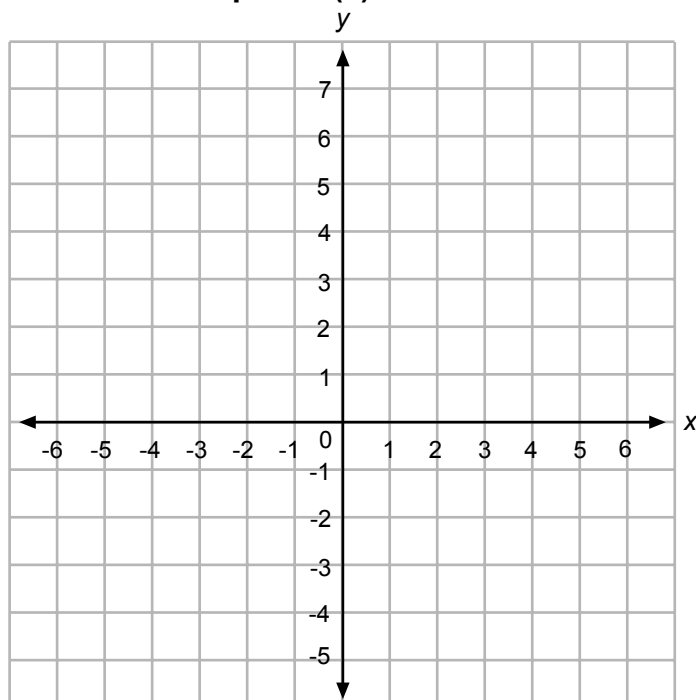
15.  $f(x) = -3x + 1$

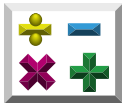
a. **Table of Values**

$f(x) = -3x + 1$	
$x$	$f(x)$
-2	
0	
2	

b. graph

**Graph of  $f(x) = -3x + 1$**





16. In this equation,  $f(x) = 92.68x$  represents the total tuition for a Florida resident at the University of Florida. The  $x$  in the equation represents the number of hours of courses an undergraduate student takes.

Calculate the tuition for a student who takes 14 hours.

Answer: \_\_\_\_\_

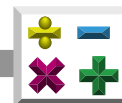
17. The University of Miami charges \$1,000 for each credit hour plus \$226.50 for various fees for full time students.
- a. Write a function to express the total cost for a semester.

$f(x) =$  \_\_\_\_\_

- b. Calculate the total cost for a student taking 15 hours.

Answer: \_\_\_\_\_





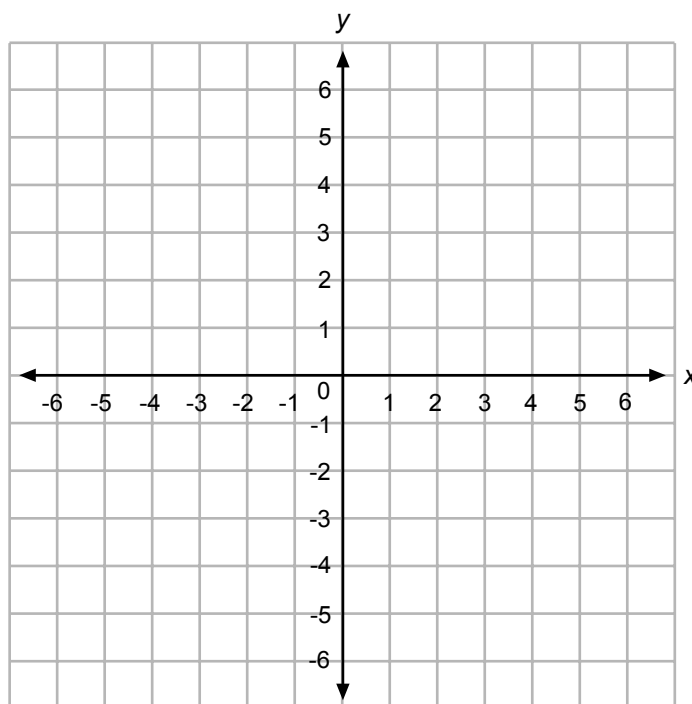
For each **function** do the following.

- Find the equation for the axis of symmetry.
- Find the coordinates of the vertex of the graph.
- Tell whether the vertex is a maximum or minimum.
- Graph the function.

18.  $f(x) = x^2 + 2x - 3$

- axis of symmetry = \_\_\_\_\_
- coordinates of vertex = \_\_\_\_\_
- maximum or minimum = \_\_\_\_\_
- graph

**Graph of  $f(x) = x^2 + 2x - 3$**

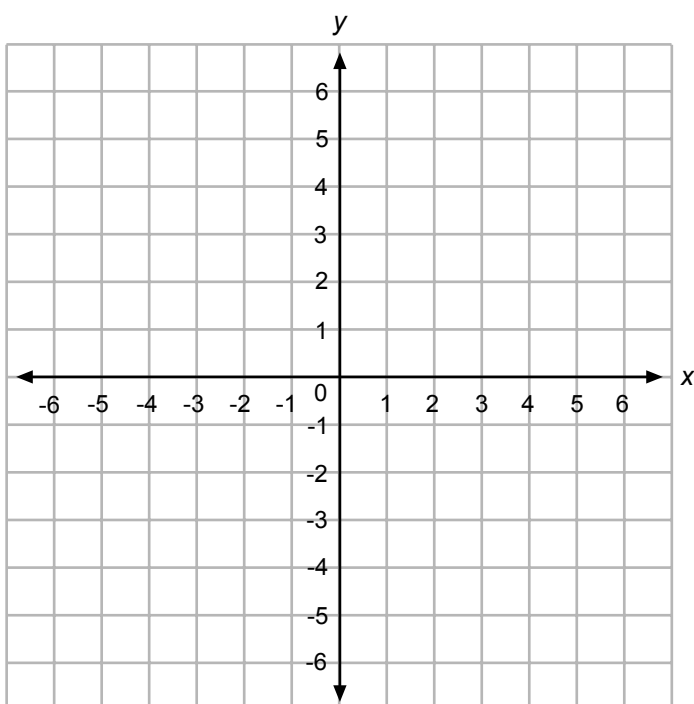


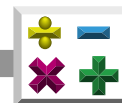


19.  $g(x) = -x^2 + x$

- a. axis of symmetry = \_\_\_\_\_
- b. coordinates of vertex = \_\_\_\_\_
- c. maximum or minimum = \_\_\_\_\_
- d. graph

**Graph of  $g(x) = -x^2 + x$**





For each **equation**, use your **graphing calculator** to do the following.

- Find the axis of symmetry.
- Find the  $x$ -intercepts.
- Graph the function.
- Find the solutions.

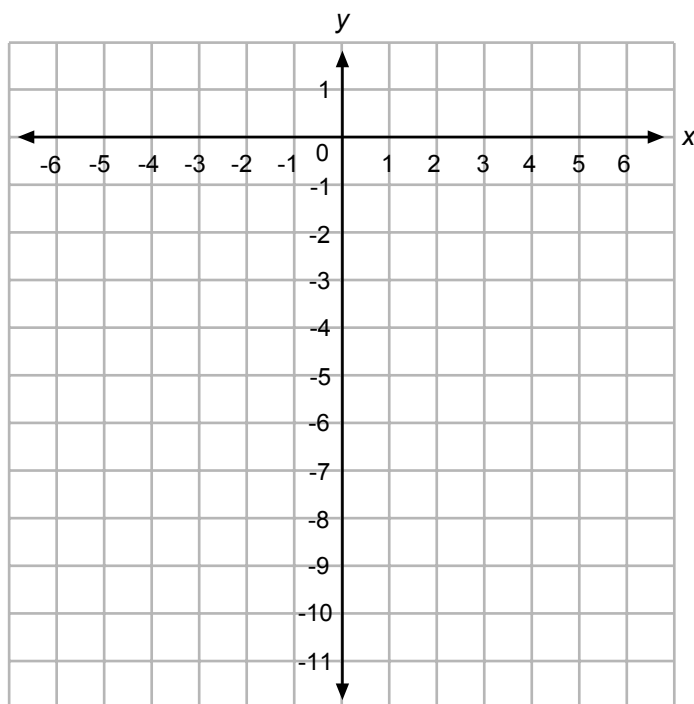
20.  $x^2 + 4x - 5 = 0$

a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = x^2 + 4x - 5$**



d. solutions = \_\_\_\_\_



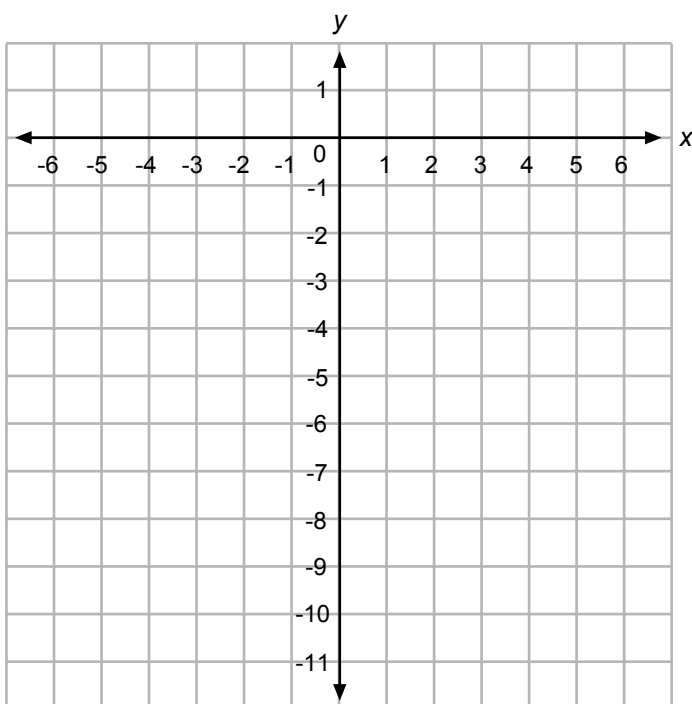
21.  $x^2 + 2x - 6 = 0$

a. axis of symmetry = \_\_\_\_\_

b.  $x$ -intercepts = \_\_\_\_\_

c. graph

**Graph of  $f(x) = x^2 + 2x - 6$**



d. solutions = \_\_\_\_\_

## **Unit 10: $X$ or $(X, Y)$ Marks the Spot!**

This unit shows students how to solve equations algebraically and graphically.

### **Unit Focus**

#### **Reading Process Strand**

##### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### **Writing Process Strand**

##### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

#### **Algebra Body of Knowledge**

##### **Standard 1: Real and Complex Number Systems**

- MA.912.A.1.8  
Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.
- MA.912.A.3.5  
Symbolically represent and solve multi-step and real-world applications that involve linear equations and inequalities.
- MA.912.A.3.12  
Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph.
- MA.912.A.3.13  
Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology.
- MA.912.A.3.14  
Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods.
- MA.912.A.3.15  
Solve real-world problems involving systems of linear equations and inequalities in two and three variables.

### **Standard 4: Polynomials**

- MA.912.A.4.3  
Factor polynomial expressions.

### **Standard 7: Quadratic Equations**

- MA.912.A.7.2  
Solve quadratic equations over the real numbers by factoring, and by using the quadratic formula.
- MA.912.A.7.8  
Use quadratic equations to solve real-world problems.

### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.
- MA.912.A.10.2  
Decide whether a solution is reasonable in the context of the original situation.
- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).







## Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

- area (A)** .....the measure, in square units, of the inside region of a closed two-dimensional figure; the number of square units needed to cover a surface  
*Example:* A rectangle with sides of 4 units by 6 units has an area of 24 square units.
- axes (of a graph)** .....the horizontal and vertical number lines used in a coordinate plane system; (singular: *axis*)
- coefficient** .....the number that multiplies the variable(s) in an algebraic expression  
*Example:* In  $4xy$ , the coefficient of  $xy$  is 4.  
If no number is specified, the coefficient is 1.
- consecutive** .....in order  
*Example:* 6, 7, 8 are consecutive whole numbers and 4, 6, 8 are consecutive even numbers.
- coordinate grid or plane** ...a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced; especially designed for locating points, displaying data, or drawing maps
- coordinates** .....numbers that correspond to points on a coordinate plane in the form  $(x, y)$ , or a number that corresponds to a point on a number line



**distributive property** .....the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products

*Examples:*  $x(a + b) = ax + bx$   
 $5(10 + 8) = 5 \cdot 10 + 5 \cdot 8$

**equation** .....a mathematical sentence stating that the two expressions have the same value

*Example:*  $2x = 10$

**equivalent expressions** ....expressions that have the same value but are presented in a different format using the properties of numbers

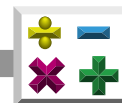
**even integer**.....any integer divisible by 2; any integer with the digit 0, 2, 4, 6, or 8 in the units place; any integer in the set  $\{\dots, -4, -2, 0, 2, 4, \dots\}$

**factor** .....a number or expression that divides evenly into another number; one of the numbers multiplied to get a product  
*Examples:* 1, 2, 4, 5, 10, and 20 are factors of 20 and  $(x + 1)$  is one of the factors of  $(x^2 - 1)$ .

**factored form** .....a number or expression expressed as the product of prime numbers and variables, where no variable has an exponent greater than 1

**factoring** .....expressing a polynomial expression as the product of monomials and polynomials

*Example:*  $x^2 - 5x + 4 = 0$   
 $(x - 4)(x - 1) = 0$



**FOIL method**.....a pattern used to multiply two binomials; multiply the first, outside, inside, and last terms:

**F** First terms  
**O** Outside terms  
**I** Inside terms  
**L** Last terms.

*Example:*

$$\begin{array}{c}
 \begin{array}{ccc}
 & 2 \text{ Outside} & \\
 \swarrow & & \searrow \\
 1 \text{ First} & & \\
 \downarrow & & \uparrow \\
 (a + b)(x - y) & = & \begin{array}{cccc} \text{F} & \text{O} & \text{I} & \text{L} \end{array} \\
 \uparrow & & \downarrow \\
 3 \text{ Inside} & & \\
 \downarrow & & \uparrow \\
 4 \text{ Last} & & 
 \end{array}
 \end{array}
 ax - ay + bx - by$$

**formula** .....a way of expressing a relationship using variables or symbols that represent numbers

**fraction** .....any part of a whole  
*Example:* One-half written in fractional form is  $\frac{1}{2}$

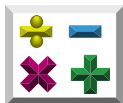
**graph** .....a drawing used to represent data  
*Example:* bar graphs, double bar graphs, circle graphs, and line graphs

**graph of an equation** .....all points whose coordinates are solutions of an equation

**inequality** .....a sentence that states one expression is greater than ( $>$ ), greater than or equal to ( $\geq$ ), less than ( $<$ ), less than or equal to ( $\leq$ ), or not equal to ( $\neq$ ) another expression  
*Examples:*  $a \neq 5$  or  $x < 7$  or  $2y + 3 \geq 11$

**infinite** .....having no boundaries or limits

**integers** .....the numbers in the set  
 $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

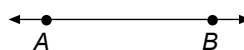


**intersect** .....to meet or cross at one point

**intersection** .....the point at which lines or curves meet

**length (*l*)** .....a one-dimensional measure that is the measurable property of line segments

**line ( $\longleftrightarrow$ )** .....a collection of an infinite number of points forming a straight path extending in opposite directions having unlimited length and no width



**monomial** .....a number, variable, or the product of a number and one or more variables; a polynomial with only *one* term

*Examples:* 8     $x$      $4c$      $2y^2$      $-3$      $\frac{xyz^2}{9}$

**negative integers** .....integers less than zero

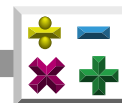
**negative numbers** .....numbers less than zero

**odd integer** .....any integer not divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set  $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$

**ordered pair** .....the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the  $x$ -axis and  $y$ -axis, respectively  
*Examples:*  $(x, y)$  or  $(3, -4)$

**parallel ( $\parallel$ )** .....being an equal distance at every point so as to never intersect

**point** .....a specific location in space that has no discernable length or width



**polynomial** ..... a monomial or sum of monomials; any rational expression with no variable in the denominator

*Examples:*  $x^3 + 4x^2 - x + 8$      $5mp^2$   
 $-7x^2y^2 + 2x^2 + 3$

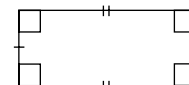
**positive integers** ..... integers greater than zero

**product** ..... the result of multiplying numbers together  
*Example:* In  $6 \times 8 = 48$ , the product is 48.

**quadratic equation** ..... an equation in the form of  $ax^2 + bx + c = 0$

**quadratic formula** ..... formula used to solve any quadratic equation;  
 if  $ax^2 + bx + c = 0$  and  $a \neq 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**rectangle** ..... a parallelogram with four right angles



**simplify an expression** ..... to perform as many of the indicated operations as possible

**solution** ..... any value for a variable that makes an equation or inequality a true statement

*Example:* In  $y = 8 + 9$   
 $y = 17$     17 is the solution.

**solution set ( { } )** ..... the set of values that make an equation or inequality true

*Example:*  $\{5, -5\}$  is the solution set for  $3x^2 = 75$ .

**solve** ..... to find all numbers that make an equation or inequality true



- standard form (of a quadratic equation)** .....  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are integers (not multiples of each other) and  $a > 0$
- substitute** ..... to replace a variable with a numeral  
*Example:*  $8(a) + 3$   
 $8(5) + 3$
- substitution** ..... a method used to solve a system of equations in which variables are replaced with known values or algebraic expressions
- sum** ..... the result of adding numbers together  
*Example:* In  $6 + 8 = 14$ , the sum is 14.
- system of equations** ..... a group of two or more equations that are related to the same situation and share variables  
*Example:* The solution to a system of equations is an ordered number set that makes all of the equations true.
- table (or chart)** ..... a data display that organizes information about a topic into categories
- term** ..... a number, variable, product, or quotient in an expression  
*Example:* In the expression  $4x^2 + 3x + x$ , the terms are  $4x^2$ ,  $3x$ , and  $x$ .
- value (of a variable)** ..... any of the numbers represented by the variable
- variable** ..... any symbol, usually a letter, which could represent a number

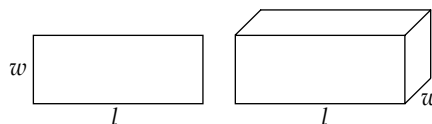


**Venn diagram** .....overlapping circles used to illustrate relationships among sets

**vertical** .....at right angles to the horizon; straight up and down



**width ( $w$ )** .....a one-dimensional measure of something side to side



**$x$ -intercept** .....the value of  $x$  at the point where a line or graph intersects the  $x$ -axis; the value of  $y$  is zero (0) at this point

**$y$ -intercept** .....the value of  $y$  at the point where a line or graph intersects the  $y$ -axis; the value of  $x$  is zero (0) at this point

**zero product property** .....for all numbers  $a$  and  $b$ , if  $ab = 0$ , then  $a = 0$  and/or  $b = 0$







## Unit 10: X or (X, Y) Marks the Spot!

### Introduction

In this unit, we will expand our knowledge of problem solving to find solutions to a variety of equations, inequalities, systems of equations, and real-world situations.

### Lesson One Purpose

#### Reading Process Strand

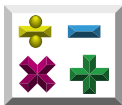
##### Standard 6: Vocabulary Development

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

#### Writing Process Strand

##### Standard 3: Prewriting

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.



## **Algebra Body of Knowledge**

### **Standard 1: Real and Complex Number Systems**

- MA.912.A.1.8  
Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.2  
Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.
- MA.912.A.3.5  
Symbolically represent and solve multi-step and real-world applications that involve linear equations and inequalities.

### **Standard 4: Polynomials**

- MA.912.A.4.3  
Factor polynomial expressions.

### **Standard 7: Quadratic Equations**

- MA.912.A.7.2  
Solve quadratic equations over the real numbers by factoring, and by using the quadratic formula.
- MA.912.A.7.8  
Use quadratic equations to solve real-world problems.

### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.
- MA.912.A.10.2  
Decide whether a solution is reasonable in the context of the original situation.



## Quadratic Equations

When we **solve** an **equation** like  $x + 7 = 12$ , we remember that we must subtract 7 from both sides of the equal sign.

$$\begin{array}{rcl} x + 7 & = & 12 \\ x + 7 - 7 & = & 12 - 7 \\ x & = & 5 \end{array} \quad \begin{array}{l} \swarrow \searrow \\ \text{subtract 7 from both sides} \end{array}$$

That leaves us with  $x = 5$ . We know that 5 is the only **solution** or **value** that can replace  $x$  and make the  $x + 7 = 12$  true.

$$\begin{array}{l} \text{If } x + 7 = 12, \text{ and} \\ x = 5 \text{ is true, then} \\ 5 + 7 = 12. \end{array}$$

Suppose you have an *equation* that looks like  $(x + 7)(x - 3) = 0$ . This means there are two numbers, one in each set of parentheses, that when multiplied together, have a **product** of 0. What kinds of numbers can be multiplied and equal 0?

Look at the following options.

$$2 \times -2 = -4 \qquad \frac{1}{5} \times 5 = 1 \qquad -\frac{4}{7} \times \frac{7}{4} = -1$$

The only way for numbers to be multiplied together with a result of zero is if one of the numbers is a 0.

$$a \times 0 = 0$$





Looking back at  $(x + 7)(x - 3) = 0$ , we understand that there are two **factors**,  $(x + 7)$  and  $(x - 3)$ . The only way to multiply them and get a *product* of 0 is if one of them is equal to zero.

This leads us to a way to *solve* the equation. Since we don't know which of the terms equals 0, we cover all the options and assume either could be equal to zero.

If  $x + 7 = 0$ ,  
then  $x = -7$ .

If  $x - 3 = 0$ ,  
then  $x = 3$ .

We now have two options which could replace  $x$  in the original equation and make it true. Let's replace  $x$  with -7 and 3, one at a time.

$(x + 7)(x - 3) = 0$	$(x + 7)(x - 3) = 0$
$(-7 + 7)(-7 - 3) = 0$	$(3 + 7)(3 - 3) = 0$
$(0)(-10) = 0$	$(10)(0) = 0$
$0 = 0$	$0 = 0$

Therefore, because either *value* of  $x$  gives us a true statement, we see that the **solution set** for  $(x + 7)(x - 3) = 0$  is  $\{-7, 3\}$ .

Now you try the items in the following practice.



## Practice

Find the **solution sets**. Refer to pages 715 and 716 as needed.

1.  $(x + 4)(x - 2) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

2.  $(x - 5)(x + 3) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

3.  $(x - 5)(x - 7) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

4.  $(x + 6)(x + 1) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

5.  $(x - 2)(x - 2) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }



6.  $(x + 18)(x - 23) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

7.  $x(x - 16) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

8.  $(x - 5)(2x + 6) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

9.  $(3x - 5)(5x + 10) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

10.  $(10x - 4)(x + 5) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }



## Factoring to Solve Equations

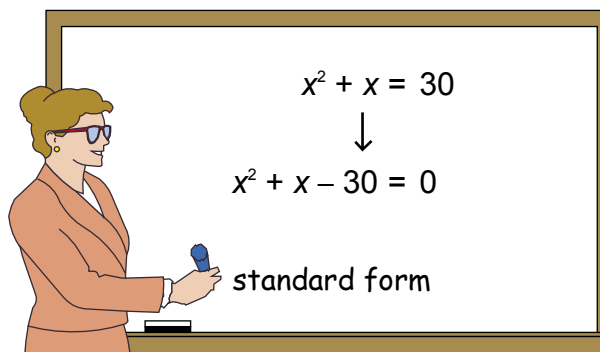
Often, equations are not given to us in **factored form** like those on the previous pages. Looking at  $x^2 + x = 30$ , we notice the  $x^2$  **term** which tells us this is a **quadratic equation** (an equation in the form  $ax^2 + bx + c = 0$ ). This term also tells us to be on the lookout for two answers in our *solution set*.

You may solve this problem by trial and error. However, we can also solve  $x^2 + x = 30$  using a format called **standard form (of a quadratic equation)**. This format is written with the *terms* in a special order:

- the  $x^2$  term first
- then the  $x$ -term
- then the numerical term followed by  $= 0$ .

For our original equation,

$$\begin{array}{lcl} x^2 + x = 30 & \longleftarrow & \text{put in } \textit{standard form} \\ x^2 + x - 30 = 30 - 30 & \longleftarrow & \text{subtract 30 from both sides} \\ x^2 + x - 30 = 0 \end{array}$$





Now that we have the proper format, we can factor the quadratic **polynomial**.



**Remember: Factoring** expresses a *polynomial* as the product of **monomials** and polynomials.

### Example 1

**Solve by factoring**

$$x^2 + x - 30 = 0$$

$$(x + 6)(x - 5) = 0$$

← factor

**Set each factor equal to 0**

If  $x + 6 = 0$ , then

$$x = -6.$$

← **zero product property**

← add -6 to each side

If  $x - 5 = 0$ , then

$$x = 5.$$

← zero product property

← add 5 to each side

Therefore, the solution set is  $\{-6, 5\}$ .

### Example 2

**Solve by factoring**

$$x^2 = 5x - 4$$

$$x^2 - 5x + 4 = 0$$

$$(x - 4)(x - 1) = 0$$

← put in standard form

← factor

**Set each factor equal to 0**

If  $x - 4 = 0$ , then

$$x = 4.$$

← zero product property

← add -4 to each side

If  $x - 1 = 0$ , then

$$x = 1.$$

← zero product property

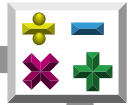
← add -1 to each side

$$\{1, 4\}$$

← write the solution set

Now it's your turn to practice on the following page.





## Practice

Find the **solution sets**. Refer to pages 719 and 720 as needed.

1.  $x^2 - x = 42$  { \_\_\_\_\_ , \_\_\_\_\_ }

2.  $x^2 - 5x = 14$  { \_\_\_\_\_ , \_\_\_\_\_ }

3.  $x^2 = -5x - 6$  { \_\_\_\_\_ , \_\_\_\_\_ }

4.  $x^2 - x = 12$  { \_\_\_\_\_ , \_\_\_\_\_ }

5.  $x^2 = 2x + 8$  { \_\_\_\_\_ , \_\_\_\_\_ }



6.  $x^2 - 2x = 15$  { \_\_\_\_\_ , \_\_\_\_\_ }

7.  $x^2 + 8x = -15$  { \_\_\_\_\_ , \_\_\_\_\_ }

8.  $x^2 - 3x = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

9.  $x^2 = -5x$  { \_\_\_\_\_ , \_\_\_\_\_ }

10.  $x^2 - 4 = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }



11.  $x^2 = 9$  { \_\_\_\_\_ , \_\_\_\_\_ }

12.  $3x^2 - 3 = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

13.  $2x^2 = 18$  { \_\_\_\_\_ , \_\_\_\_\_ }



## Practice

Use the list below to write the correct term for each definition on the line provided.

<b>equation</b>	<b>solution</b>
<b>factor</b>	<b>solve</b>
<b>product</b>	<b>value (of a variable)</b>

- \_\_\_\_\_ 1. a mathematical sentence stating that the two expressions have the same value
- \_\_\_\_\_ 2. to find all numbers that make an equation or inequality true
- \_\_\_\_\_ 3. any of the numbers represented by the variable
- \_\_\_\_\_ 4. any value for a variable that makes an equation or inequality a true statement
- \_\_\_\_\_ 5. the result of multiplying numbers together
- \_\_\_\_\_ 6. a number or expression that divides evenly into another number; one of the numbers multiplied to get a product



## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |       |   |                             |
|-------|---|-----------------------------|
| _____ | 1. an equation in the form of $ax^2 + bx + c = 0$   | A. factored form            |
| _____ | 2. a monomial or sum of monomials; any rational expression with no variable in the denominator                                      | B. factoring                |
| _____ | 3. the set of values that make an equation or inequality true   | C. monomial                 |
| _____ | 4. expressing a polynomial expression as the product of monomials and polynomials   | D. polynomial               |
| _____ | 5. a number, variable, or the product of a number and one or more variables; a polynomial with only <i>one</i> term                 | E. quadratic equation       |
| _____ | 6. a number or expression expressed as the product of prime numbers and variables, where no variable has an exponent greater than 1 | F. solution set ( $\{ \}$ ) |



## Solving Word Problems

We can also use the processes on pages 715-716 and 719-720 to solve word problems. Let's see how.

### Example 1

Two **consecutive** (in order) **positive integers** (integers greater than zero) have a product of 110. Find the integers.

let the 1<sup>st</sup> integer =  $x$   
and the 2<sup>nd</sup> integer =  $x + 1$

$$\begin{aligned}x(x + 1) &= 110 \\x^2 + x &= 110 \\x^2 + x - 110 &= 0 \\(x - 10)(x + 11) &= 0 \\x - 10 &= 0 \quad \text{or} \quad x + 11 = 0 \\x &= 10 \quad \text{or} \quad x = -11\end{aligned}$$

Since the problem asked for *positive integers*, we must eliminate -11 as an answer. Therefore, the two integers are  $x = 10$  and  $x + 1 = 11$ .



**Remember: Integers** are the numbers in the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ .



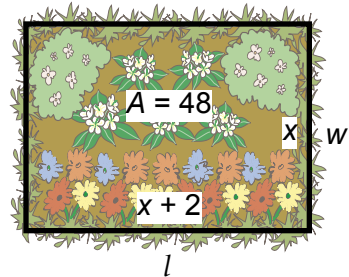
### Example 2

Billy has a garden that is 2 feet longer than it is wide. If the **area** ( $A$ ) of his garden is 48 square feet, what are the dimensions of his garden?

If we knew the **width** ( $w$ ), we could find the **length** ( $l$ ), which is 2 feet longer. Since we don't know the *width*, let's represent it with  $x$ . The *length* will then be  $x + 2$ .

$$\begin{aligned}\text{width} &= x \\ \text{length} &= x + 2\end{aligned}$$

The *area* ( $A$ ) of a **rectangle** can be found using the **formula** length ( $l$ ) times width ( $w$ ).

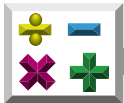


$$\begin{aligned}A &= lw \\ A &= 48\end{aligned}$$

$$\begin{aligned}\text{So, } x(x + 2) &= 48 \\ x^2 + 2x &= 48 \\ x^2 + 2x - 48 &= 0 \\ (x + 8)(x - 6) &= 0 \\ x + 8 &= 0 \quad \text{or} \quad x - 6 = 0 \\ x &= -8 \quad \text{or} \quad x = 6\end{aligned}$$

A garden cannot be -8 feet long, so we must use only the 6 as a value for  $x$ .

So, the width of the garden is 6 feet and the length is 8 feet.



## Practice

Solve each problem. Refer to pages 726 and 727 as needed.

1. The product of two consecutive positive integers is 72. Find the integers.

Answer: \_\_\_\_\_

2. The product of two consecutive positive integers is 90. Find the integers.

Answer: \_\_\_\_\_

3. The product of two consecutive **negative odd integers** is 35. Find the integers.

Answer: \_\_\_\_\_

4. The product of two consecutive *negative odd integers* is 143. Find the integers.

Answer: \_\_\_\_\_





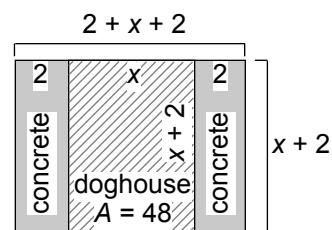
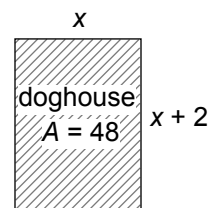
5. Sara's photo is 5 inches by 7 inches. When she adds a frame, she gets an area of 63 square inches. What is the width of the frame?

Answer: \_\_\_\_\_ inches



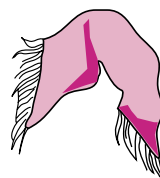
6. Bob wants to build a doghouse that is 2 feet longer than it is wide. He's building it on a concrete slab that will leave 2 feet of concrete slab visible on each side. If the area of the doghouse is 48 square feet, what are the dimensions of the concrete slab?

Answer: \_\_\_\_\_ feet x \_\_\_\_\_ feet



7. Marianne's scarf is 5 inches longer than it is wide. If the area of her scarf is 84 square inches, what are its dimensions?

Answer: \_\_\_\_\_ inches x \_\_\_\_\_ inches





## Practice

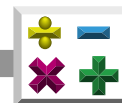
Use the list below to complete the following statements.

<b>area (<math>A</math>)</b>	<b>length (<math>l</math>)</b>	<b>positive integers</b>
<b>consecutive</b>	<b>negative integers</b>	<b>rectangle</b>
<b>integers</b>	<b>odd integers</b>	<b>width (<math>w</math>)</b>

1. Integers that are less than zero are \_\_\_\_\_ .
2. Integers that are greater than zero are \_\_\_\_\_ .
3. Integers that are *not* divisible by 2 are \_\_\_\_\_ .
4. The measure in square units of the inside region of a closed two-dimensional figure is called the \_\_\_\_\_ .
5. A parallelogram with four right angles is called a(n) \_\_\_\_\_ .
6. Numbers in order are \_\_\_\_\_ .
7. To find the area ( $A$ ) of a rectangle, you multiply the \_\_\_\_\_ by the \_\_\_\_\_ .
8. Numbers in the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$  are called \_\_\_\_\_ .

Answer the following.

9. The length of a rectangle can \_\_\_\_\_ (always, sometimes, never) be a negative number.



## Using the Quadratic Formula

Sometimes an equation seems difficult to factor. When this happens, you may need to use the **quadratic formula**. Remember that quadratic equations use the format below.

$$ax^2 + bx + c = 0$$

The *quadratic formula* uses the information from the equation and looks like the following.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Compare the equation with the formula and notice how all the same letters are just in different places.

Let's see how the quadratic formula is used to solve the following equation.

In the equation,  $a = 4$ ,  $b = 1$ , and  $c = -5$ . These values are **substituted** into the quadratic formula below.

$$4x^2 + x - 5 = 0$$

← original equation

$a = 4$	$b = 1$	$c = -5$
---------	---------	----------

← values

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

← quadratic formula

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(4)(-5)}}{2(4)}$$

← values  $a = 4$ ,  $b = 1$ , and  $c = -5$  substituted

$$x = \frac{-1 \pm \sqrt{1 - (-80)}}{8}$$

← simplify

$$x = \frac{-1 \pm \sqrt{81}}{8}$$

$$x = \frac{-1 \pm 9}{8}$$

$$x = \frac{-1 + 9}{8} \quad \text{or} \quad x = \frac{-1 - 9}{8}$$

$$x = \frac{8}{8} \quad \text{or} \quad x = \frac{-10}{8}$$

$$x = 1 \quad \text{or} \quad x = -\frac{5}{4} \quad \leftarrow \text{the solution set is } \{1, -\frac{5}{4}\}$$



**Remember:** The symbol  $\pm$  means plus or minus. Therefore,  $\pm$  means we have two factors. One is found by adding and the other by subtracting.



Let's look at another example.

$$2x^2 + 5x + 3 = 0$$

← original equation

$$a = 2 \quad b = 5 \quad c = 3$$

← values

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

← quadratic formula

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)}$$

← values  $a = 2$ ,  $b = 5$ , and  $c = 3$  *substituted*

$$x = \frac{-5 \pm \sqrt{25 - 24}}{4}$$

← simplify

$$x = \frac{-5 \pm \sqrt{1}}{4}$$

$$x = \frac{-5 \pm 1}{4}$$

$$x = \frac{-5 + 1}{4} \quad \text{or} \quad x = \frac{-5 - 1}{4}$$

$$x = \frac{-4}{4} \quad \text{or} \quad x = \frac{-6}{4}$$

$$x = -1 \quad \text{or} \quad x = \frac{-3}{2} \quad \leftarrow \text{the solution set is } \{-1, \frac{-3}{2}\}$$

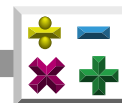
Often your answer will not **simplify** all the way to a **fraction** or *integer*.

Some answers will look like  $\frac{-3 \pm \sqrt{13}}{6}$ . You can check your work by using a graphing calculator or another advanced-level calculator.

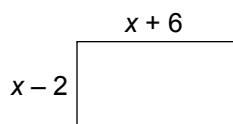
We can even use the quadratic formula when solving word problems.

However, before you start using the quadratic formula, it is important to remember to put the equation you are working with in the correct format. The equation must look like the following.

$$ax^2 + bx + c = 0$$



Look at this example.



If a rectangle has an area of 20 and its dimensions are as shown, find the actual length and width of the rectangle.

$$(x + 6)(x - 2) = 20$$

← set up the equation

$$x^2 + 4x - 12 = 20$$

← **FOIL**—First, Outside, Inside, Last

$$x^2 + 4x - 32 = 0$$

← format ( $ax^2 + bx + c = 0$ )

$$(x + 8)(x - 4) = 0$$

← factor

$$x + 8 = 0 \quad \text{or} \quad x - 4 = 0$$

← solve

$$x = -8 \quad \text{or} \quad x = 4$$

$x \neq -8$  because that would result in negative lengths.

← first check to see *if* solutions are reasonable



**Remember:** The symbol  $\neq$  means is *not* equal to.

$$x + 6 \longrightarrow 4 + 6 = 10$$

$$x - 2 \longrightarrow 4 - 2 = 2$$

← then check answer by replacing  $x$  with 4

The length and width of the rectangle are 10 and 2.



## Practice

Use the **quadratic formula** below to solve the following equations.



Check your work using a calculator.

**quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

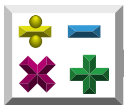
1.  $2x^2 + 5x + 3 = 0$

2.  $2x^2 + 3x + 1 = 0$



3.  $4x^2 + 3x - 1 = 0$

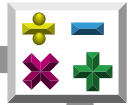
4.  $6x^2 + 5x + 1 = 0$



5.  $4x^2 - 11x + 6 = 0$

6.  $2x^2 - x - 3 = 0$





7.  $2x^2 - 3x + 1 = 0$

8.  $9x^2 - 3x - 5 = 0$



9.  $8x^2 - 6x - 2 = 0$

10.  $9x^2 + 9x - 4 = 0$



11. Jacob wants to build a deck that is  $(x + 7)$  units long and  $(x + 3)$  units wide. If the area of his deck is 117 square units, what are the dimensions of his deck?

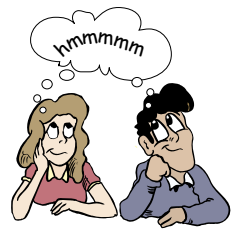
Answer: \_\_\_\_\_ units x \_\_\_\_\_ units



12. Cecilia and Roberto are thinking of 2 consecutive odd integers whose product is 143. Find the integers.

First use the guess and check method. Then check by solving.

Answer: \_\_\_\_\_ and \_\_\_\_\_





## Practice

Use the **quadratic formula** below to solve the following equations.



Show work and  
check your work  
with a calculator.

**quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.  $2x^2 + 7x + 6 = 0$

2.  $5x^2 + 16x + 3 = 0$



3.  $6x^2 + 5x + 1 = 0$

4.  $9x^2 + 9x + 2 = 0$



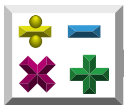
5.  $10x^2 + 7x + 1 = 0$

6.  $10x^2 - 7x + 1 = 0$



7.  $x^2 + 2x - 3 = 0$

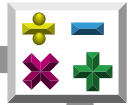
8.  $x^2 + 2x - 15 = 0$



9.  $x^2 - 5x + 6 = 0$

10.  $x^2 - 9x + 20 = 0$





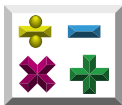
11. If the sides of a rectangular walkway are  $(x + 3)$  units and  $(x - 6)$  units, and the area is 10 square units, find the dimensions of the walkway.

Answer: \_\_\_\_\_ units x \_\_\_\_\_ units

12. Jordan is thinking of 2 consecutive even integers. If the product of her integers is 168, find the numbers.

**Hint:** You could use the guess and check problem-solving strategy to solve.

Answer: \_\_\_\_\_ and \_\_\_\_\_



## **Lesson Two Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Writing Process Strand**

#### **Standard 3: Prewriting**

- LA.910.3.1.3  
The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

## **Algebra Body of Knowledge**

### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.12  
Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph.
- MA.912.A.3.13  
Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology.



- MA.912.A.3.14  
Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods.
- MA.912.A.3.15  
Solve real-world problems involving systems of linear equations and inequalities in two and three variables.

### **Standard 10: Mathematical Reasoning and Problem Solving**

- MA.912.A.10.1  
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.



## Systems of Equations

When we look at an equation like  $x + y = 5$ , we see that because there are two **variables**, there are many possible solutions. For instance,

- if  $x = 5$ , then  $y = 0$
- if  $x = 2$ , then  $y = 3$
- if  $x = -4$ , then  $y = 9$
- if  $x = 2.5$ , then  $y = 2.5$ , etc.

Another equation such as  $x - y = 1$  allows a specific solution to be determined. Taken together, these two equations help to limit the possible solutions.

When taken together, we call this a **system of equations**. A *system of equations* is a group of two or more equations that are related to the same situation and share the same variables. Look at the equations below.

$$\begin{aligned}x + y &= 5 \\x - y &= 1\end{aligned}$$

One possible way to solve the system of equations above is to **graph each equation** on the same set of **axes**. Use a **table of values** like those on the following page to help determine two possible **points** for each **line** ( $\longleftrightarrow$ ).

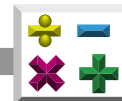


Table of Values

$x + y = 5$	
$x$	$y$
0	5
5	0

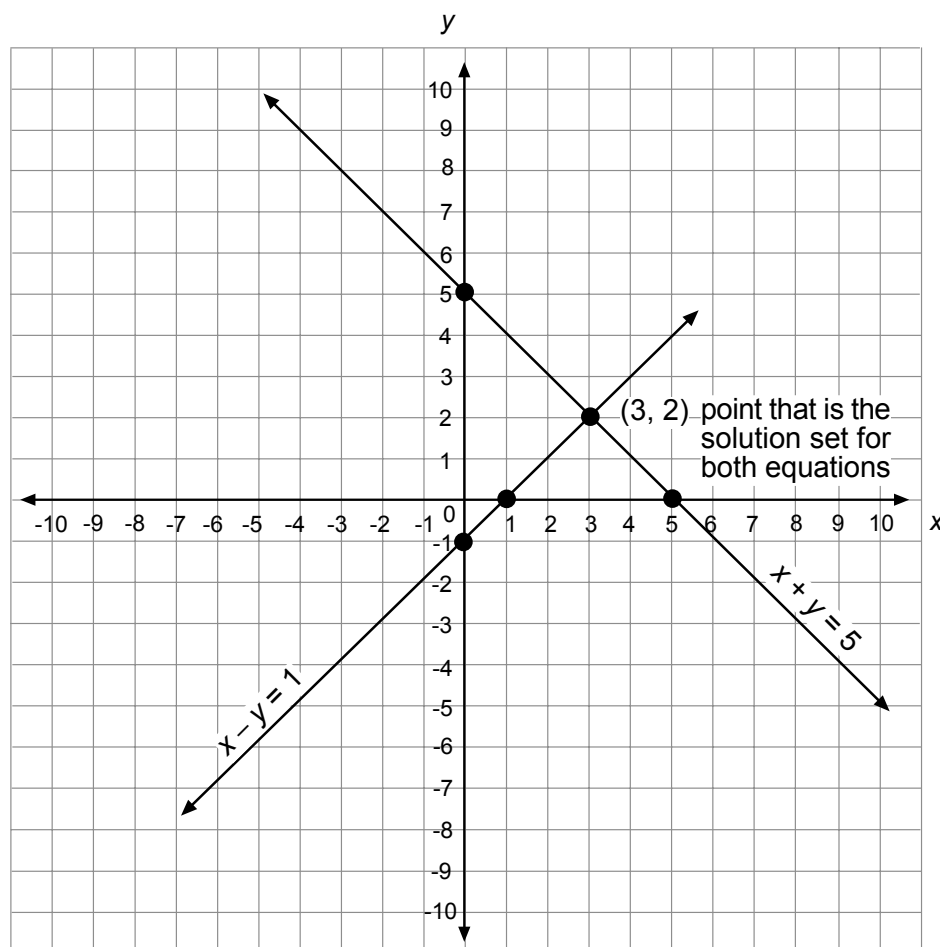
Table of Values

$x - y = 1$	
$x$	$y$
0	-1
1	0

Notice that the values in the table represent the  $x$ -intercepts and  $y$ -intercepts.

Plot the *points* for the first equation on the **coordinate grid** or **plane** below, then draw a *line* connecting them. Do the same for the second set of points.

Graph of  $x + y = 5$  and  $x - y = 1$



We see from the **graph** above that the two lines **intersect** or *cross at a point*. That point  $(3, 2)$  is the solution set for both equations. It is the only point



that makes both equations true. You can check your work by replacing  $x$  with 3 and  $y$  with 2 in both equations to see if they produce true statements.

You can also produce this graph on your *graphing calculator*. To closely estimate the **coordinates** of the points of the graph, move the cursor, the blinking dot, along one line until it gets to the point of **intersection**.



Although graphing is one way to deal with systems of equations; however, it is *not* always the most accurate method. If our graph paper is *not* perfect, our pencil is *not* super-sharp, or the point of *intersection* is *not* at a corner on the grid, we may *not* get the correct answer.

The system can also be solved algebraically with more accuracy. Let's see how that works.

We know from past experience that we can solve problems more easily when there is only one *variable*. So, our job is to eliminate a variable. If we look at the two equations **vertically** (straight up and down), we see that, by adding in columns, the  $y$ 's will disappear.

$$\begin{array}{r} x + y = 5 \\ x - y = 1 \\ \hline 2x + 0 = 6 \end{array}$$

This leaves us with a new equation to solve:

$$\begin{array}{r} 2x + 0 = 6 \\ 2x = 6 \\ \frac{2x}{2} = \frac{6}{2} \quad \leftarrow \text{divide both sides by 2} \\ x = 3 \end{array}$$

We've found the value for  $x$ ; now we must find the value of  $y$ . Use either of the original equations and replace the  $x$  with 3. The example below uses the first one.

$$\begin{array}{r} x + y = 5 \\ 3 + y = 5 \\ 3 - 3 + y = 5 - 3 \quad \leftarrow \text{subtract 3 from both sides} \\ y = 2 \end{array}$$

So, our solution set is  $\{3, 2\}$ .



Let's try another! We'll solve and then graph this time.

$$\begin{array}{rcl} 2x + y & = & 6 \\ -2x + 2y & = & -12 \\ \hline 0 + 3y & = & -6 \\ \frac{3y}{3} & = & \frac{-6}{3} = \frac{-2}{1} \\ y & = & -2 \end{array}$$

← add to eliminate the x's

← solve

$$\begin{array}{rcl} 2x + -2 & = & 6 \\ 2x + -2 + 2 & = & 6 + 2 \\ 2x & = & 8 \\ \frac{2x}{2} & = & \frac{8}{2} = \frac{4}{1} \\ x & = & 4 \end{array}$$

← replace y with -2 in one equation and solve for x

Our solution set is  $\{4, -2\}$ .

Now let's see how graphing the two equations is done on the following page.



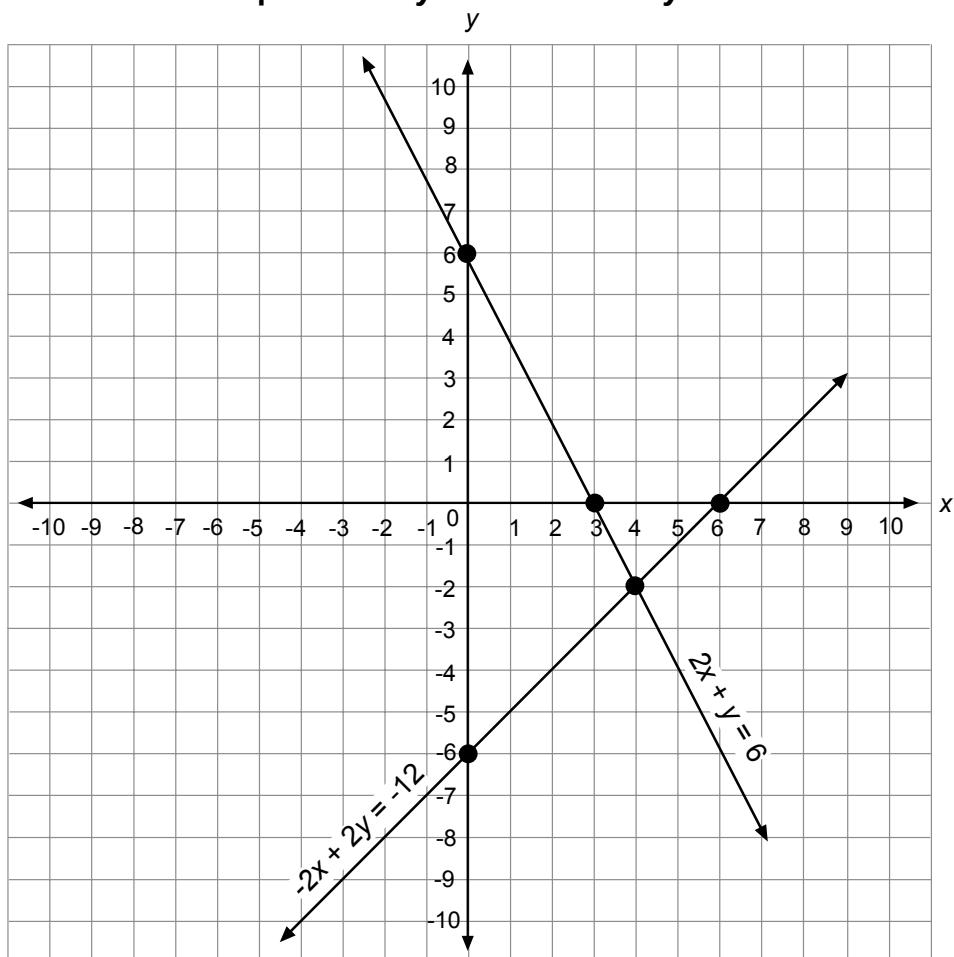
Table of Values

$2x + y = 6$	
$x$	$y$
0	6
3	0

Table of Values

$-2x + 2y = -12$	
$x$	$y$
0	-6
6	0

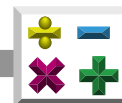
Graph of  $2x + y = 6$  and  $-2x + 2y = -12$



**Note:** Watch for these special situations.

- If the graphs of the equations are the same line, then the two equations are *equivalent* and have an **infinite** (that is, limitless) number of possible solutions.
- If the graphs do not *intersect* at all, they are **parallel** ( $\parallel$ ), and are an equal distance at every point. They have *no* possible solutions. The solution set would be empty— $\{ \}$ .





## Practice

Solve each **system of equations** algebraically. Use the **table of values** to solve and **graph** both equations on the graphs provided. Refer to pages 748-752 as needed.



Check your work with a **graphing calculator** by replacing  $x$  and  $y$  in both equations with the coordinates of the point of intersection if one exists.

**Hint:** Two of the following sets of equations are **equivalent expressions** and will have the same line with an *infinite* number of possible solutions. See note on the previous page.

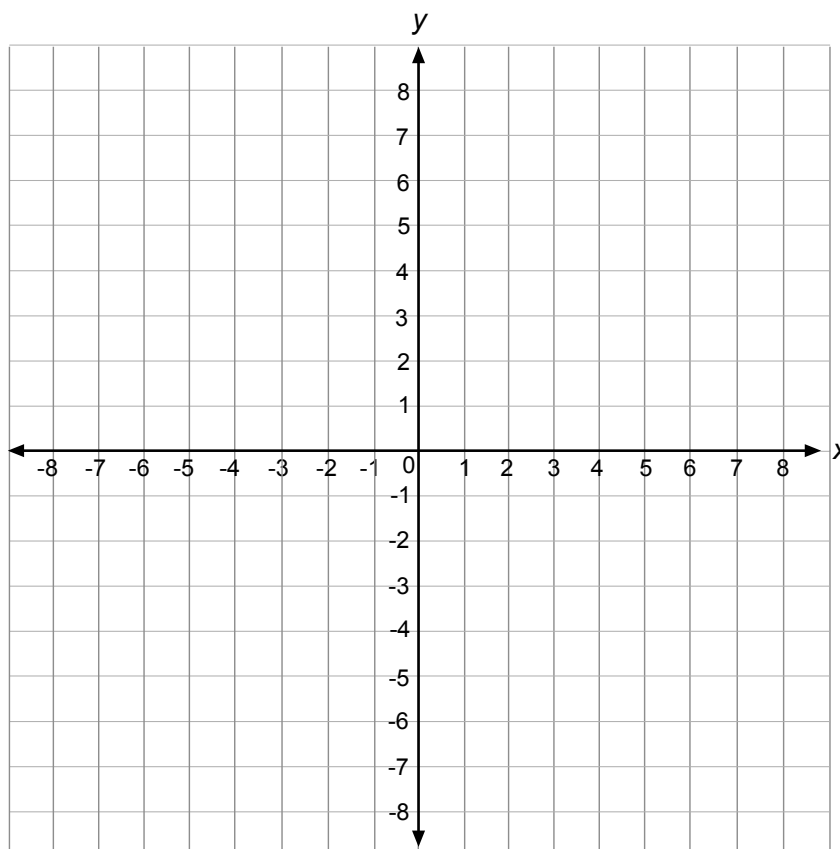
1.  $x - y = -1$   
 $x + y = 7$

**Table of Values**   **Table of Values**

$x - y = -1$	
$x$	$y$

$x + y = 7$	
$x$	$y$

**Graph of  $x - y = -1$  and  $x + y = 7$**





2.  $2x - y = 4$   
 $x + y = 5$

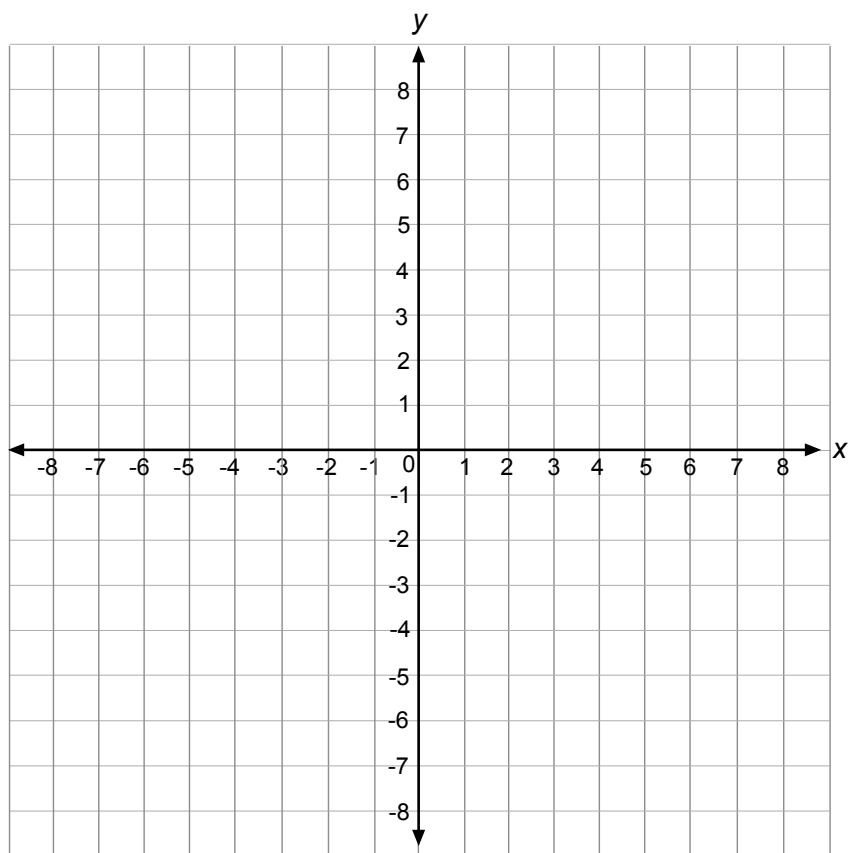
**Table of Values**

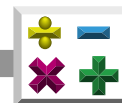
$2x - y = 4$	
$x$	$y$

**Table of Values**

$x + y = 5$	
$x$	$y$
0	5
5	0

**Graph of  $2x - y = 4$  and  $x + y = 5$**





3.  $4x - y = 2$   
 $-2x + y = 0$

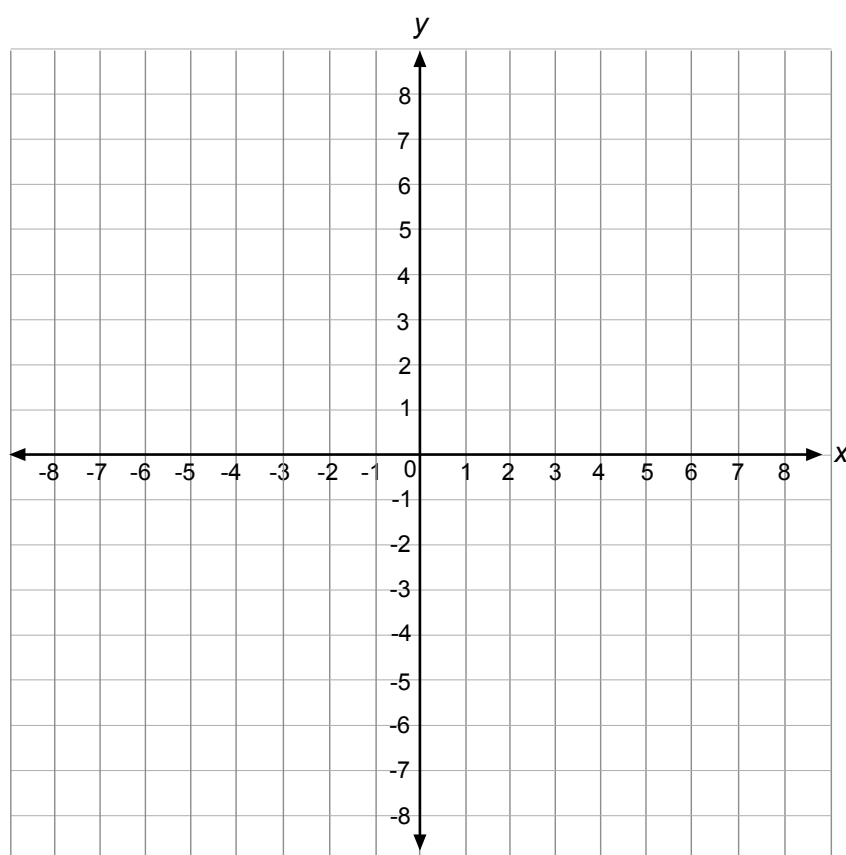
Table of Values

$4x - y = 2$	
$x$	$y$

Table of Values

$-2x + y = 0$	
$x$	$y$

Graph of  $4x - y = 2$  and  $-2x + y = 0$





4.  $x - 2y = 4$   
 $2x - 4y = 8$

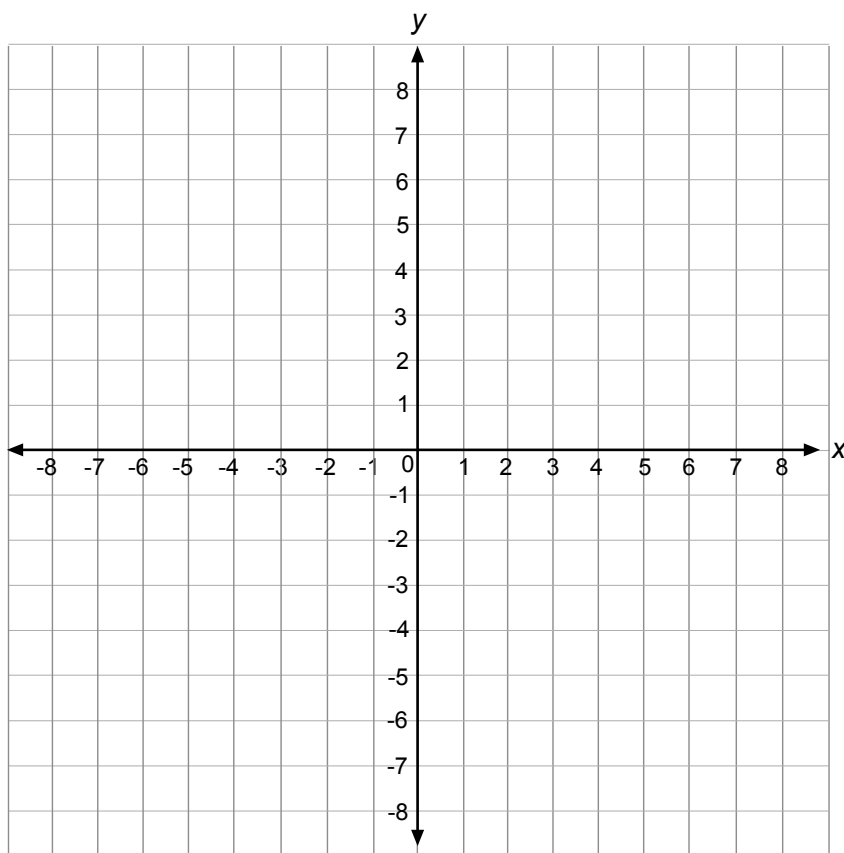
Table of Values

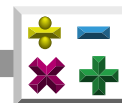
$x - 2y = 4$	
$x$	$y$

Table of Values

$2x - 4y = 8$	
$x$	$y$

Graph of  $x - 2y = 4$  and  $2x - 4y = 8$





5.  $2x + y = 8$   
 $-2x + y = -4$

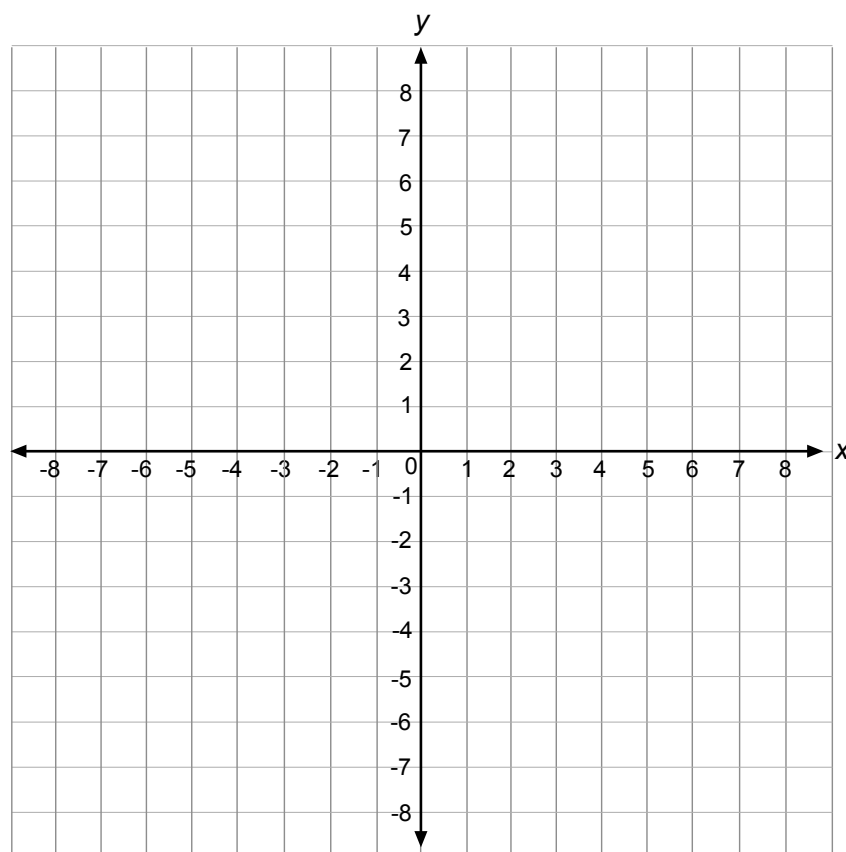
Table of Values

$2x + y = 8$	
$x$	$y$

Table of Values

$-2x + y = -4$	
$x$	$y$

Graph of  $2x + y = 8$  and  $-2x + y = -4$





6.  $3x - 2y = -1$   
 $-6x + 4y = 2$

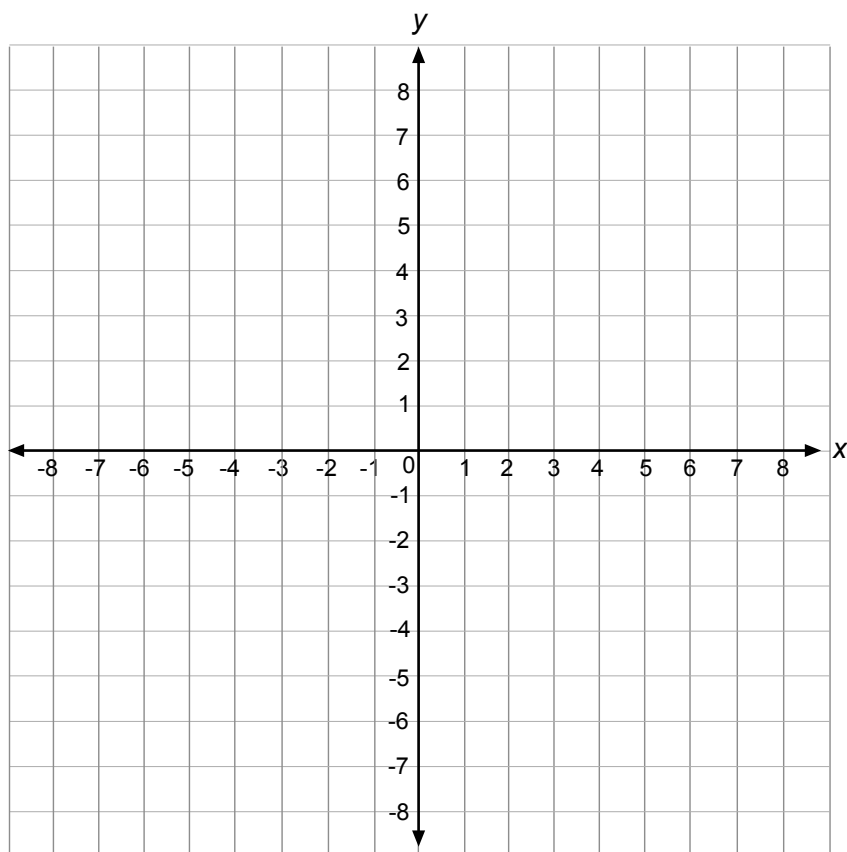
Table of Values

$3x - 2y = -1$	
$x$	$y$

Table of Values

$-6x + 4y = 2$	
$x$	$y$

Graph of  $3x - 2y = -1$  and  $-6x + 4y = 2$





## Using Substitution to Solve Equations

There are other processes we can use to solve systems of equations. Let's take a look at some of the options.

### Example 1

Suppose our two equations are as follows.

$$\begin{aligned}2x + 3y &= 14 \\ x &= 4\end{aligned}$$

To solve this system, we could use a method called **substitution**. We simply put the value of  $x$  from the second equation in for the  $x$  in the first equation.

$$\begin{aligned}2x + 3y &= 14 && \swarrow \text{substitute 4 for } x \\ 2(4) + 3y &= 14 && \swarrow \text{simplify} \\ 8 + 3y &= 14 && \swarrow \\ 8 - 8 + 3y &= 14 - 8 && \swarrow \text{subtract} \\ 3y &= 6 && \swarrow \text{divide} \\ \frac{3y}{3} &= \frac{6}{3} \\ y &= 2\end{aligned}$$

The solution set is  $\{4, 2\}$ .



## Example 2

This one is a little more complex.

Below are our two equations.

$$\begin{aligned}4x - y &= -2 \\ x &= y + 4\end{aligned}$$

We can substitute  $(y + 4)$  from the second equation in for  $x$  in the first equation.

$$\begin{aligned}4x - y &= -2 && \leftarrow \text{substitute } (y + 4) \text{ for } x \\ 4(y + 4) - y &= -2 && \leftarrow \text{distribute} \\ 4y + 16 - y &= -2 && \leftarrow \text{simplify} \\ 3y + 16 &= -2 && \leftarrow \text{subtract} \\ 3y &= -18 && \leftarrow \text{divide} \\ y &= -6\end{aligned}$$

Notice that  $(y + 4)$  is in parentheses. This helps us remember to *distribute* when the time comes.

Now we must find the value of  $x$ . Use an original equation and substitute  $-6$  for  $y$  and then solve for  $x$ .

$$\begin{aligned}4x - y &= -2 && \leftarrow \text{original equation} \\ 4x - (-6) &= -2 && \leftarrow \text{substitute } (-6) \text{ for } y \\ 4x + 6 &= -2 && \leftarrow \text{simplify} \\ 4x &= -8 && \leftarrow \text{subtract} \\ x &= -2 && \leftarrow \text{divide}\end{aligned}$$

Now try the practice on the following page.





## Practice

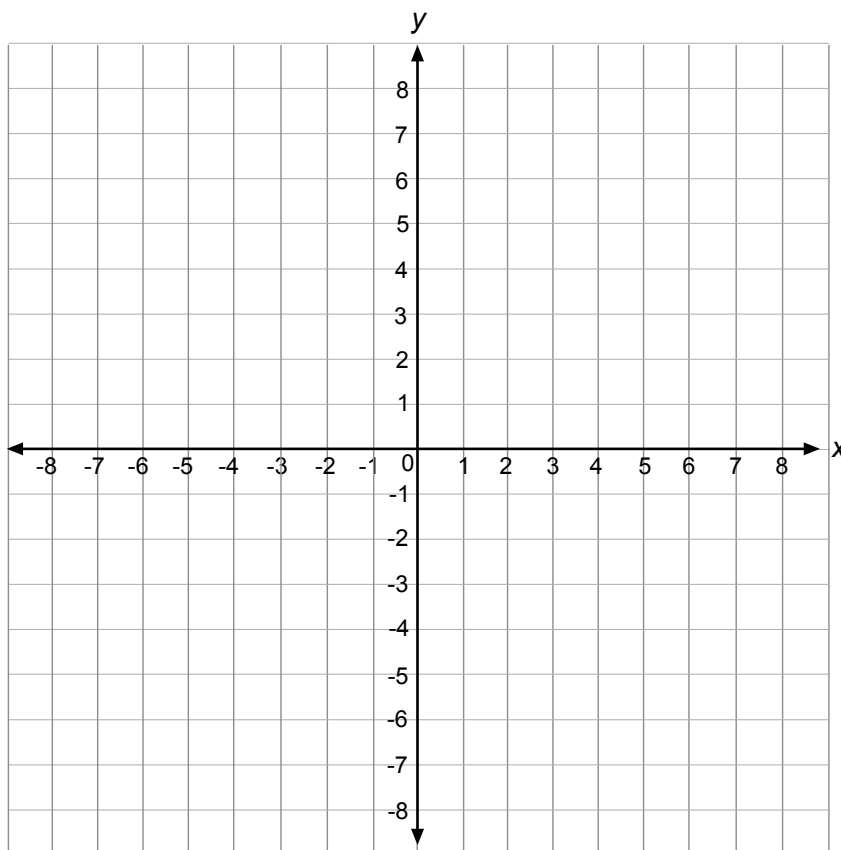
Solve each **system of equations** algebraically. Use the **substitution method** to solve and **graph** both equations on the graphs provided. Refer to pages 759 and 760 as needed.



Check your work by graphing on a calculator or by replacing  $x$  and  $y$  with the coordinates of your solution.

1.  $3x - 2y = 6$   
 $x = 4$

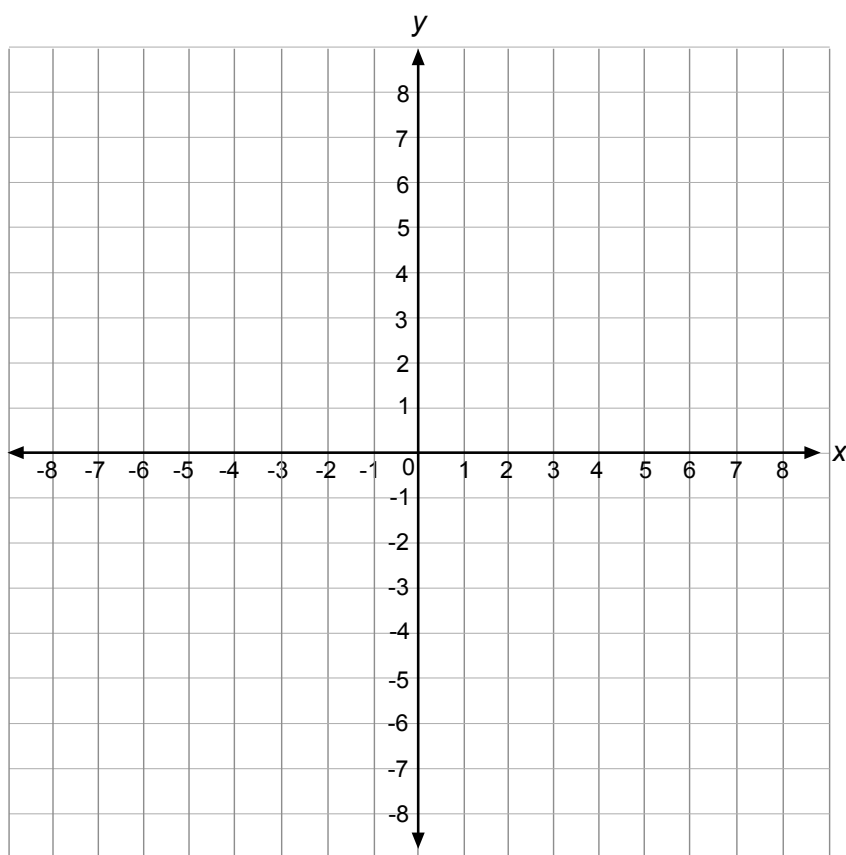
Graph of  $3x - 2y = 6$  and  $x = 4$





2.  $5x - y = 9$   
 $x = 2y$

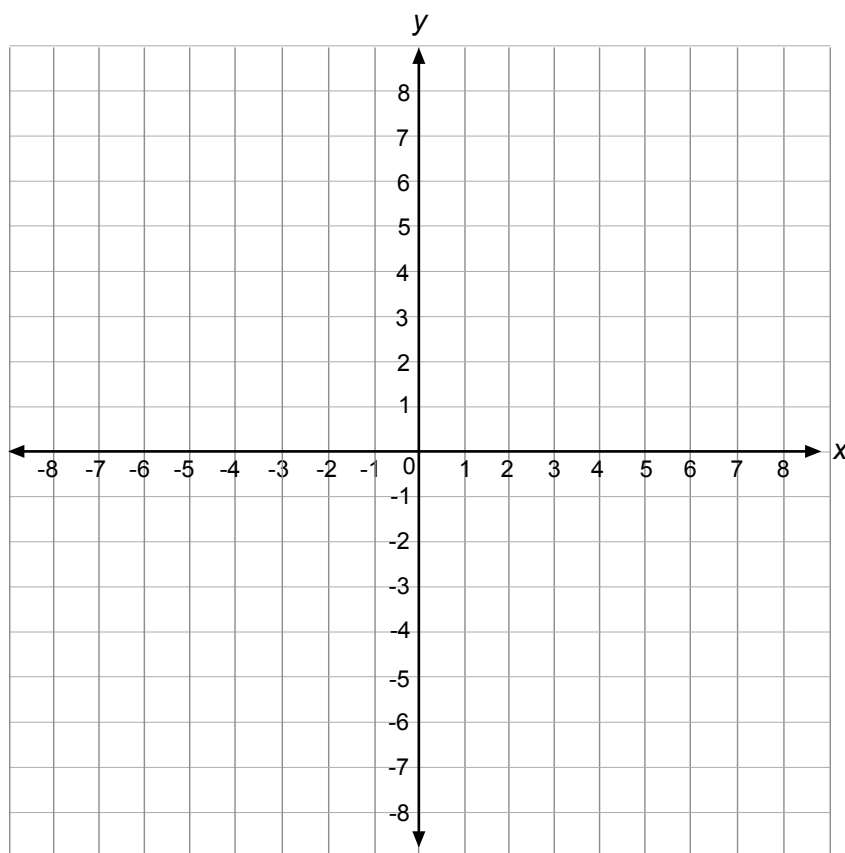
**Graph of  $5x - y = 9$  and  $x = 2y$**





3.  $x + y = 5$   
 $x = y + 1$

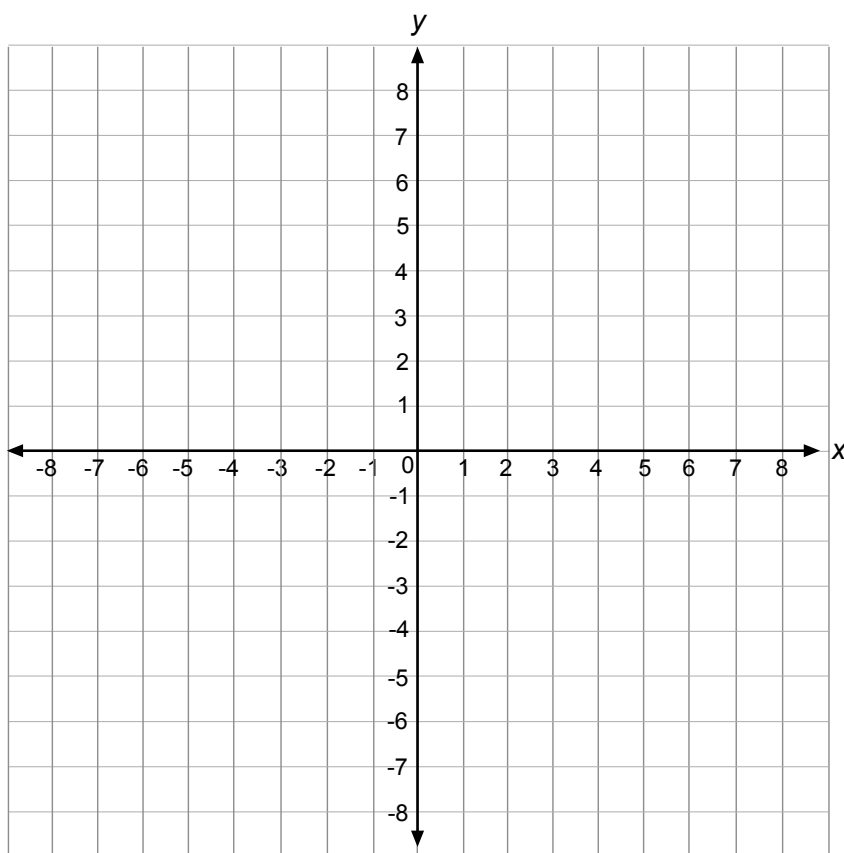
**Graph of  $x + y = 5$  and  $x = y + 1$**





4.  $5x + y = -15$   
 $y = 1 - x$

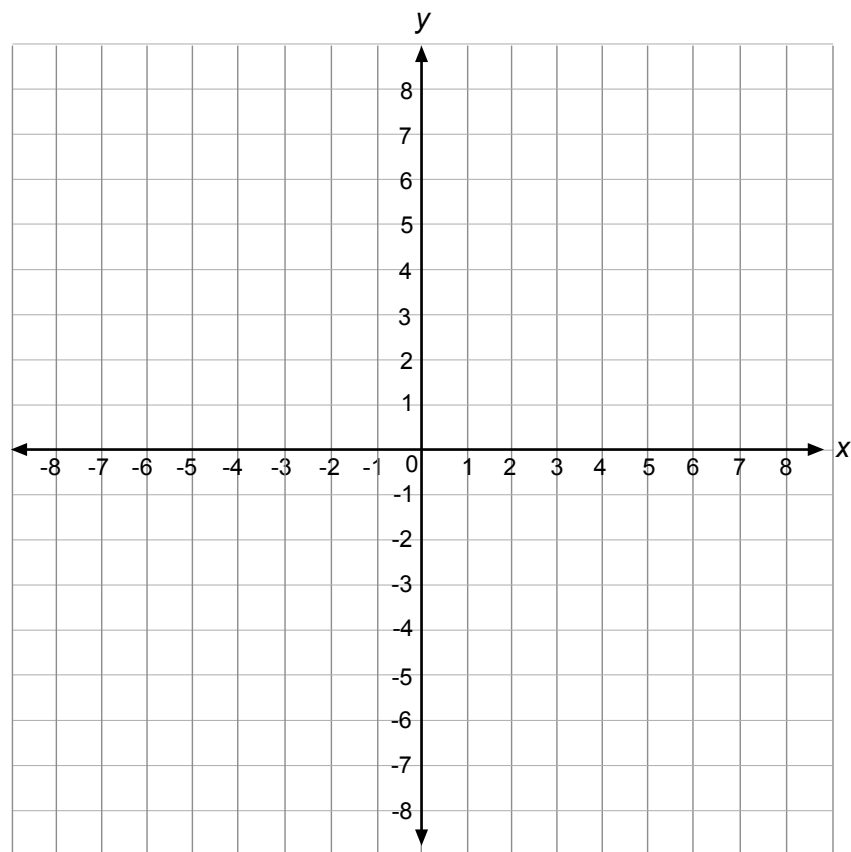
**Graph of  $5x + y = -15$  and  $y = 1 - x$**





5.  $x = 2y + 15$   
 $4x + 2y = 10$

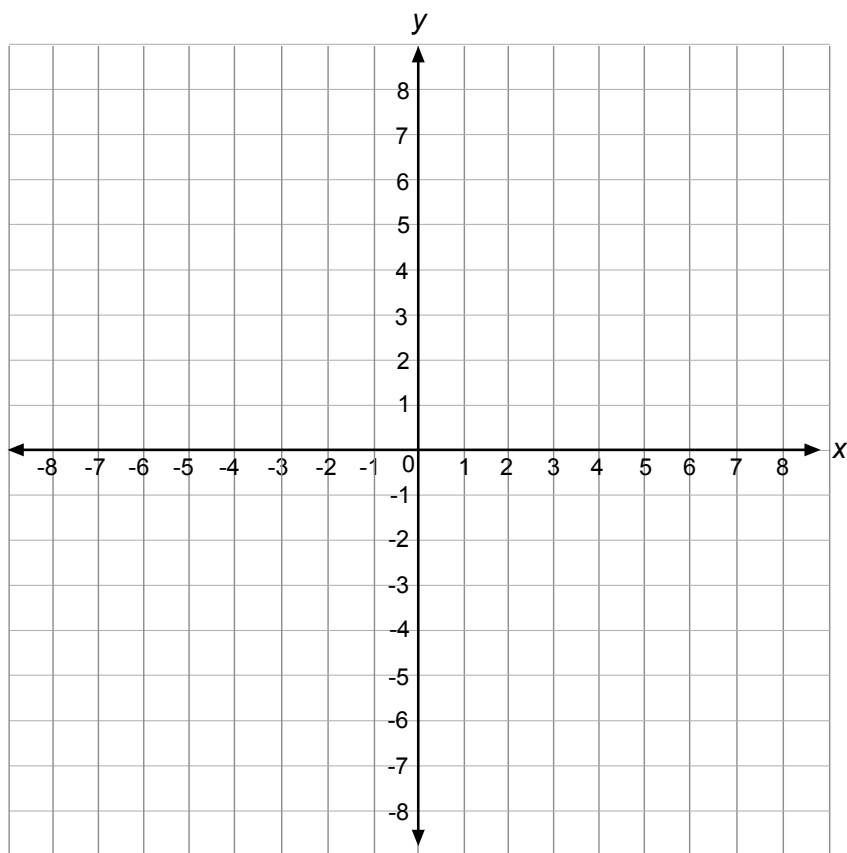
**Graph of  $x = 2y + 15$  and  $4x + 2y = 10$**

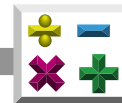




6.  $x + 2y = 14$   
 $x = 3y - 11$

**Graph of  $x + 2y = 14$  and  $x = 3y - 11$**





## Using Magic to Solve Equations

There are times when neither the algebraic or substitution method seems like a good option. If the equations should look similar to these, we have another option.

### Example 1

$$\begin{aligned} 5x + 12y &= 41 \\ 9x + 4y &= 21 \end{aligned}$$



We have to perform a little “math-magic” to solve this problem. When looking at these equations, you should see that if the  $4y$  were  $-12y$  instead, we could add vertically and the  $y$ ’s would disappear from the equation.

So, our job is to make that  $4y$  into  $-12y$ . We could do that by multiplying  $4y$  by  $-3$ . The only catch is that we must multiply the whole equation by  $-3$  to keep everything balanced.

$$\begin{aligned} 9x + 4y &= 21 && \leftarrow \text{original equation} \\ -3(9x + 4y) &= 21(-3) && \leftarrow \text{multiply equation by } -3 \\ -27x + (-12y) &= -63 && \leftarrow \text{new 2}^{\text{nd}} \text{ equation} \end{aligned}$$

Now line up the equations, replacing the second one with the new equation.

$$\begin{array}{rcl} 5x + 12y & = & 41 \quad \leftarrow \text{original 1}^{\text{st}} \text{ equation} \\ -27x + (-12y) & = & -63 \quad \leftarrow \text{new 2}^{\text{nd}} \text{ equation} \\ \hline -22x + 0 & = & -22 \quad \leftarrow \text{subtract vertically} \\ -22x & = & -22 \quad \leftarrow \text{simplify} \\ x & = & 1 \quad \leftarrow \text{divide} \end{array}$$

Now that we know the value of  $x$ , we can replace  $x$  with  $1$  in the original equation and solve for  $y$ .

$$\begin{aligned} 5x + 12y &= 41 && \leftarrow \text{original 1}^{\text{st}} \text{ equation} \\ 5(1) + 12y &= 41 && \leftarrow \text{substitute (1) for } x \\ 5 + 12y &= 41 && \leftarrow \text{simplify} \\ 12y &= 36 && \leftarrow \text{subtract} \\ y &= 3 && \leftarrow \text{divide} \end{aligned}$$

Our solution set is  $\{1, 3\}$ . Be sure to put the answers in the *correct order* because they are an **ordered pair**, where the first and second value represent a position on a *coordinate grid* or *system*.



Sometimes you may have to perform “math-magic” on both equations to get numbers to “disappear.”

### Example 2

$$\begin{aligned}3x - 4y &= 2 \\ 2x + 3y &= 7\end{aligned}$$

After close inspection, we see that this will take double magic. If the **coefficients** of the  $x$ 's could be made into a  $6x$  and a  $-6x$ , this problem might be solvable. Let's try!

Multiply the first equation by 2 and the second equation by -3.

$$\begin{array}{rclcl}2(3x - 4y = 2) & \longrightarrow & 6x - 8y = 4 & \longleftarrow & 1^{\text{st}} \text{ equation} \cdot 2 \\ -3(2x + 3y = 7) & \longrightarrow & -6x - 9y = -21 & \longleftarrow & 2^{\text{nd}} \text{ equation} \cdot -3 \\ \hline & & 0 - 17y = -17 & & \\ & & -17y = -17 & & \\ & & y = 1 & & \end{array}$$

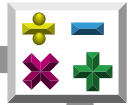
Use  $y = 1$  to find the value of  $x$  using an original equation.

$$\begin{aligned}3x - 4y &= 2 & \longleftarrow & \text{original } 1^{\text{st}} \text{ equation} \\ 3x - 4(1) &= 2 & \longleftarrow & \text{substitute (1) for } y \\ 3x - 4 &= 2 \\ 3x &= 6 \\ x &= 2\end{aligned}$$

The solution set is  $\{2, 1\}$ .

Now it's your turn to practice on the next page.





## Practice

Solve each of the following **systems of equations**. Refer to pages 767 and 768 as needed. Check your work.

1. 
$$\begin{aligned} 3x + y &= 7 \\ 2x - 3y &= 12 \end{aligned}$$

2. 
$$\begin{aligned} 3x + y &= 11 \\ x + 2y &= 12 \end{aligned}$$

3. 
$$\begin{aligned} 9x + 8y &= -45 \\ 6x + y &= 9 \end{aligned}$$



4. 
$$\begin{aligned} -5x + 4y &= 4 \\ 4x - 7y &= 12 \end{aligned}$$

5. 
$$\begin{aligned} -2x - 11y &= 4 \\ 5x + 9y &= 27 \end{aligned}$$

6. 
$$\begin{aligned} 2x + 3y &= 20 \\ 3x + 2y &= 15 \end{aligned}$$



## Solving More Word Problems

Let's see how we might use the methods we've learned to solve word problems.

### Example 1

Twice the **sum** of two integers is 20. The larger integer is 1 more than twice the smaller. Find the integers.

Let  $S$  = the smaller integer

Let  $L$  = the larger integer

Now, write equations to fit the wording in the problem.

$$\begin{array}{llll} 2(S + L) = 20 & \text{and} & L = 2S + 1 & \\ 2S + 2L = 20 & \leftarrow & \text{simplify} & \\ 2S + 2(2S + 1) = 20 & \leftarrow & \text{substitute } (2S + 1) \text{ for } L & \\ 2S + 4S + 2 = 20 & \leftarrow & \text{distribute} & \\ 6S + 2 = 20 & \leftarrow & \text{simplify} & \\ 6S = 18 & \leftarrow & \text{subtract} & \\ S = 3 & \leftarrow & \text{divide} & \end{array}$$

Since the smaller integer is 3, the larger one is  $2(3) + 1$  or 7. The integers are 3 and 7.



## Example 2

Three tennis lessons and three golf lessons cost \$60. Nine tennis lessons and six golf lessons cost \$147. Find the cost of one tennis lesson and one golf lesson.

Let  $T$  = the cost of 1 tennis lesson

Let  $G$  = the cost of 1 golf lesson

Use the variables to interpret the sentences and make equations.

$$3T + 3G = 60$$

$$9T + 6G = 147$$

Make the *coefficients* of  $G$  match by multiplying the first equation by  $-2$ .

$$\begin{array}{rcl} -6T + -6G = -120 & \longleftarrow & -2(3T + 3G = 60) \\ 9T + 6G = 147 & \longleftarrow & \text{bring in the 2}^{\text{nd}} \text{ equation} \\ \hline 3T + 0 = 27 & \longleftarrow & \text{subtract} \\ 3T = 27 & \longleftarrow & \text{divide} \\ T = 9 & \longleftarrow & \end{array}$$



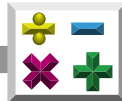
We know that one tennis lesson costs \$9, so let's find the cost of one golf lesson.

$$\begin{array}{rcl} 3T + 3G = 60 & \longleftarrow & \text{original 1}^{\text{st}} \text{ equation} \\ 3(9) + 3G = 60 & \longleftarrow & \text{substitute (9) for } T \\ 27 + 3G = 60 & \longleftarrow & \text{simplify} \\ 3G = 33 & \longleftarrow & \text{divide} \\ G = 11 & \longleftarrow & \end{array}$$



So, one tennis lesson costs \$9 and one golf lesson costs \$11.

Now you try a few items on the next page.



## Practice

*Solve each of the following. Refer to pages 754 and 755 as needed.*

1. The sum of two numbers is 35. The larger one is 4 times the smaller one. Find the two numbers.

Answer: \_\_\_\_\_ and \_\_\_\_\_

2. A 90-foot cable is cut into two pieces. One piece is 18 feet longer than the shorter one. Find the lengths of the two pieces.

Answer: \_\_\_\_\_ feet and \_\_\_\_\_ feet

3. Joey spent \$98 on a pair of jeans and a shirt. The jeans cost \$20 more than the shirt. How much did each cost?



Answer: jeans = \$ \_\_\_\_\_ and shirt = \$ \_\_\_\_\_



4. Four sandwiches and three drinks cost \$13. Two drinks cost \$0.60 more than one sandwich. Find the cost of one drink and one sandwich.



Answer: \$ \_\_\_\_\_

5. The football team at Leon High School has 7 more members than the team from Central High School. Together the two teams have 83 players. How many players does each team have?



Answer: Central = \_\_\_\_\_ and Leon = \_\_\_\_\_

6. Andre earns \$40 a week less than Sylvia. Together they earn \$360 each week. How much does each earn?

Answer: Sylvia = \$ \_\_\_\_\_ and Andre = \$ \_\_\_\_\_



## Practice

*Match each definition with the correct term. Write the letter on the line provided.*

- |   |                             |
|---|-----------------------------|
| _____ 1. a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced | A. axes (of a graph)        |
| _____ 2. all points whose coordinates are solutions of an equation                                      | B. coordinate grid or plane |
| _____ 3. a group of two or more equations that are related to the same situation and share variables    | C. graph                    |
| _____ 4. a drawing used to represent data   | D. graph of an equation     |
| _____ 5. at right angles to the horizon; straight up and down   | E. intersect                |
| _____ 6. any symbol, usually a letter, which could represent a number                                   | F. system of equations      |
| _____ 7. the horizontal and vertical number lines used in a coordinate plane system                     | G. table (or chart)         |
| _____ 8. a data display that organizes information about a topic into categories                        | H. variable                 |
| _____ 9. to meet or cross at one point  | I. vertical                 |



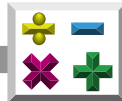
## Practice

Use the list below to write the correct term for each definition on the line provided.

<b>coefficient</b>	<b>parallel (  )</b>	<b>substitution</b>
<b>infinite</b>	<b>simplify an expression</b>	<b>sum</b>
<b>ordered pair</b>	<b>substitute</b>	

- \_\_\_\_\_ 1. a method used to solve a system of equations in which variables are replaced with known values or algebraic expressions
- \_\_\_\_\_ 2. the result of adding numbers together
- \_\_\_\_\_ 3. the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the  $x$ -axis and  $y$ -axis, respectively
- \_\_\_\_\_ 4. to perform as many of the indicated operations as possible
- \_\_\_\_\_ 5. to replace a variable with a numeral
- \_\_\_\_\_ 6. having no boundaries or limits
- \_\_\_\_\_ 7. being an equal distance at every point so as to never intersect
- \_\_\_\_\_ 8. the number that multiplies the variable(s) in an algebraic expression





## **Lesson Three Purpose**

### **Reading Process Strand**

#### **Standard 6: Vocabulary Development**

- LA.910.1.6.1  
The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2  
The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5  
The student will relate new vocabulary to familiar words.

### **Algebra Body of Knowledge**

#### **Standard 3: Linear Equations and Inequalities**

- MA.912.A.3.12  
Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph.
- MA.912.A.3.13  
Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology.
- MA.912.A.3.14  
Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods.



## Standard 10: Mathematical Reasoning and Problem Solving

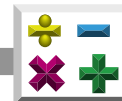
- MA.912.A.10.3  
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities rational or radical expressions or logarithmic or exponential functions).

### Graphing Inequalities

When graphing **inequalities**, you use much the same processes you used when graphing equations. The difference is that *inequalities* give you infinitely larger sets of solutions. In addition, your results with inequalities are always expressed using the following terms in relation to another expression:

- greater than ( $>$ )
- greater than or equal to ( $\geq$ )
- less than ( $<$ )
- less than or equal to ( $\leq$ )
- not equal to ( $\neq$ ).

Therefore, we cannot graph an inequality as a line or a point. We must illustrate the entire set of answers by *shading* our graphs.

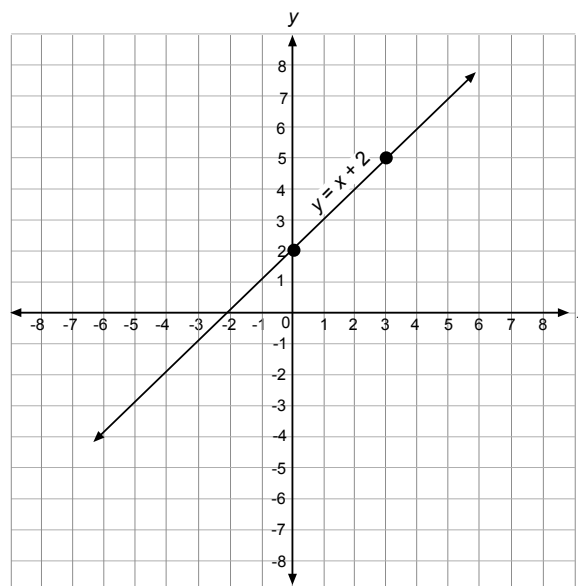


For instance, when we graph  $y = x + 2$  using points, we found by using the table of values below, we get the line seen in Graph 1 below.

**Table of Values**

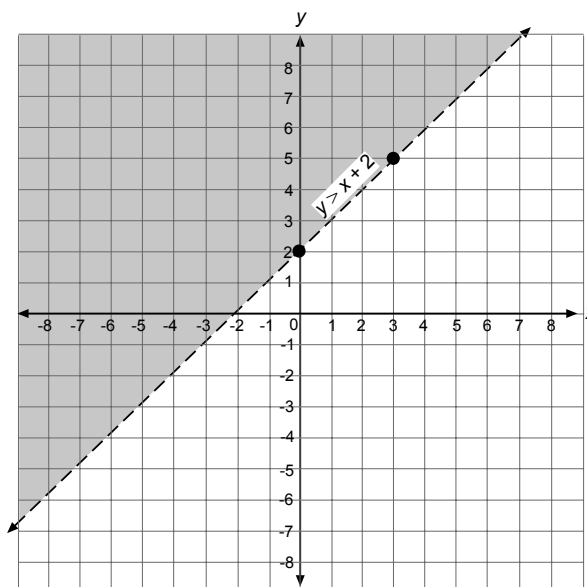
$y = x + 2$	
$x$	$y$
0	2
3	5

**Graph 1 of  $y = x + 2$**



But when we graph  $y > x + 2$ , we use the line we found in Graph 1 as a *boundary*. Since  $y \neq x + 2$ , we show that by making the boundary line *dotted* ( $\cdots$ ). Then we shade the appropriate part of the grid. Because this is a “greater than” ( $>$ ) problem, we shade *above* the dotted boundary line. See Graph 2 below.

**Graph 2 of  $y > x + 2$**



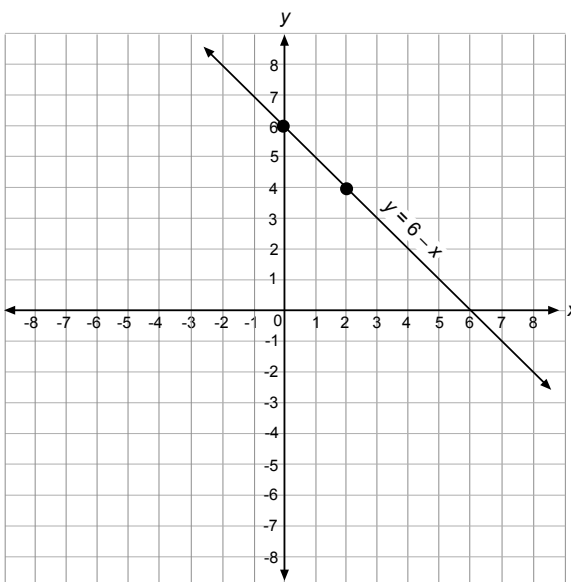


Suppose we wanted to graph  $x + y \leq 6$ . We first transform the inequality so that  $y$  is alone on the left side:  $y \leq 6 - x$ . We find a pair of points using a table of values, then graph the boundary line. Use the equation  $y = 6 - x$  to find two pairs of points in the table of values. Graph the line that goes through points  $(0, 6)$  and  $(2, 4)$  from the table of values.

**Table of Values**

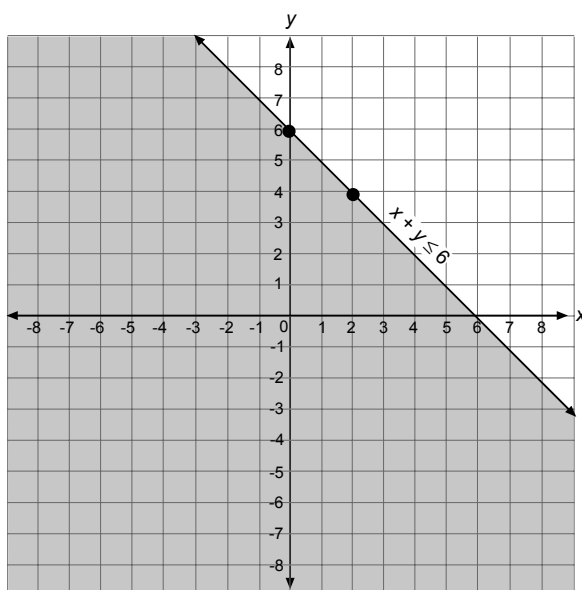
$y = 6 - x$	
$x$	$y$
0	6
2	4

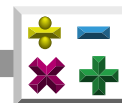
**Graph 3 of  $y = 6 - x$**



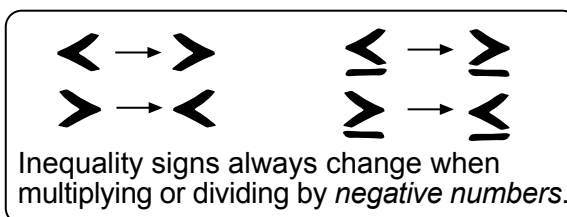
Now look at the inequality again. The symbol was  $\leq$ , so we leave the line solid and shade below the line.

**Graph 4 of  $x + y \leq 6$**





**Remember:** Change the inequality sign whenever you multiply or divide the inequality by a **negative number**.



**Note:**

- **Greater than** ( $>$ ) means to shade **above** or to the **right** of the line.
- **Less than** ( $<$ ) means to shade **below** or to the **left** of the line.

### Test for Accuracy Before You Shade

You can test your graph for accuracy before you shade by *choosing a point that satisfies the inequality*. Choose a point that falls in the area you are about to shade. Do *not* choose a point on the boundary line.

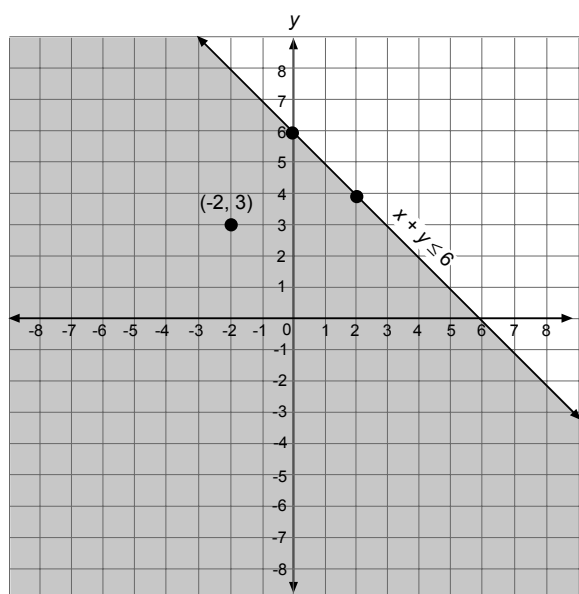
For example, suppose you chose  $(-2, 3)$ .

$$\begin{aligned} -2 + 3 &\leq 6 \\ 1 &\leq 6 \end{aligned}$$

1. The ordered pair  $(-2, 3)$  *satisfies* the inequality.
2. The ordered pair  $(-2, 3)$  falls in the area about to be shaded.

Thus, the shaded area for the graph  $x + y \leq 6$  is correct.

**Graph 4 of  $x + y \leq 6$  with Test Point**





## Practice

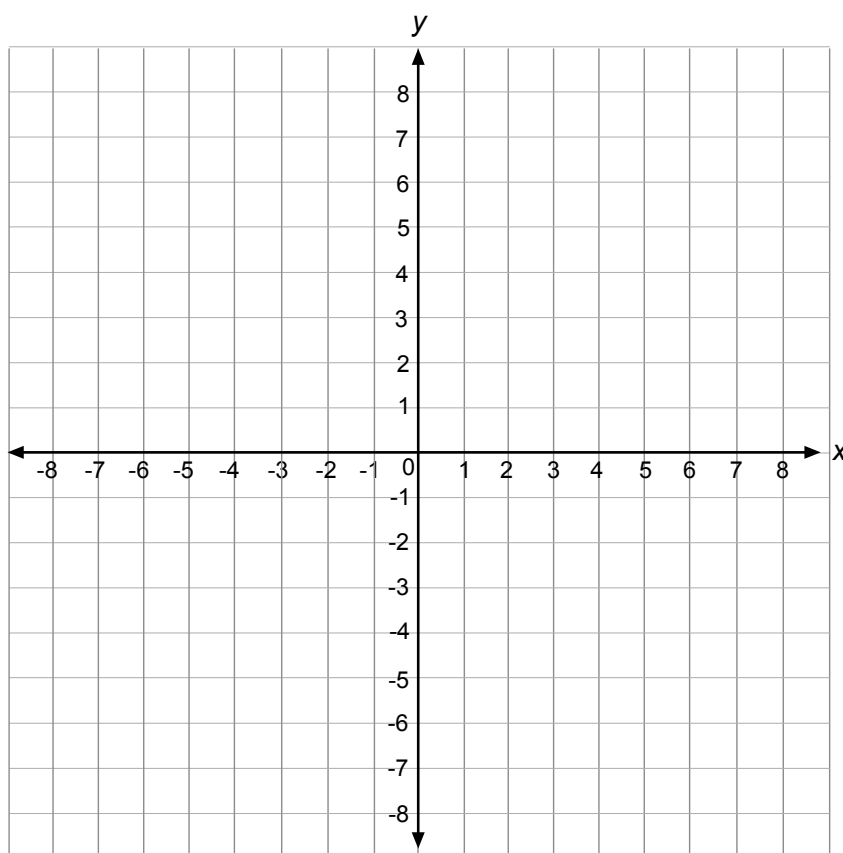
Graph each of the following **inequalities** on the graphs provided. Refer to pages 778-781 as needed.

1.  $y \geq 2x - 3$

**Table of Values**

$y \geq 2x - 3$	
$x$	$y$

**Graph of  $y \geq 2x - 3$**



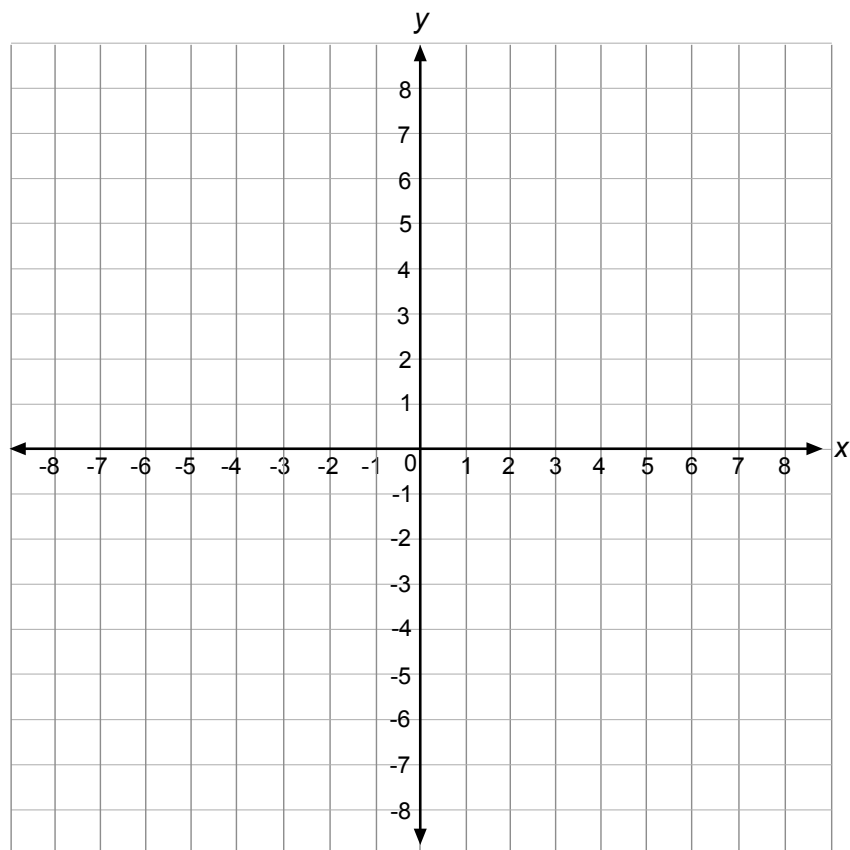


2.  $y < x + 4$

Table of Values

$y < x + 4$	
$x$	$y$

Graph of  $y < x + 4$



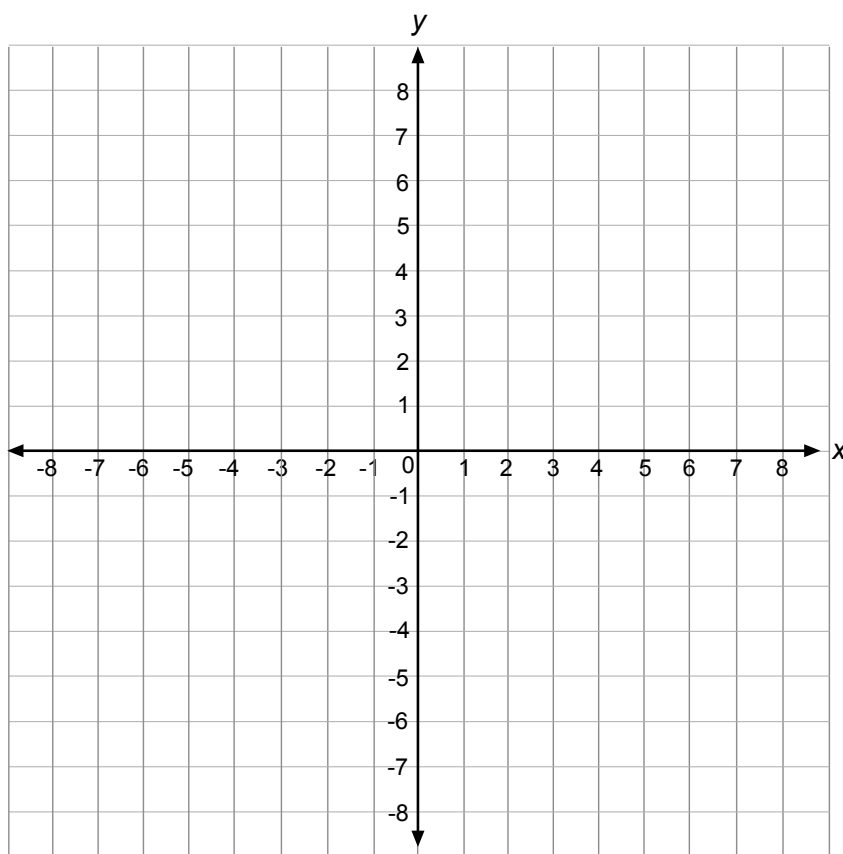


3.  $y \leq 3x + 1$

**Table of Values**

$y \leq 3x + 1$	
$x$	$y$

**Graph of  $y \leq 3x + 1$**



Is the point  $(-3, 7)$  part of the solution? \_\_\_\_\_



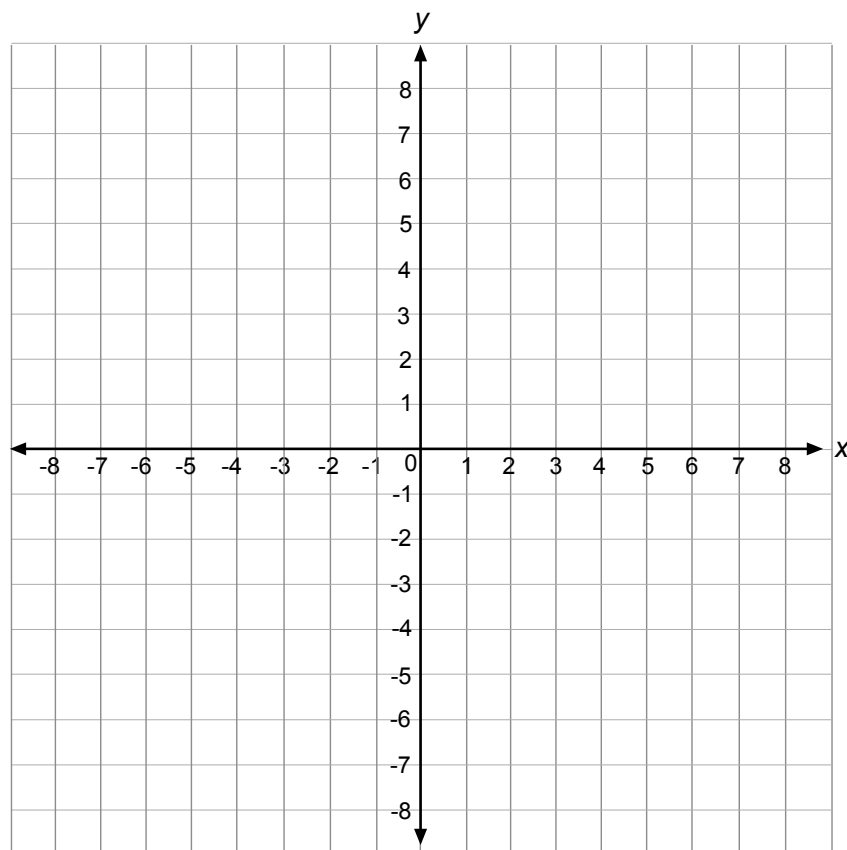


4.  $y > x$

**Table of Values**

$y > x$	
$x$	$y$

**Graph of  $y > x$**



Is the point (3, 6) part of the solution? \_\_\_\_\_

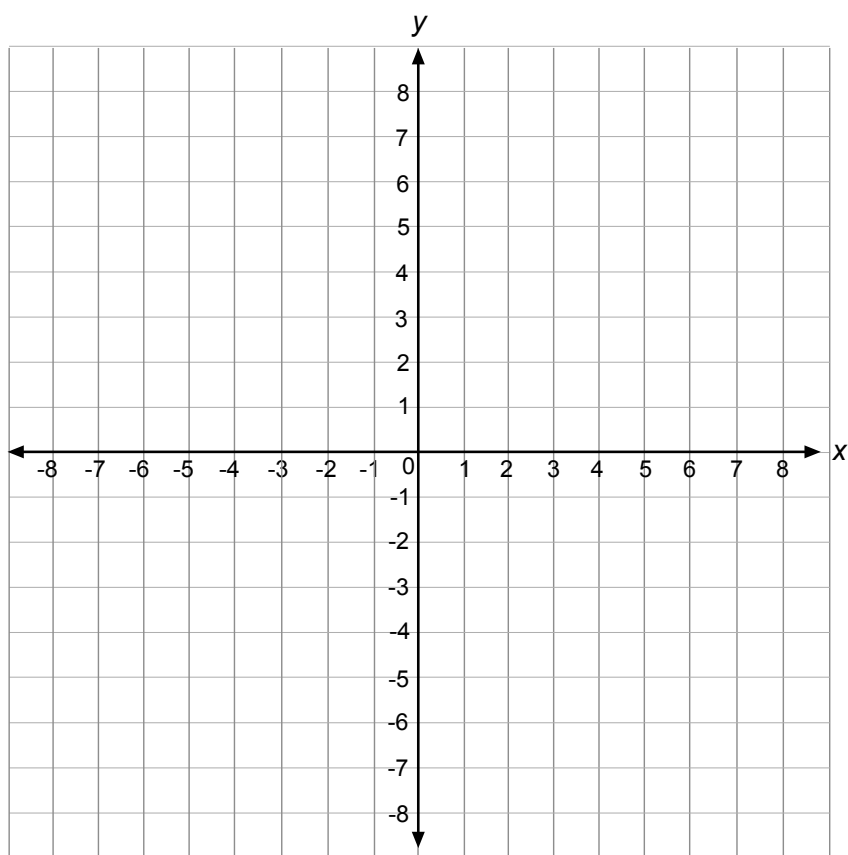


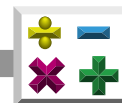
5.  $x + y < -5$

**Table of Values**

$x + y < -5$	
$x$	$y$

**Graph of  $x + y < -5$**



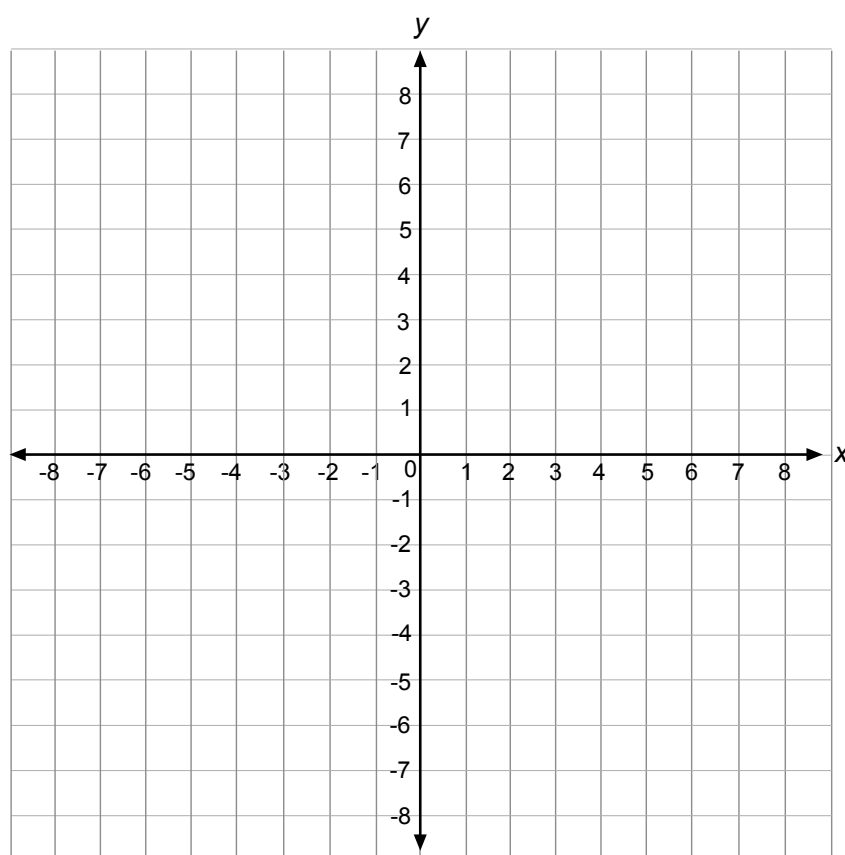


6.  $x - 5y \geq 10$

**Table of Values**

$x - 5y \geq 10$	
$x$	$y$

**Graph of  $x - 5y \geq 10$**



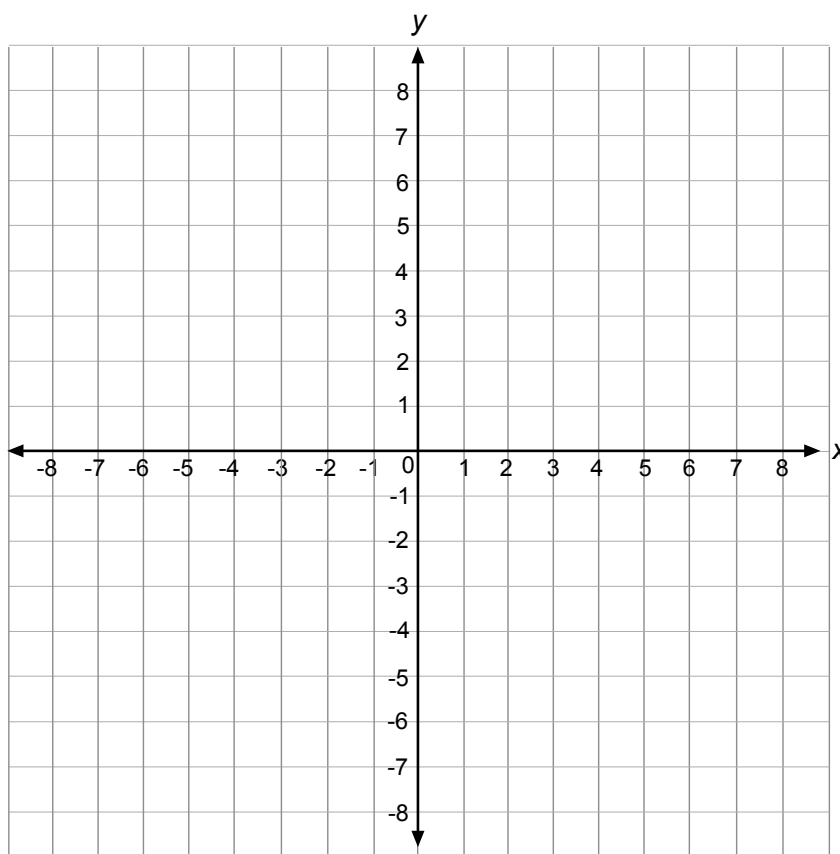


7.  $x - 5y \leq 10$

**Table of Values**

$x - 5y \leq 10$	
$x$	$y$

**Graph of  $x - 5y \leq 10$**



Is the point  $(0, 0)$  part of the solution? \_\_\_\_\_

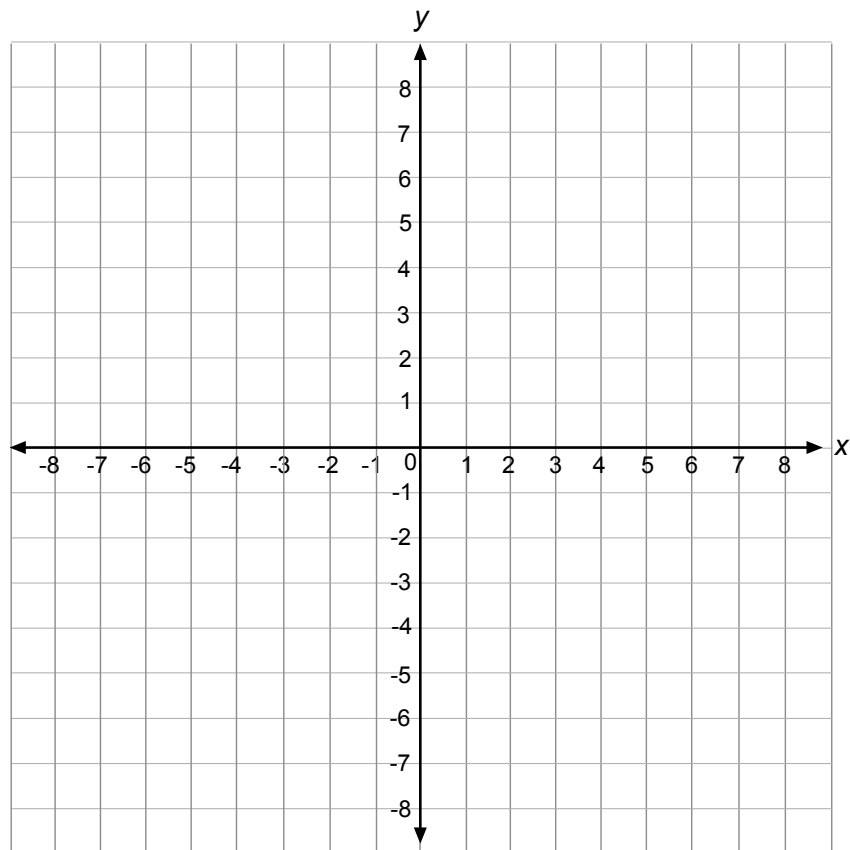


8.  $y \leq -3$

**Table of Values**

$y \leq -3$	
$x$	$y$

**Graph of  $y \leq -3$**





## Graphing Multiple Inequalities

We can graph two or more inequalities on the same grid to find which solutions the two inequalities have in common or to find those solutions that work in one inequality or the other. The key words are “and” and “or.” Let’s see how these small, ordinary words affect our graphing.

### Example 1

Graphically show the solutions for  $2x + 3y > 6$  and  $y \leq 2x$ .

**Note:** See how the inequality  $2x + 3y > 6$  is transformed in the table of values into the equivalent inequality  $y > 2 - \frac{2}{3}x$ . Refer to pages 778-781 as needed.

Step 1. Find the boundary lines for the two inequalities and draw them. Remember to make the line for the first inequality *dotted*.

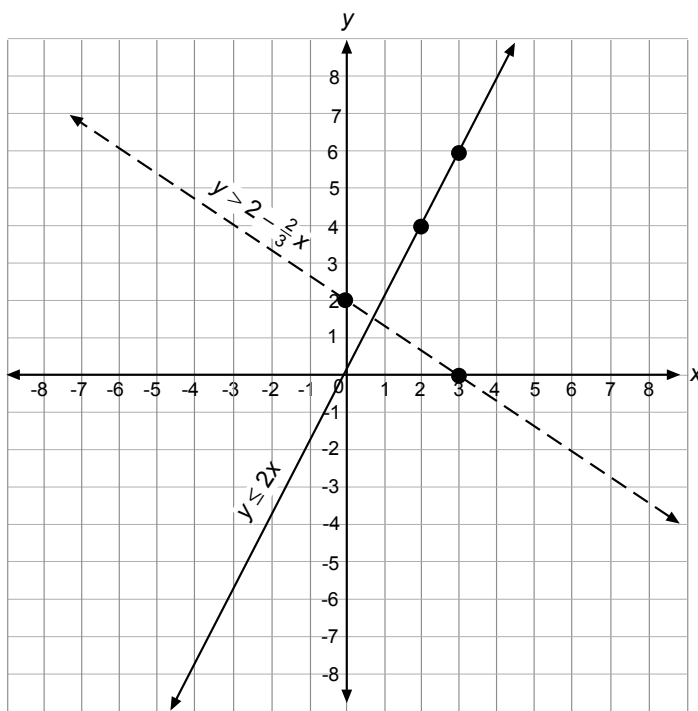
**Table of Values**

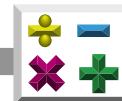
$y > 2 - \frac{2}{3}x$	
$x$	$y$
0	2
3	0

**Table of Values**

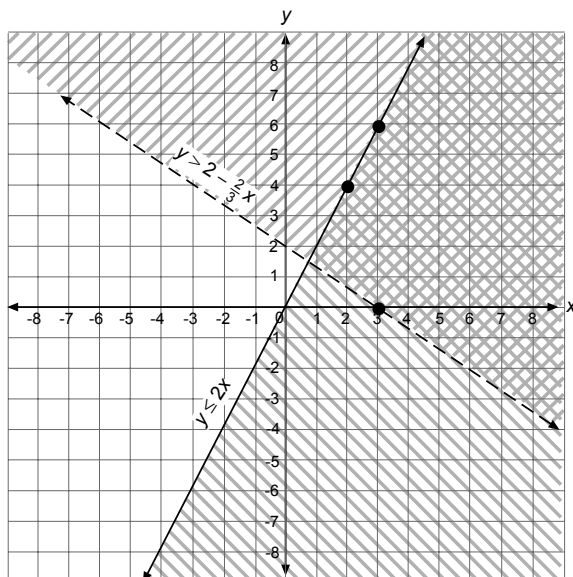
$y \leq 2x$	
$x$	$y$
3	6
2	4

**Graph Shows Boundary Lines of  $2x + 3y > 6$  and  $y \leq 2x$**





### Graph of $2x + 3y > 6$ and $y \leq 2x$ with Both Inequalities Shaded



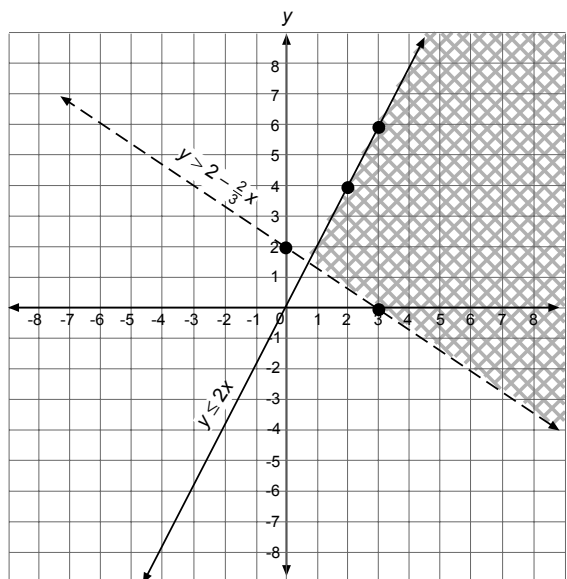
Step 2. Since the 1<sup>st</sup> inequality is *greater than*, shade *above* the dotted line.

Step 3. Shade the 2<sup>nd</sup> inequality *below* the solid line using a different type shading or different color.

Step 4. Because this is an “and” problem, we want to have as our solution *only* the parts where both shadings appear at the same time (in other words, where the shadings *overlap*, just as in the **Venn diagrams** in a previous unit). We want to show *only* those solutions that are *valid in both inequalities at the same time*.

Step 5. The solution for  $2x + 3y > 6$  **and**  $y \leq 2x$  is shown to the right.

### Final Solution for Graph of $2x + 3y > 6$ and $y \leq 2x$



Look at the finished graph above.  
The point  $(-1, 1)$  is *not* in the shaded region. Therefore, the point  $(-1, 1)$  is *not* a solution of the intersection of  $2x + 3y > 6$  and  $y \leq 2x$ .



## Example 2

Let's see how the graph of the solution would look if the problem had been  $2x + 3y > 6$  or  $y \leq 2x$ .

We follow the same steps from 1 and 2 of the previous example.

Step 1. Find the boundary lines for the two inequalities and draw them. Remember to make the line for the first inequality *dotted*.

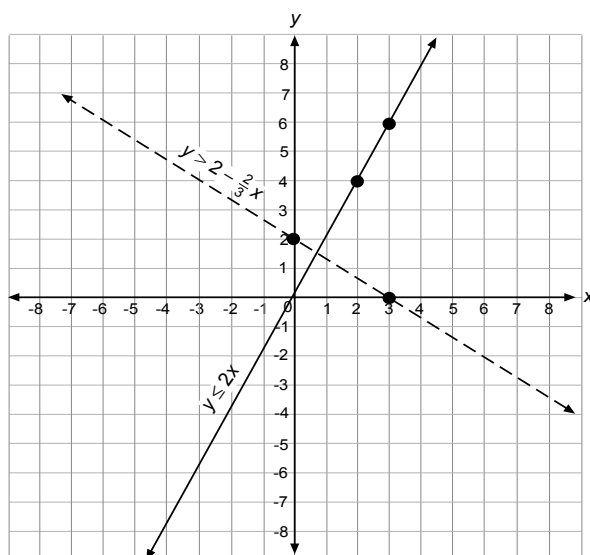
**Table of Values**

$y > 2 - \frac{2}{3}x$	
$x$	$y$
0	2
3	0

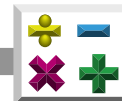
**Table of Values**

$y \leq 2x$	
$x$	$y$
3	6
2	4

**Graph Shows Boundary Lines of  $2x + 3y > 6$  or  $y \leq 2x$**

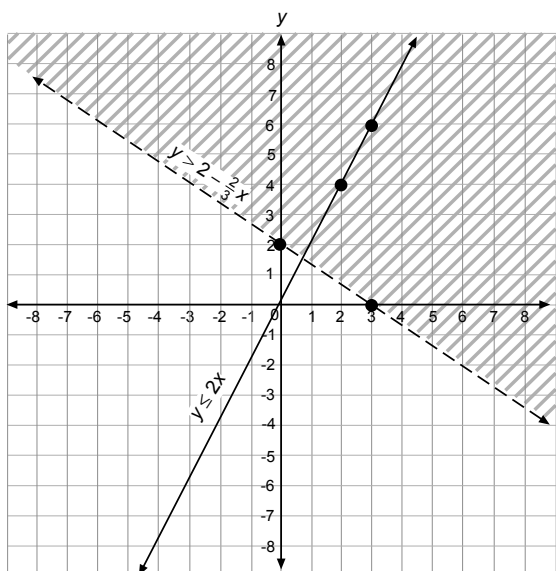






Step 2. Since the 1<sup>st</sup> inequality is *greater than*, shade *above* the dotted line.

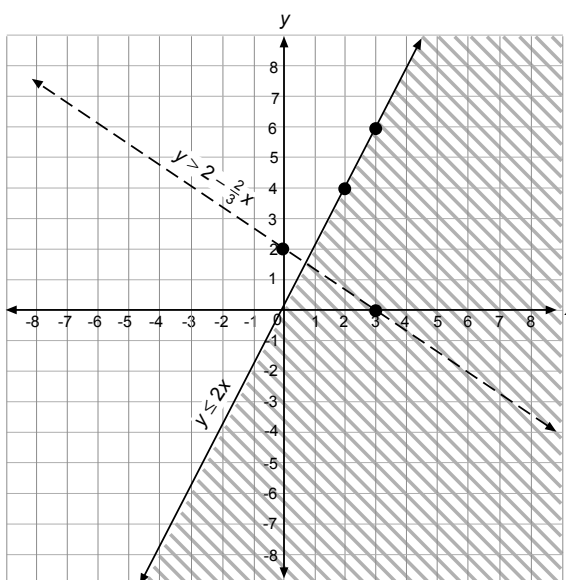
**Graph of  $2x + 3y > 6$  or  $y \leq 2x$  with 1<sup>st</sup> Inequality Shaded**



Now we change the process to fit the “or.”

Step 3. Shade the 2<sup>nd</sup> inequality *below* the solid line using the same shading as in step 2.

**Graph of  $2x + 3y > 6$  or  $y \leq 2x$  with  $y \leq 2x$  Shaded**

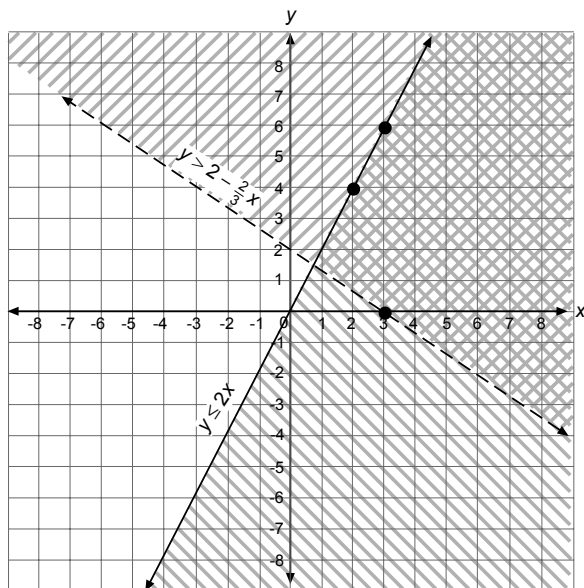




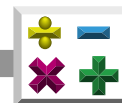
Step 4. Because this is now an “or” problem, we want to have as our solution *all* the parts that are shaded. This shows that a solution to *either* inequality is *acceptable*.

Step 5. The solution for  $2x + 3y > 6$  or  $y \leq 2x$  is shown below.

**Final Solution for Graph of  $2x + 3y > 6$  or  $y \leq 2x$**



Now it's your turn to practice.



## Practice

Graph the following **inequalities** on the graphs provided. Refer to pages 778-781 and 790-794 as needed.

1.  $x \geq -2$  and  $-x + y \geq 1$

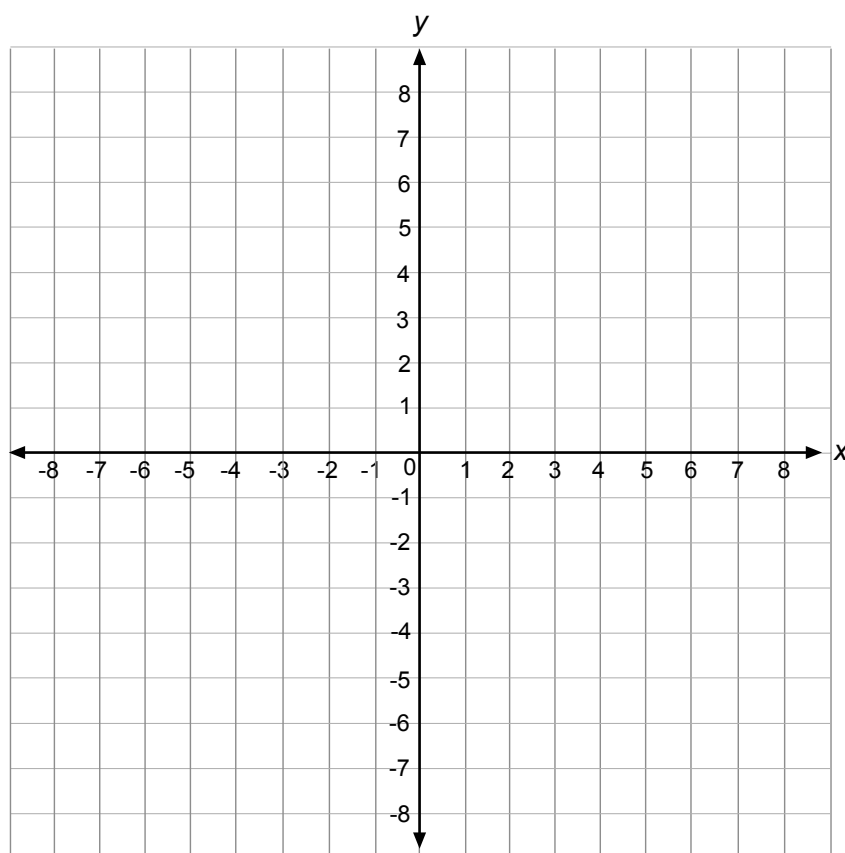
Table of Values

$x \geq -2$	
$x$	$y$

Table of Values

$-x + y \geq 1$	
$x$	$y$

Graph of  $x \geq -2$  and  $-x + y \geq 1$





2.  $y < -3$  or  $y \geq 2$

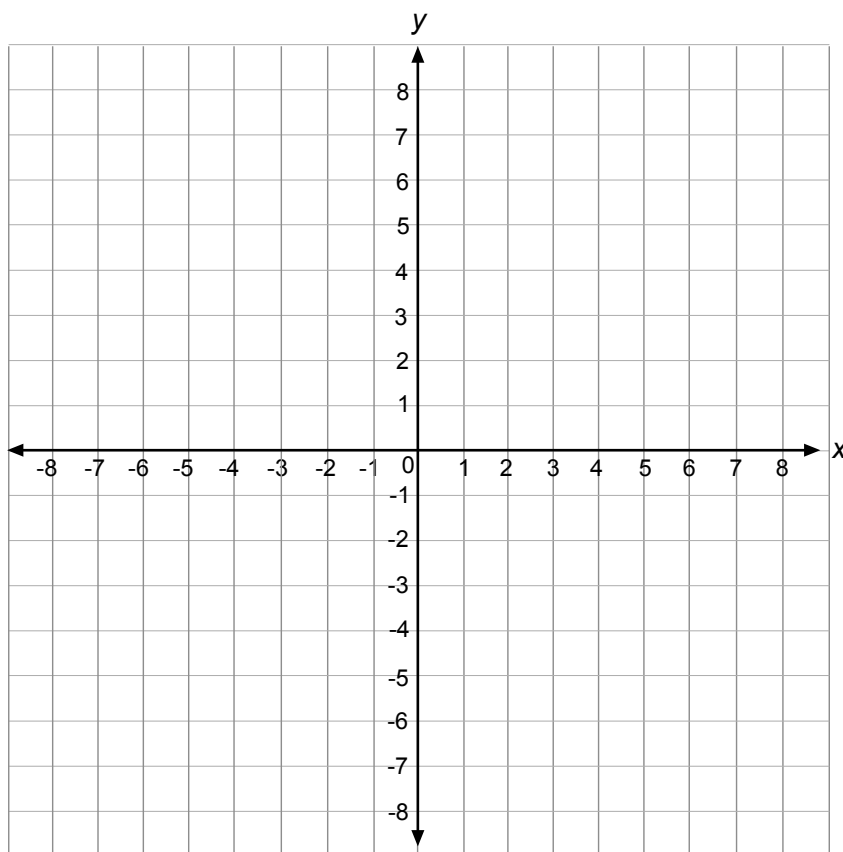
Table of Values

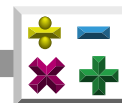
$y < -3$	
$x$	$y$

Table of Values

$y \geq 2$	
$x$	$y$

Graph of  $y < -3$  or  $y \geq 2$





3.  $x + 2y > 0$  and  $x - y \leq 5$

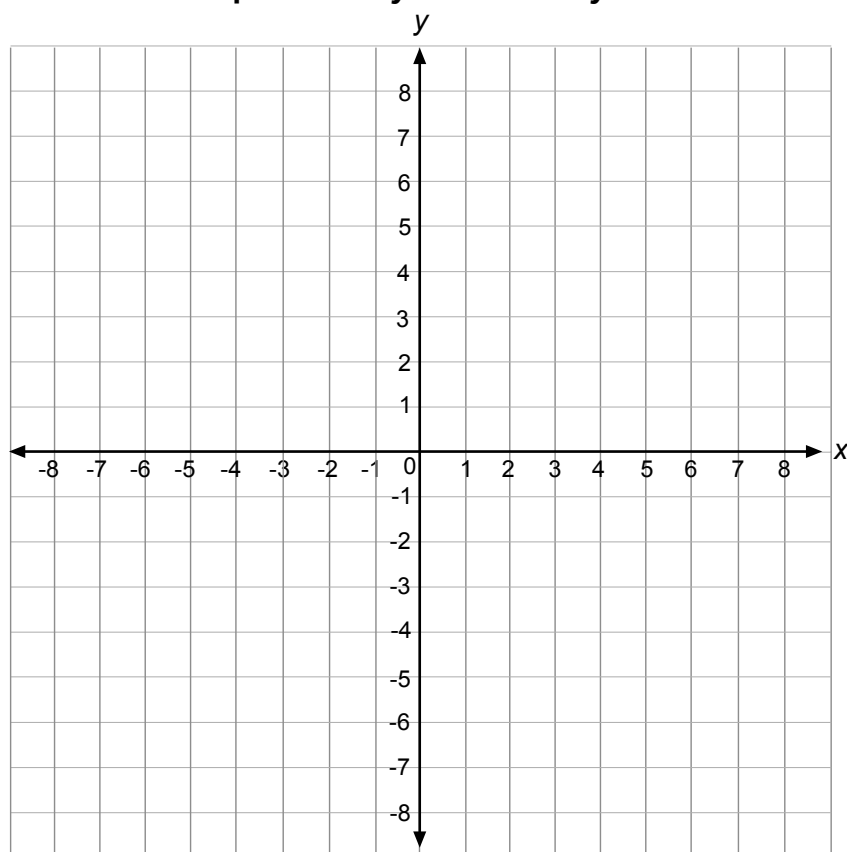
**Table of Values**

$x + 2y > 0$	
$x$	$y$

**Table of Values**

$x - y \leq 5$	
$x$	$y$

**Graph of  $x + 2y > 0$  and  $x - y \leq 5$**



Is the point (4, 4) part of the solution? \_\_\_\_\_



4.  $x + y > 1$  or  $x - y > 1$

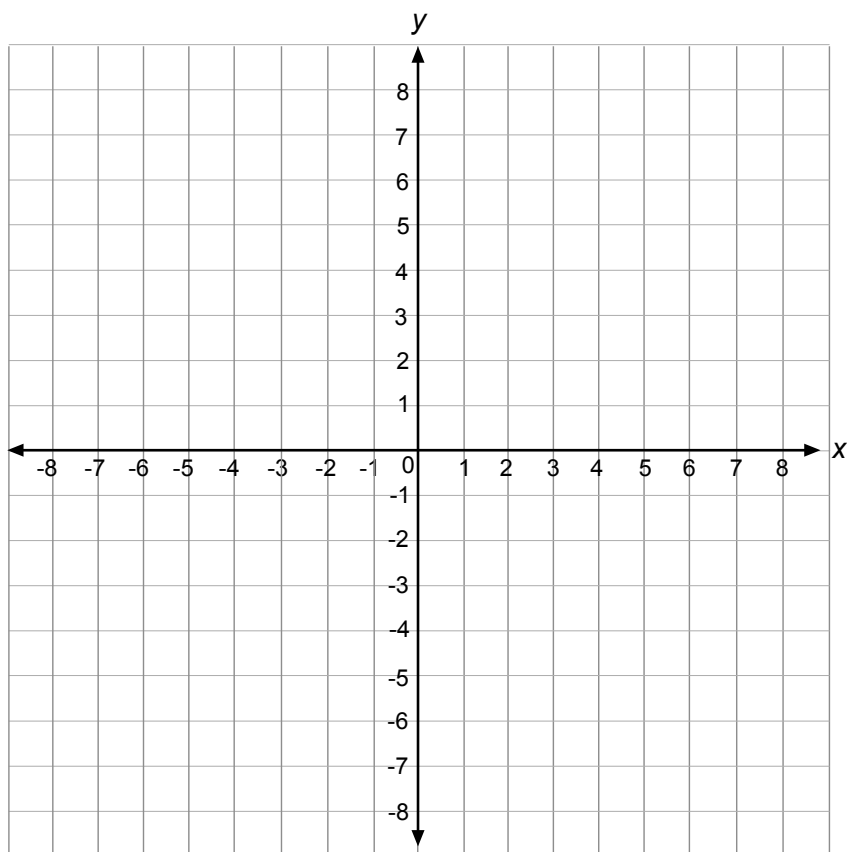
Table of Values

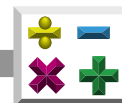
$x + y > 1$	
$x$	$y$

Table of Values

$x - y > 1$	
$x$	$y$

Graph of  $x + y > 1$  or  $x - y > 1$





5.  $x + y > 1$  and  $x - y > 1$

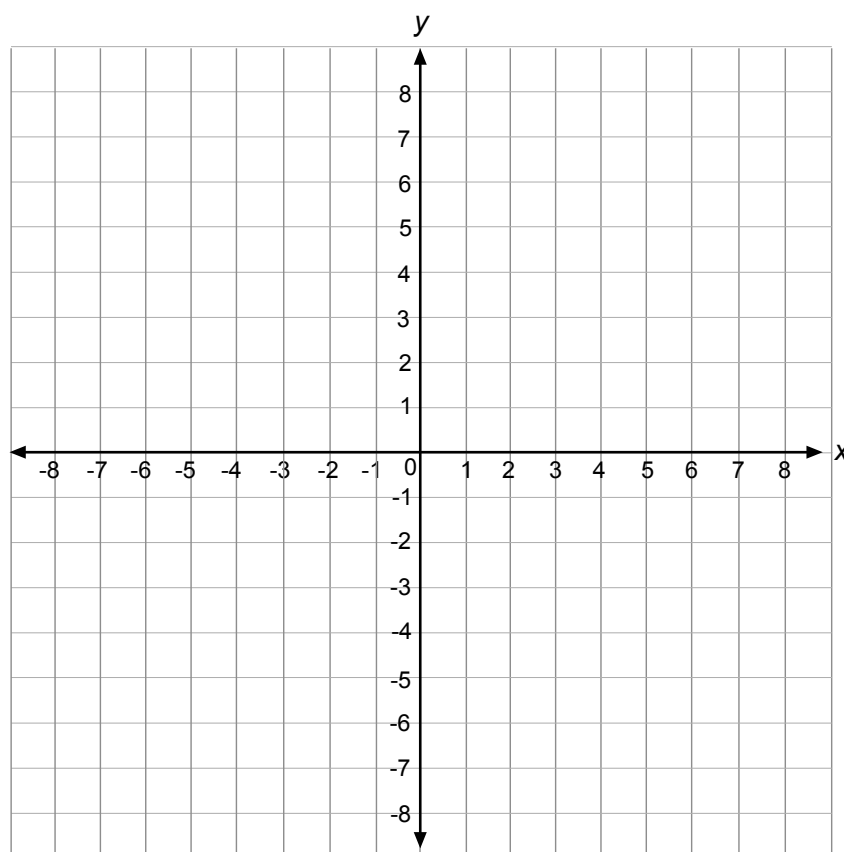
Table of Values

$x + y > 1$	
$x$	$y$

Table of Values

$x - y > 1$	
$x$	$y$

Graph of  $x + y > 1$  and  $x - y > 1$



Is the point  $(0, 0)$  part of the solution? \_\_\_\_\_



6.  $y \leq 3$  or  $x + y > 4$

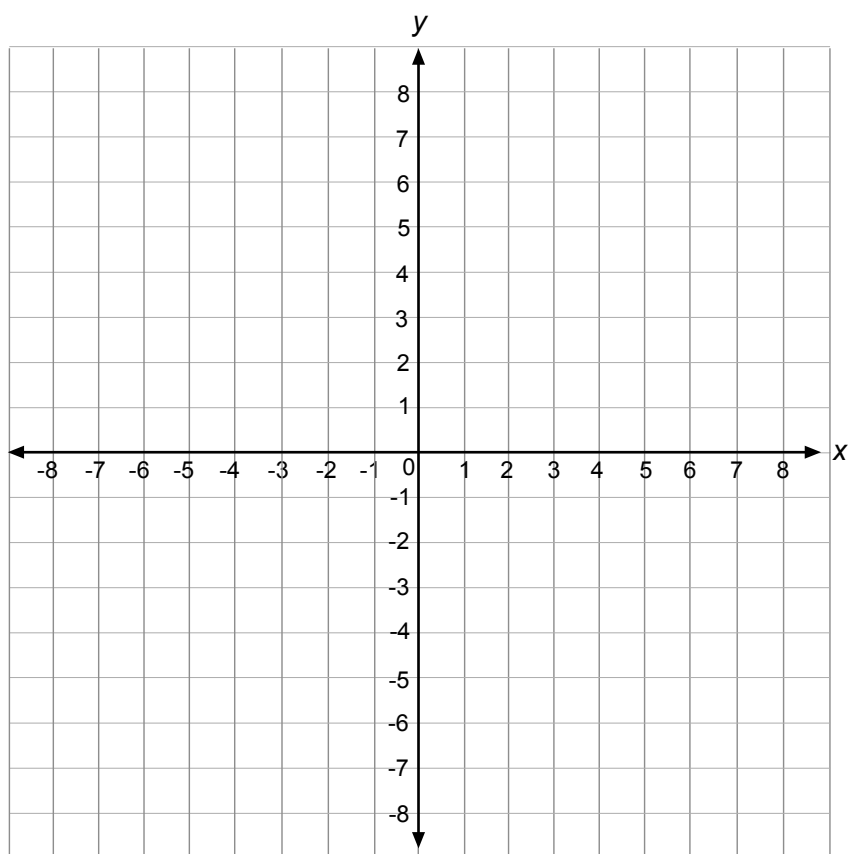
Table of Values

$y \leq 3$	
$x$	$y$

Table of Values

$x + y > 4$	
$x$	$y$

Graph of  $y \leq 3$  and  $x + y > 4$







## Practice

Match each definition with the correct term. Write the letter on the line provided.

- |       |   |  |
|-------|---|--|
| _____ | 1. a monomial or sum of monomials; any rational expression with no variable in the denominator  | A. factoring                               |
| _____ | 2. numbers less than zero   | B. inequality                              |
| _____ | 3. a sentence that states one expression is greater than ( $>$ ), greater than or equal to ( $\geq$ ), less than ( $<$ ), less than or equal to ( $\leq$ ), or not equal to ( $\neq$ ) another expression | C. negative numbers                        |
| _____ | 4. a group of two or more equations that are related to the same situation and share variables  | D. polynomial                              |
| _____ | 5. a method used to solve a system of equations in which variables are replaced with known values or algebraic expressions  | E. standard form (of a quadratic equation) |
| _____ | 6. $ax^2 + bx + c = 0$ , where $a$ , $b$ , and $c$ are integers (not multiples of each other) and $a > 0$   | F. substitution                            |
| _____ | 7. expressing a polynomial expression as the product of monomials and polynomials   | G. system of equations                     |



## Unit Review

Find the **solution sets**.

1.  $(x + 5)(x - 7) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

2.  $(3x - 2)(3x - 6) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

3.  $x(x - 7) = 0$  { \_\_\_\_\_ , \_\_\_\_\_ }

4.  $x^2 + x = 42$  { \_\_\_\_\_ , \_\_\_\_\_ }

5.  $x^2 - 10x = -16$  { \_\_\_\_\_ , \_\_\_\_\_ }



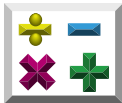
*Solve each of the following. Show all your work.*

6. Max has a garden 4 feet longer than it is wide. If the area of his garden is 96 square feet, find the dimensions of Max's garden.

Answer: \_\_\_\_\_ feet x \_\_\_\_\_ feet

7. The product of two consecutive positive **even integers** (integers divisible by 2) is 440. Find the integers.

Answer: \_\_\_\_\_ and \_\_\_\_\_



Use the **quadratic formula** below to solve the following equations.



Check  
your work  
using a  
calculator.

**quadratic formula**

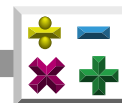
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

8.  $2x^2 - 7x - 15 = 0$

9.  $x^2 + 4x - 30 = -9$

10. The sides of a rose garden are  $(x + 8)$  units and  $(x - 3)$  units. If the area of the garden is 12 square units, find the dimensions of the rose garden.

Answer: \_\_\_\_\_ units x \_\_\_\_\_ units



Solve **algebraically**, then graph each **system of equations** on the graphs provided. Refer to pages 748-752, 759-760, 778-781, and 790-794 as needed.

11.  $2x - y = 6$   
 $x + y = 9$

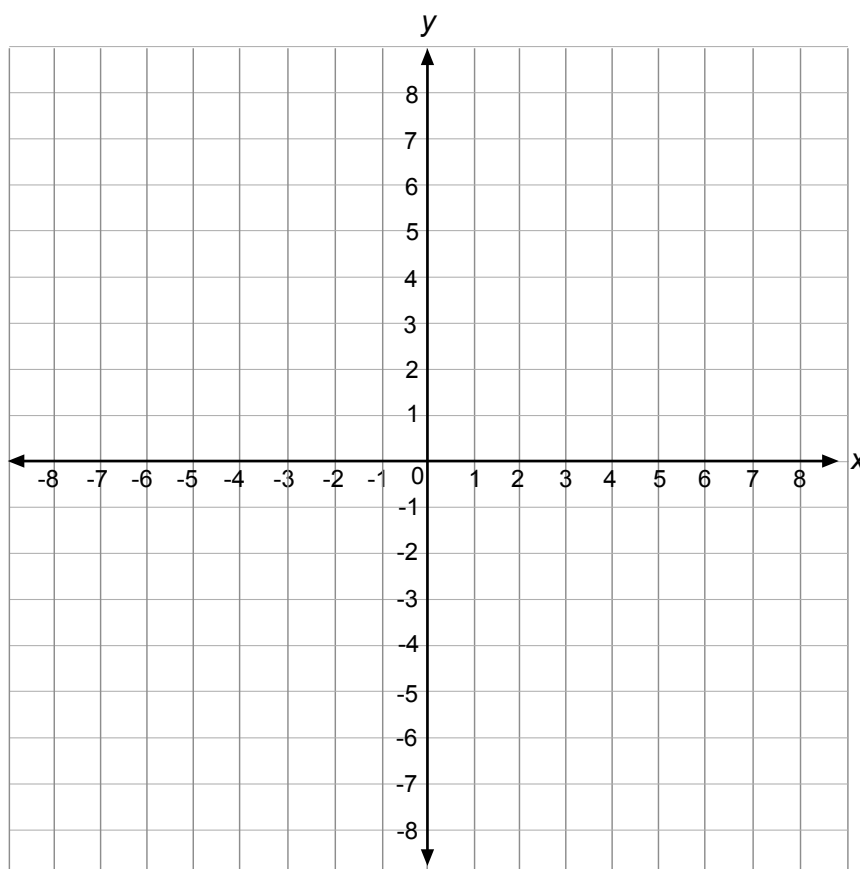
Table of Values

$2x - y = 6$	
$x$	$y$

Table of Values

$x + y = 9$	
$x$	$y$

Graph of  $2x - y = 6$  and  $x + y = 9$





12.  $x + y = 7$   
 $3x - 4y = 7$

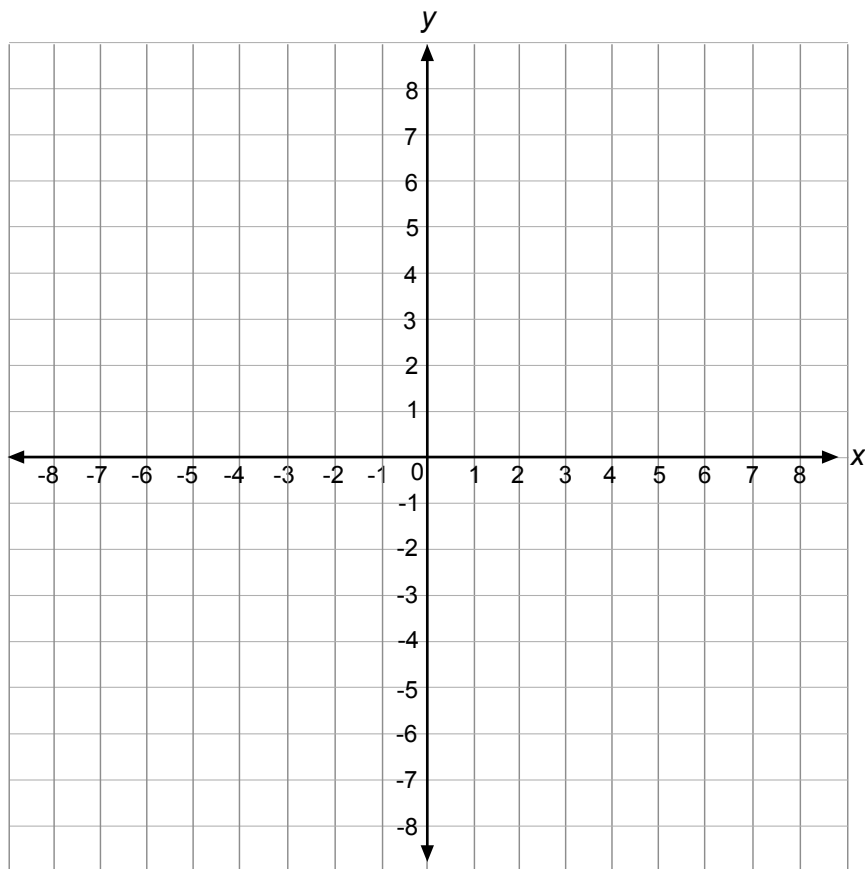
Table of Values

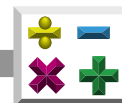
$x + y = 7$	
$x$	$y$

Table of Values

$3x - 4y = 7$	
$x$	$y$

Graph of  $x + y = 7$  and  $3x - 4y = 7$





13.  $2x - 4y = 8$   
 $x + 4y = 10$

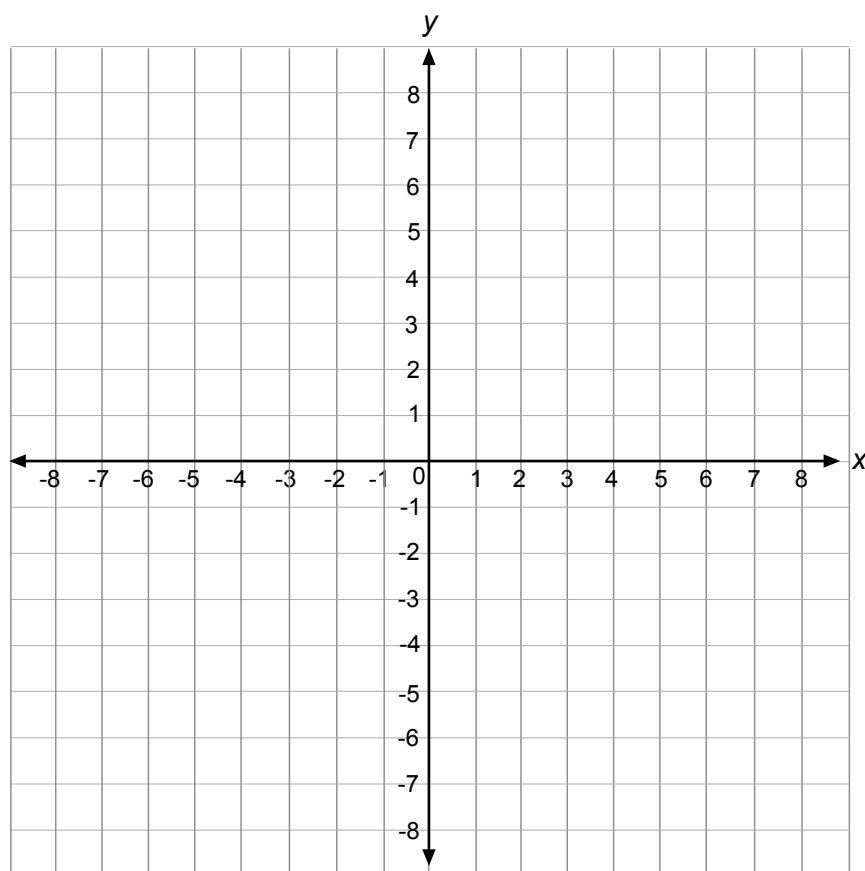
Table of Values

$2x - 4y = 8$	
$x$	$y$

Table of Values

$x + 4y = 10$	
$x$	$y$

Graph of  $2x - 4y = 8$  and  $x + 4y = 10$





14.  $3x + 2y = 8$   
 $y = -2$

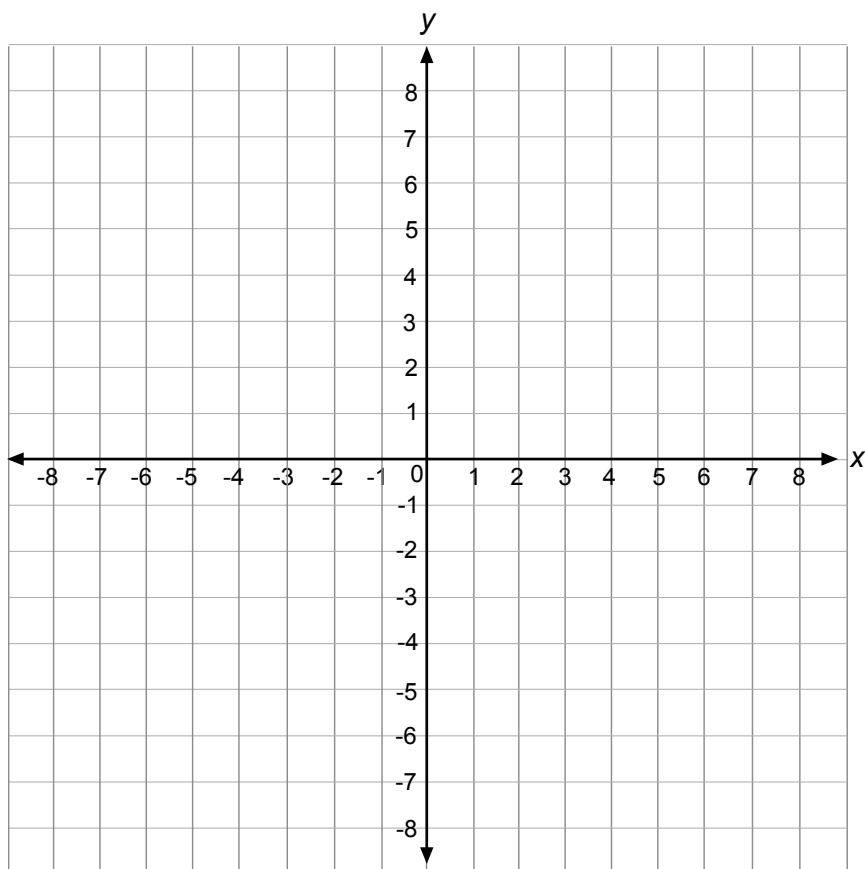
**Table of Values**

$3x + 2y = 8$	
$x$	$y$

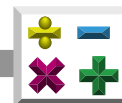
**Table of Values**

$y = -2$	
$x$	$y$

**Graph of  $3x + 2y = 8$  and  $y = -2$**







15.  $3x + 5y = 26$   
 $2x - 2y = -20$

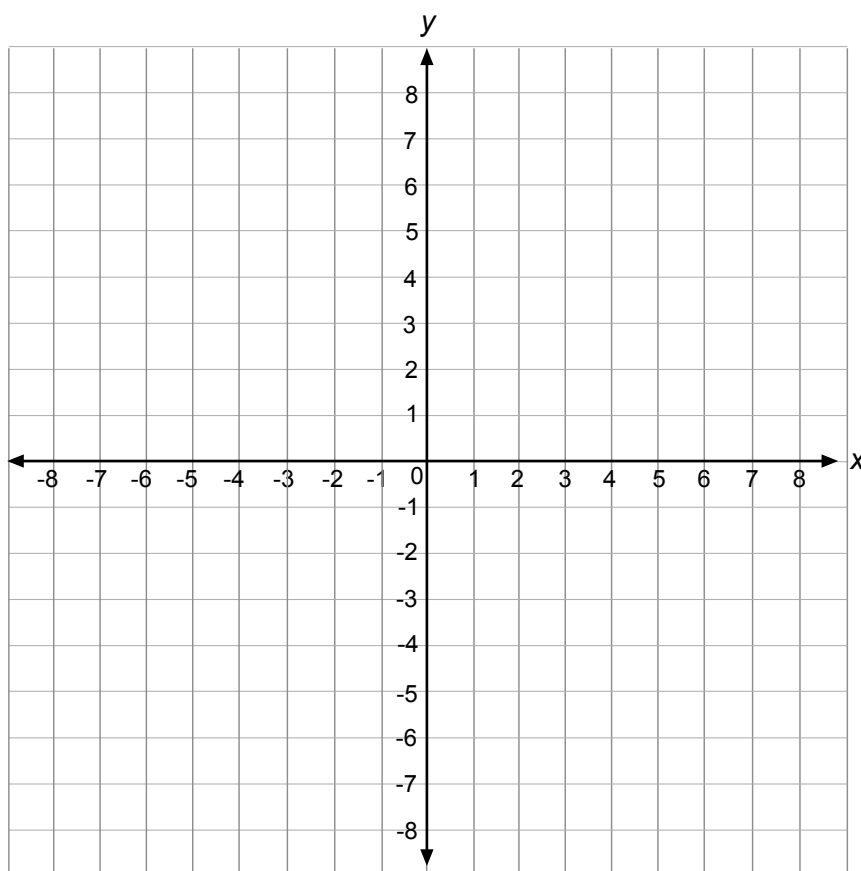
Table of Values

$3x + 5y = 26$	
$x$	$y$

Table of Values

$2x - 2y = -20$	
$x$	$y$

Graph of  $3x + 5y = 26$  and  $2x - 2y = -20$





*Solve each of the following. Show all your work.*

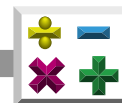
16. The sum of two numbers is 52. The larger number is 2 more than 4 times the smaller number. Find the two numbers.

Answer: \_\_\_\_\_ and \_\_\_\_\_

17. The band has 8 more than twice the number of students as the chorus. Together there are 119 students in both programs. How many are in each?



Answer: chorus = \_\_\_\_\_ and band = \_\_\_\_\_



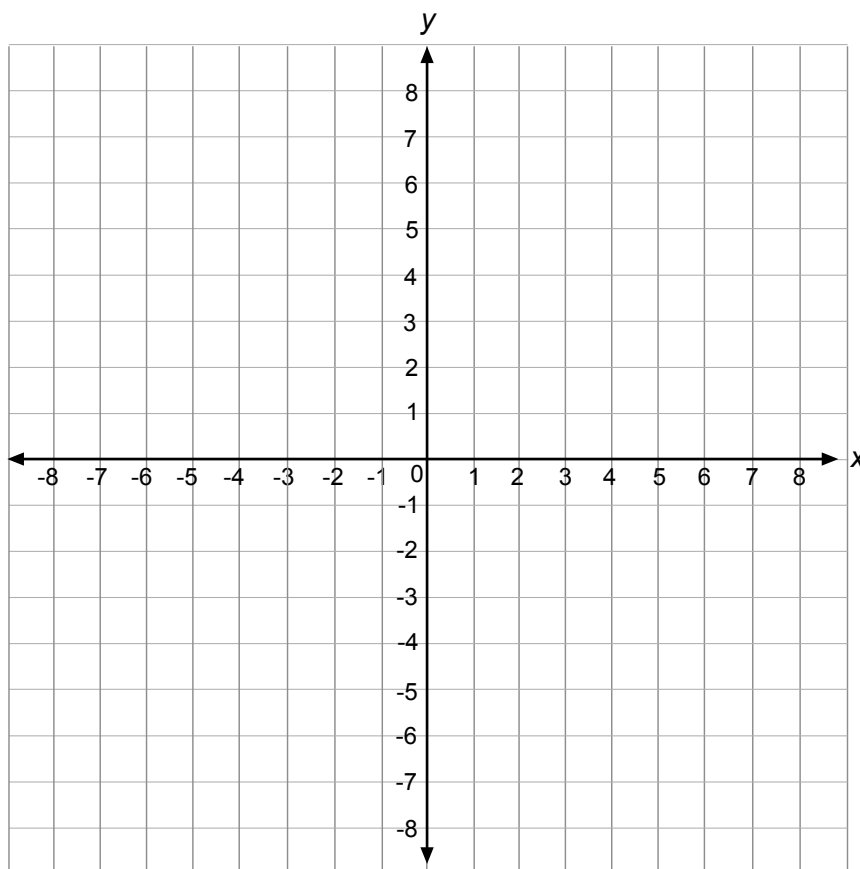
Graph the following **inequalities** on the graphs provided.

18.  $y > x - 6$

**Table of Values**

$y > x - 6$	
$x$	$y$

**Graph of  $y > x - 6$**



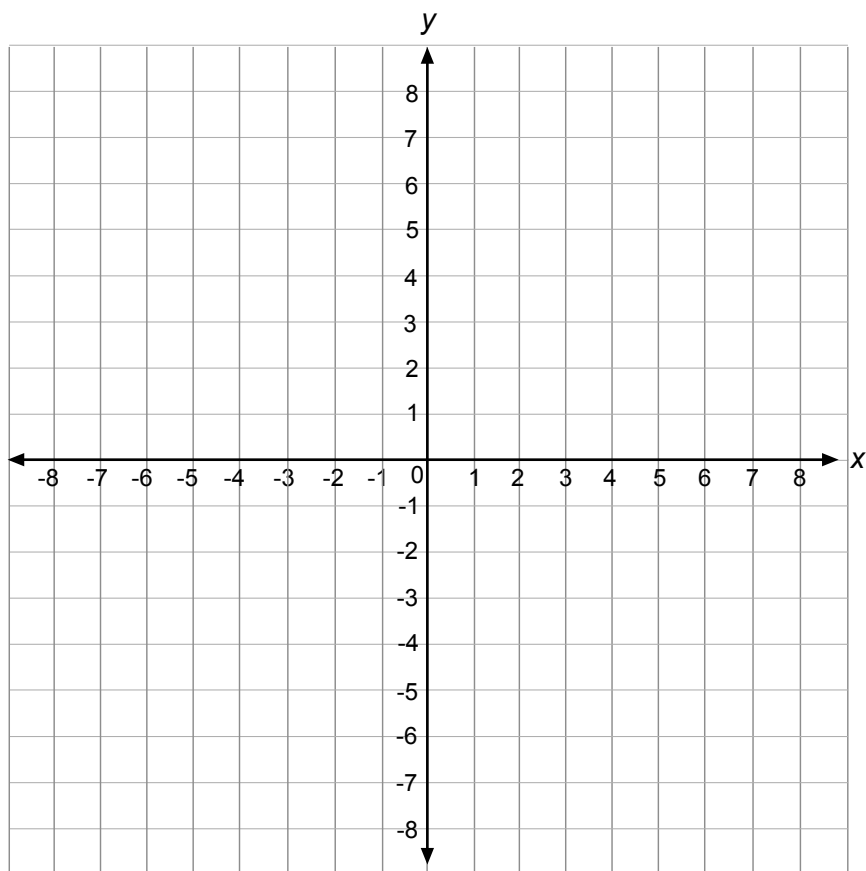


19.  $8x - 4y \leq 12$

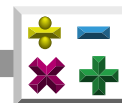
**Table of Values**

$8x - 4y \leq 12$	
$x$	$y$

**Graph of  $8x - 4y \leq 12$**



Is the point  $(0, 0)$  part of the solution? \_\_\_\_\_



20.  $y \geq 4x - 3$  and  $x + y < 0$

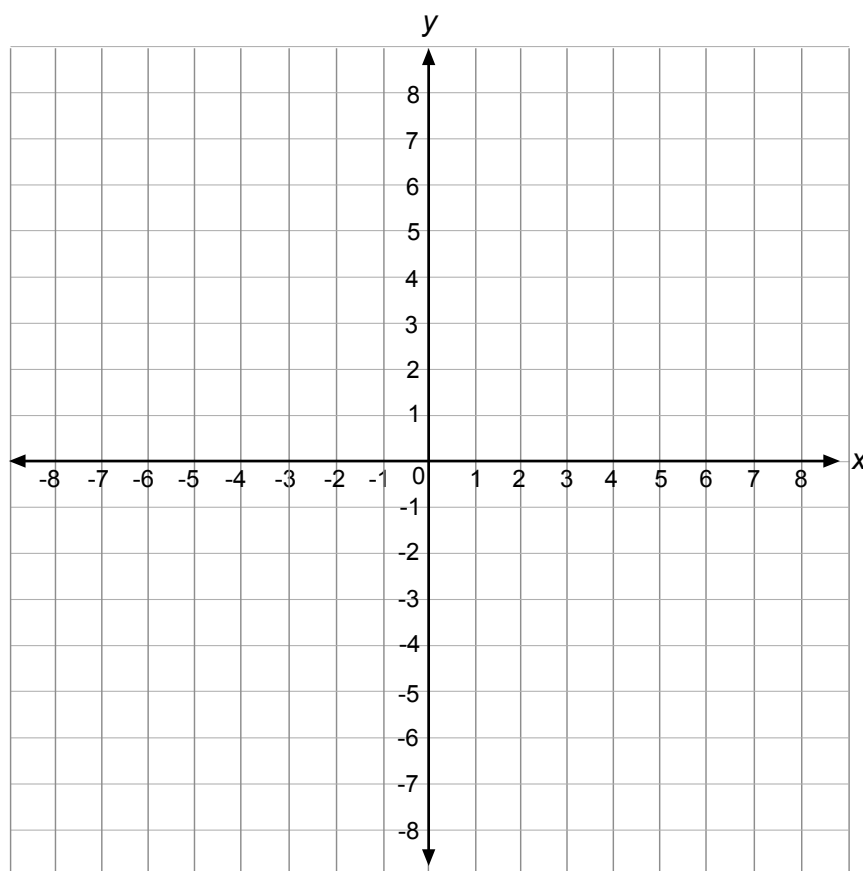
Table of Values

$y \geq 4x - 3$	
$x$	$y$

Table of Values

$x + y < 0$	
$x$	$y$

Graph of  $y \geq 4x - 3$  and  $x + y < 0$





21.  $x + y > 4$  or  $y \geq x - 2$

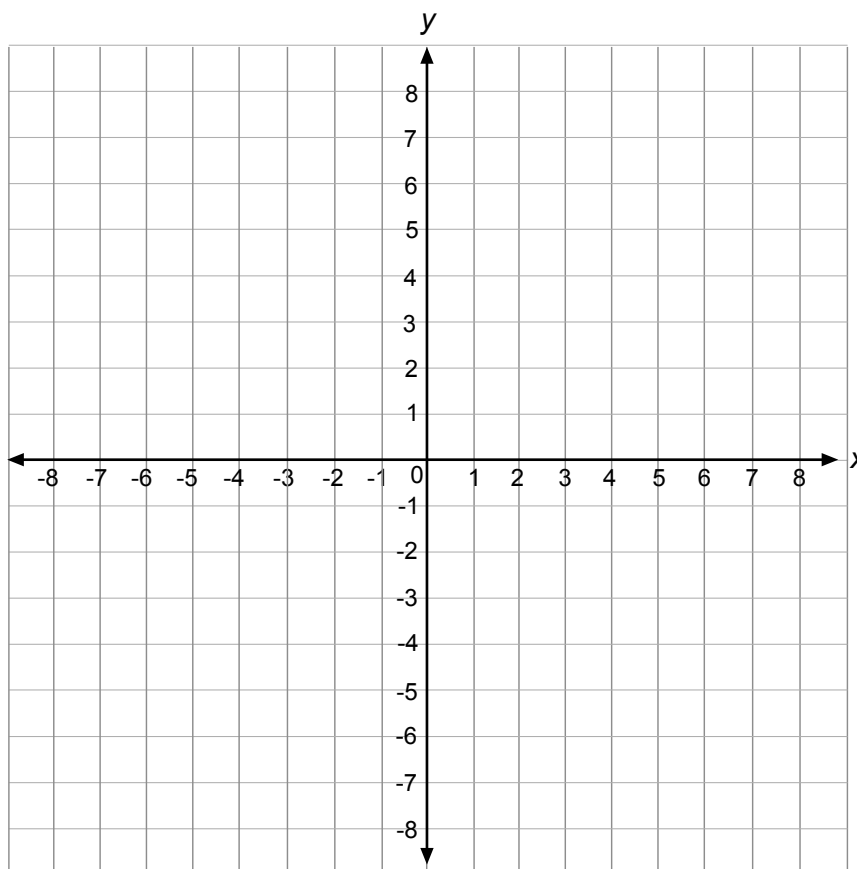
**Table of Values**

$x + y > 4$	
$x$	$y$

**Table of Values**

$y \geq x - 2$	
$x$	$y$

**Graph of  $x + y > 4$  or  $y \geq x - 2$**



Is the point  $(3, -6)$  part of the solution? \_\_\_\_\_

# Appendices





## Table of Squares and Approximate Square Roots

$n$	$n^2$	$\sqrt{n}$	$n$	$n^2$	$\sqrt{n}$
1	1	1.000	51	2,601	7.141
2	4	1.414	52	2,704	7.211
3	9	1.732	53	2,809	7.280
4	16	2.000	54	2,916	7.348
5	25	2.236	55	3,025	7.416
6	36	2.449	56	3,136	7.483
7	49	2.646	57	3,249	7.550
8	64	2.828	58	3,364	7.616
9	81	3.000	59	3,481	7.681
10	100	3.162	60	3,600	7.746
11	121	3.317	61	3,721	7.810
12	144	3.464	62	3,844	7.874
13	169	3.606	63	3,969	7.937
14	196	3.742	64	4,096	8.000
15	225	3.873	65	4,225	8.062
16	256	4.000	66	4,356	8.124
17	289	4.123	67	4,489	8.185
18	324	4.243	68	4,624	8.246
19	361	4.359	69	4,761	8.307
20	400	4.472	70	4,900	8.367
21	441	4.583	71	5,041	8.426
22	484	4.690	72	5,184	8.485
23	529	4.796	73	5,329	8.544
24	576	4.899	74	5,476	8.602
25	625	5.00	75	5,625	8.660
26	676	5.099	76	5,776	8.718
27	729	5.196	77	5,929	8.775
28	784	5.292	78	6,084	8.832
29	841	5.385	79	6,241	8.888
30	900	5.477	80	6,400	8.944
31	961	5.568	81	6,561	9.000
32	1,024	5.657	82	6,724	9.055
33	1,089	5.745	83	6,889	9.110
34	1,156	5.831	84	7,056	9.165
35	1,225	5.916	85	7,225	9.220
36	1,296	6.000	86	7,396	9.274
37	1,369	6.083	87	7,569	9.327
38	1,444	6.164	88	7,744	9.381
39	1,521	6.245	89	7,921	9.434
40	1,600	6.325	90	8,100	9.487
41	1,681	6.403	91	8,281	9.539
42	1,764	6.481	92	8,464	9.592
43	1,849	6.557	93	8,649	9.644
44	1,936	6.633	94	8,836	9.695
45	2,025	6.708	95	9,025	9.747
46	2,116	6.782	96	9,216	9.798
47	2,209	6.856	97	9,409	9.849
48	2,304	6.928	98	9,604	9.899
49	2,401	7.000	99	9,801	9.950
50	2,500	7.071	100	10,000	10.000



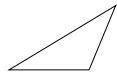
## Mathematical Symbols

$\div$ or $/$	divide	$\parallel$	is parallel to
$\times$ or $\bullet$	times	$\approx$	is approximately equal to
$=$	is equal to	$\cong$	is congruent to
$-$	negative	$\sim$	is similar to
$+$	positive	$\sqrt{\quad}$	nonnegative square root
$\pm$	positive or negative	$\%$	percent
$\neq$	is not equal to	$\pi$	pi
$>$	is greater than	$\overleftrightarrow{AB}$	line $AB$
$<$	is less than	$\overline{AB}$	line segment $AB$
$\nless$	is not greater than	$\overrightarrow{AB}$	ray $AB$
$\lessdot$	is not less than	$\triangle ABC$	triangle $ABC$
$\gtrsim$	is greater than or equal to	$\angle ABC$	angle $ABC$
$\lesssim$	is less than or equal to	$m\overline{AB}$	measure of line segment $AB$
$^{\circ}$	degrees	$m\angle ABC$	measure of angle $ABC$
$\perp$	is perpendicular to		



## FCAT Mathematics Reference Sheet

### Formulas



**triangle**

$$A = \frac{1}{2}bh$$



**rectangle**

$$A = lw$$



**trapezoid**

$$A = \frac{1}{2}h(b_1 + b_2)$$



**parallelogram**

$$A = bh$$



**circle**

$$A = \pi r^2$$

### Key

$b$  = base

$h$  = height

$l$  = length

$w$  = width

$\ell$  = slant height

S.A. = surface area

$d$  = diameter

$r$  = radius

$A$  = area

$C$  = circumference

$V$  = volume

Use 3.14 or  $\frac{22}{7}$  for  $\pi$ .

### circumference

$$C = \pi d \text{ or } C = 2\pi r$$



**right circular cone**

### Volume

$$V = \frac{1}{3}\pi r^2 h$$

### Total Surface Area

$$S.A. = \frac{1}{2}(2\pi r)\ell + \pi r^2 \quad \text{or} \quad S.A. = \pi r\ell + \pi r^2$$



**square pyramid**

$$V = \frac{1}{3}lwh$$

$$S.A. = 4\left(\frac{1}{2}l\ell\right) + l^2 \quad \text{or} \quad S.A. = 2l\ell + l^2$$



**sphere**

$$V = \frac{4}{3}\pi r^3$$

$$S.A. = 4\pi r^2$$



**right circular cylinder**

$$V = \pi r^2 h$$

$$S.A. = 2\pi r h + 2\pi r^2$$



**rectangular solid**

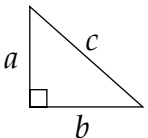
$$V = lwh$$

$$S.A. = 2(lw) + 2(hw) + 2(lh)$$

In the following formulas,  $n$  represents the number of sides.

- In a polygon, the sum of the measures of the interior angles is equal to  $180(n - 2)$ .
- In a regular polygon, the measure of an interior angle is equal to  $\frac{180(n - 2)}{n}$ .

## FCAT Mathematics Reference Sheet

 <p>Pythagorean theorem:</p> $a^2 + b^2 = c^2$	<p>Distance between two points</p> <p><math>P_1 (x_1, y_1)</math> and <math>P_2 (x_2, y_2)</math>:</p> $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
<p>Slope-intercept form of an equation of a line:</p> $y = mx + b$ <p>where <math>m</math> = slope and <math>b</math> = the <math>y</math>-intercept.</p>	<p>Midpoint between two points</p> <p><math>P_1 (x_1, y_1)</math> and <math>P_2 (x_2, y_2)</math>:</p> $\left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$
<p>Distance, rate, time formula:</p> $d = rt$ <p>where <math>d</math> = distance, <math>r</math> = rate, <math>t</math> = time.</p>	<p>Simple interest formula:</p> $I = prt$ <p>where <math>p</math> = principal, <math>r</math> = rate, <math>t</math> = time.</p>

### Conversions

1 yard = 3 feet = 36 inches

1 mile = 1,760 yards = 5,280 feet

1 acre = 43,560 square feet

1 hour = 60 minutes

1 minute = 60 seconds

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

1 liter = 1000 milliliters = 1000 cubic centimeters

1 meter = 100 centimeters = 1000 millimeters

1 kilometer = 1000 meters

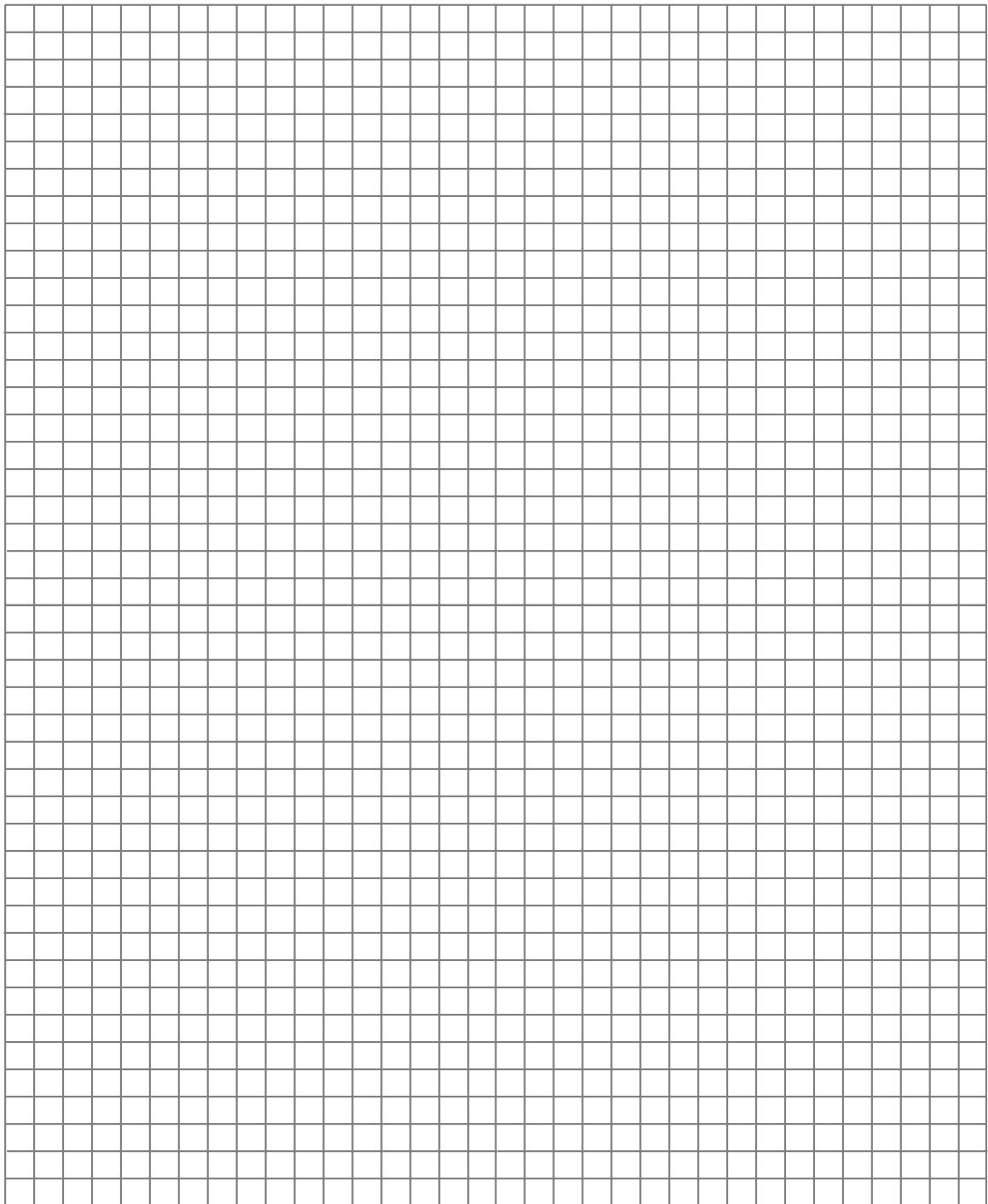
1 gram = 1000 milligrams

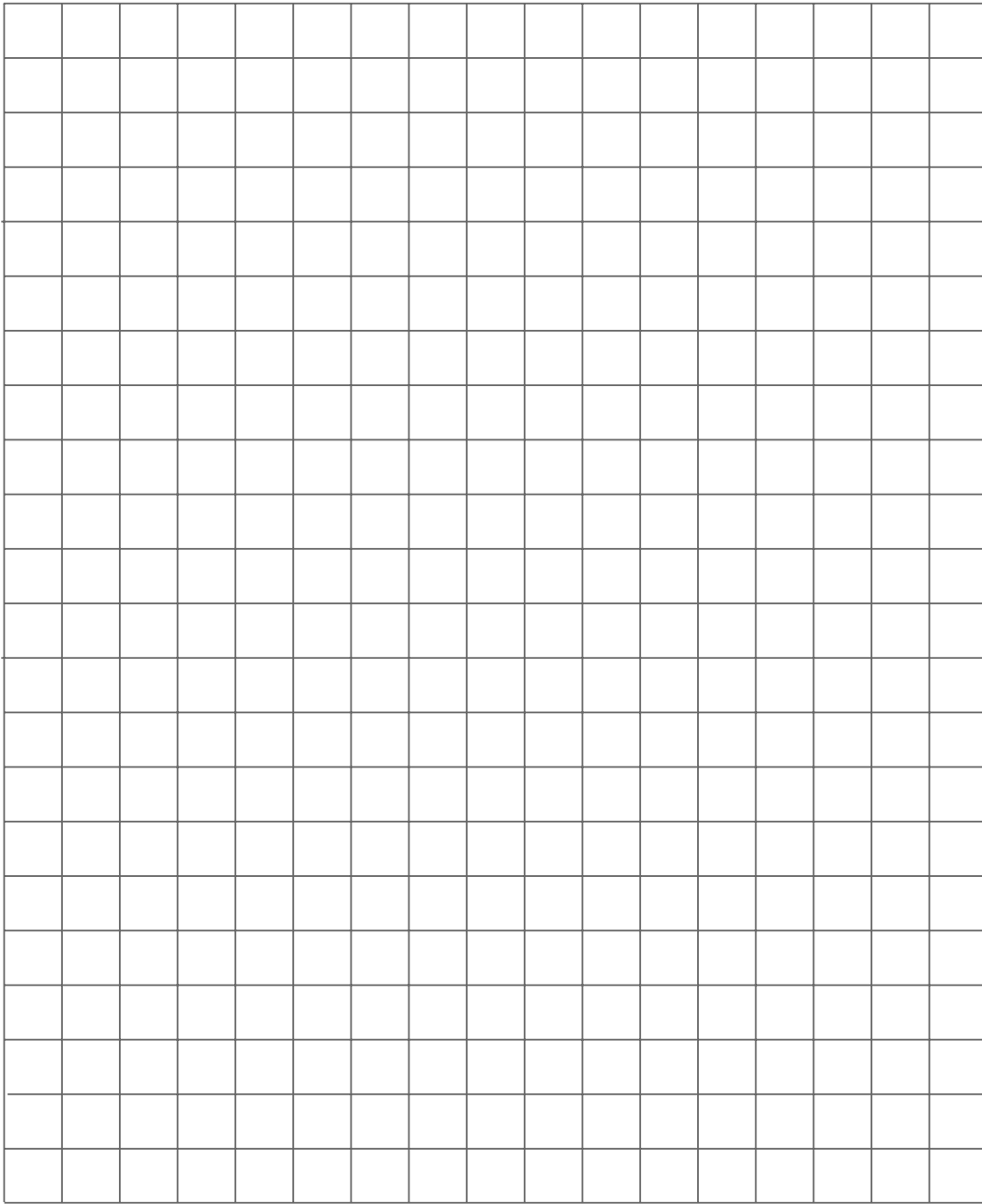
1 kilogram = 1000 gram

1 pound = 16 ounces

1 ton = 2,000 pounds

Metric numbers with four digits are presented without a comma (e.g., 9960 kilometers). For metric numbers greater than four digits, a space is used instead of a comma (e.g., 12 500 liters).







# Index

## A

absolute value ..... 3, 28, 501, 512  
 addend ..... 3, 27  
 additive identity ..... 3, 27, 73, 108  
 additive inverses ..... 3, 13, 73, 99, 191, 207  
 algebraic expression ..... 3, 54  
 angle ( $\angle$ ) ..... 73, 122, 413, 429  
 area ( $A$ ) ..... 73, 129, 705, 727  
 associative property ..... 3, 27, 73, 108  
 axes (of a graph) ..... 705, 748  
 axis of symmetry ..... 619, 667

## B

base (of an exponent) (algebraic)  
 ..... 191, 216  
 binomial ..... 191, 200  
 braces  $\{ \}$  ..... 4, 13, 451, 456

## C

canceling ..... 191, 223, 279, 290  
 Cartesian cross product ..... 451, 479  
 circle ..... 413, 429  
 coefficient ..... 191, 216, 361, 376,  
 ..... 619, 666, 705, 768  
 common denominator .... 279, 304, 501, 596  
 common factor ..... 191, 223, 279, 290  
 common multiple ..... 279, 304  
 commutative property ..... 4, 27, 73, 108,  
 ..... 192, 248  
 complement ..... 451, 473  
 composite number ..... 192, 246  
 congruent ( $\cong$ ) ..... 413, 429  
 conjugate ..... 361, 399  
 consecutive ..... 74, 88, 705, 726  
 constant ..... 501, 559  
 coordinate grid or plane ..... 451, 479,  
 ..... 501, 511, 705, 749  
 coordinate plane ..... 501, 542  
 coordinates ..... 501, 538, 619, 627, 705, 750  
 corresponding ..... 413, 433  
 corresponding angles and sides .... 413, 429

counting numbers (natural numbers)

..... 4, 12, 192, 200, 451, 456  
 cross multiplication ..... 280, 290, 413, 421  
 cube (power) ..... 4, 54, 74, 88  
 cubic units ..... 74, 143

## D

data ..... 619, 649  
 decimal number .... 4, 14, 280, 325, 361, 366  
 decrease ..... 74, 88  
 degree ( $^{\circ}$ ) ..... 74, 122, 414, 429, 501, 586  
 denominator .... 192, 200, 280, 289, 361, 366,  
 ..... 414, 420, 502, 552  
 difference ..... 4, 54, 74, 88, 280, 307  
 digit ..... 4, 13, 361, 383  
 distance ..... 502, 511  
 distributive property ..... 74, 107, 192, 219,  
 ..... 281, 306, 361, 391, 414, 421, 706, 760  
 domain ..... 619, 627

## E

element ..... 619, 627  
 element or member ..... 4, 12, 451, 456  
 empty set or null set ( $\emptyset$ ) ..... 4, 13, 451, 456  
 endpoint ..... 502, 538  
 equation ..... 4, 54, 74, 83, 281, 323, 414,  
 ..... 421, 502, 515, 619, 636, 706, 715  
 equiangular ..... 414, 429  
 equilateral ..... 414, 429  
 equivalent (forms of a number)  
 ..... 74, 112, 281, 290  
 equivalent expressions ..... 706, 753  
 estimation ..... 619, 652  
 even integer ..... 5, 13, 74, 156,  
 ..... 451, 457, 706, 803  
 exponent (exponential form)  
 ..... 5, 23, 192, 216  
 expression ..... 5, 22, 75, 83, 193, 200,  
 ..... 281, 289, 362, 366, 451, 479,  
 ..... 502, 516, 620, 636

## F

factor ..... 193, 216, 281, 290, 362, 366,  
..... 502, 516, 620, 677, 706, 716  
factored form ..... 193, 247, 706, 719  
factoring ..... 281, 290, 620, 666, 706, 720  
finite set ..... 5, 12, 452, 456  
FOIL method ..... 193, 232, 362, 395,  
..... 620, 676, 707, 733  
formula ..... 75, 178, 502, 529, 707, 727  
fraction ..... 5, 14, 193, 200, 282, 289,  
..... 362, 371, 414, 420, 707, 732  
function notation ..... 620, 636  
function (of  $x$ ) ..... 620, 627

## G

graph ..... 503, 511, 707, 749  
graph of an equation ..... 707, 748  
graph of a number ..... 75, 159  
graph of a point ..... 503, 511  
greatest common factor (GCF) ..... 194, 248  
grouping symbols ..... 5, 22, 194, 206

## H

height ( $h$ ) ..... 414, 439  
horizontal ..... 503, 511, 621, 642  
hypotenuse ..... 503, 514

## I

increase ..... 75, 88  
inequality ..... 75, 159, 282, 340, 707, 778  
infinite ..... 707, 752  
infinite set ..... 5, 12, 452, 456  
integers ..... 5, 13, 75, 83, 194, 249, 282, 289,  
..... 415, 425, 452, 457, 503, 558, 707, 726  
intersect ..... 503, 586, 621, 667, 708, 749  
intersection ( $\cap$ ) ..... 452, 463  
intersection ..... 708, 750  
inverse operation ..... 75, 124, 282, 327  
irrational number ..... 5, 14, 75, 161,  
..... 282, 289, 362, 371

## L

least common denominator (LCD) 282, 304  
least common multiple (LCM) ..... 282, 304  
leg ..... 503, 515  
length ( $l$ ) ..... 75, 121, 415, 426, 503, 515,  
..... 708, 727  
like terms ..... 76, 112, 194, 206, 283, 327,  
..... 363, 372  
line ( $\leftrightarrow$ ) ..... 503, 511, 621, 641, 708, 748  
linear equation ..... 503, 558  
linear function ..... 621, 641  
line segment ( $\text{---}$ ) ..... 504, 538

## M

maximum ..... 621, 668  
mean (or average) ..... 621, 662  
measure ( $m$ ) of an angle ( $\angle$ ) ..... 76, 122  
member or element ..... 6, 12, 452, 456  
midpoint (of a line segment) ..... 504, 538  
minimum ..... 283, 305, 621, 667  
monomial ..... 194, 200, 708, 720  
multiples ..... 6, 13  
multiplicative identity ..... 76, 108, 283, 330  
multiplicative inverse (reciprocals) ... 76, 97  
multiplicative property of -1  
..... 76, 99, 283, 330  
multiplicative property of zero ..... 76, 108

## N

natural numbers (counting numbers)  
..... 6, 12, 194, 200, 452, 456  
negative integers ... 6, 28, 504, 558, 708, 728  
negative numbers ..... 6, 13, 76, 83, 283, 341,  
..... 504, 512, 708, 781  
null set ( $\emptyset$ ) or empty set ..... 6, 13, 452, 456  
number line ..... 6, 25, 76, 159, 504, 538  
numerator ..... 194, 200, 283, 289, 363, 371,  
..... 415, 420, 504, 552

## O

odd integer ..... 6, 14, 76, 186, 708, 728  
opposites ..... 6, 13, 195, 207  
ordered pair ..... 452, 479, 504, 511,  
..... 621, 627, 708, 767  
order of operations ..... 7, 22, 77, 106, 195, 217,  
..... 283, 327  
origin ..... 621, 641

## P

parabola ..... 621, 666  
parallel (  $\parallel$  ) ..... 504, 586, 708, 752  
parallel lines ..... 504, 586  
pattern (relationship) ..... 7, 14, 452, 456  
perfect square ..... 363, 366  
perimeter ( $P$ ) ..... 77, 121, 415, 434  
perpendicular ( $\perp$ ) ..... 505, 586  
perpendicular lines ..... 505, 586  
pi ( $\pi$ ) ..... 7, 14  
point ... 453, 463, 505, 511, 621, 631, 708, 748  
polygon ..... 415, 429  
polynomial ..... 195, 200, 284, 289, 709, 720  
positive integers ..... 7, 28, 453, 457, 709, 726  
positive numbers ..... 7, 12, 77, 83, 284, 343,  
..... 505, 512  
power (of a number)  
..... 7, 23, 77, 88, 195, 200  
prime factorization ..... 195, 246  
prime number ..... 195, 246  
product ..... 8, 40, 77, 88, 196, 200, 284, 290,  
..... 363, 395, 505, 586, 709, 715  
proportion ..... 415, 420  
Pythagorean theorem ..... 505, 515

## Q

quadratic equation ..... 622, 677, 709, 719  
quadratic formula ..... 709, 731  
quadratic function ..... 622, 666  
quotient ..... 8, 43, 77, 88, 196, 200, 284, 289

## R

radical ..... 363, 371, 505, 516  
radical expression ..... 363, 366, 505, 516  
radical sign ( $\sqrt{\phantom{x}}$ ) ..... 363, 366, 506, 516

radicand ..... 506, 516  
range ..... 622, 627  
ratio ..... 8, 14, 77, 162, 284, 289, 415, 420  
rational expression ..... 196, 200, 284, 289  
rationalizing the denominator ..... 364, 371  
rational number ..... 8, 14, 77, 161, 284, 289,  
..... 364, 371  
real numbers ..... 8, 14, 77, 161, 284, 289  
reciprocals ..... 78, 97, 284, 328, 506, 586  
rectangle ..... 78, 121, 709, 727  
regular polygon ..... 416, 429  
relation ..... 453, 479, 622, 627  
repeating decimal ..... 8, 14  
right angle ..... 506, 586  
right triangle ..... 506, 514  
rise ..... 506, 549  
root ..... 8, 23, 506, 516  
roots ..... 622, 677  
roster ..... 453, 456  
rounded number ..... 416, 444, 622, 656  
rule ..... 453, 456  
run ..... 506, 549

## S

scale factor ..... 416, 429  
set ..... 8, 12, 453, 456, 622, 627  
side ..... 78, 121, 416, 429, 507, 515  
similar figures ( $\sim$ ) ..... 417, 429  
simplest form (of a fraction) ..... 285, 337  
simplest form (of an expression) ... 196, 201  
simplest radical form ..... 364, 367, 507, 517  
simplify a fraction ..... 507, 543  
simplify an expression ..... 8, 39, 78, 107,  
..... 285, 290, 364, 377, 709, 732  
slope ..... 507, 549, 622, 641  
slope-intercept form ..... 507, 568  
solution  
..... 78, 84, 285, 327, 623, 677, 709, 715  
solution set ( $\{ \}$ ) ..... 709, 716  
solve ..... 9, 54, 78, 84, 417, 421, 623, 677,  
..... 709, 715  
square ..... 78, 155  
square (of a number) ..... 9, 54, 78, 88,  
..... 507, 515  
square root ..... 364, 366, 507, 516  
square units ..... 79, 129  
standard form (of a linear equation)  
..... 508, 558

standard form (of a quadratic equation)  
 ..... 196, 206, 710, 719  
 substitute ..... 79, 84, 285, 327, 710, 731  
 substitution ..... 710, 759  
 substitution property of equality .... 79, 109  
 sum ..... 9, 27, 79, 88, 196, 208, 285, 307,  
 ..... 508, 515, 710, 771  
 symmetric property of equality ..... 79, 109  
 system of equations ..... 710, 748

## T

table (or chart) ..... 79, 148, 710, 748  
 term ... 196, 200, 285, 295, 364, 376, 710, 719  
 terminating decimal ..... 9, 14  
 trapezoid ..... 417, 433  
 triangle ..... 79, 121, 417, 429, 508, 515  
 trinomial ..... 196, 201

## U

union ( $\cup$ ) ..... 453, 462

## V

value (of a variable) ..... 9, 49, 417, 421,  
 ..... 508, 529, 623, 627, 710, 715  
 variable ..... 9, 49, 79, 83, 197, 200, 285, 289,  
 ..... 364, 376, 417, 421, 508, 559,  
 ..... 623, 646, 710, 748  
 Venn diagram ..... 9, 15, 453, 464, 711, 791  
 vertex ..... 623, 667  
 vertical ..... 508, 512, 623, 631, 711, 750  
 vertical line test ..... 623, 631

## W

whole numbers ..... 9, 13, 197, 246, 364, 367  
 width ( $w$ ) ..... 79, 121, 711, 727

## X

$x$ -axis ..... 508, 511, 623, 642  
 $x$ -coordinate ..... 453, 479, 508, 542  
 $x$ -intercept ..... 508, 559, 623, 642, 711, 749

## Y

$y$ -axis ..... 508, 511, 623, 642  
 $y$ -coordinate ..... 453, 479, 508, 542  
 $y$ -intercept ..... 508, 559, 624, 641, 711, 749

## Z

zero product property ..... 197, 219,  
 ..... 624, 677, 711, 720  
 zero property of multiplication ..... 197, 219  
 zeros ..... 624, 677

## References

- Bailey, Rhonda, et al. *Glencoe Mathematics: Applications and Concepts*. New York: McGraw-Hill Glencoe, 2004.
- Boyd, Cindy J., et al. *Glencoe Mathematics: Geometry*. New York: McGraw-Hill Glencoe, 2004.
- Brooks, Jane, et al., eds. *Pacemaker Geometry, First Edition*. Parsippany, NJ: Globe Fearon, 2003.
- Collins, Williams, et al. *Glencoe Algebra 1: Integration, Applications, and Connections*. New York: McGraw-Hill Glencoe, 1998.
- Cummins, Jerry, et al. *Glencoe Algebra: Concepts and Applications*. New York: McGraw-Hill Glencoe, 2004.
- Cummins, Jerry, et al. *Glencoe Geometry: Concepts and Applications*. New York: McGraw-Hill Glencoe, 2005.
- Florida Department of Education. *Florida Course Descriptions*. Tallahassee, FL: State of Florida, 2008.
- Florida Department of Education. *Florida Curriculum Framework: Mathematics*. Tallahassee, FL: State of Florida, 1996.
- Haenisch, Siegfried. *AGS Publishing: Algebra*. Circle Pines, MN: AGS Publishing, 2004.
- Hirsch, Christian R., et al. *Contemporary Mathematics in Context Course 1, Parts A and B*. Columbus, New York: McGraw-Hill Glencoe, 2003.
- Holiday, Berchie, et al. *Glencoe Mathematics: Algebra 1*. New York: McGraw-Hill, 2005.
- Lappan, Glenda, et al. *Frogs, Fleas, and Painted Cubes*. White Plains, NY: McGraw-Hill Glencoe, 2004.
- Lappan, Glenda, et al. *Looking for Pythagoras: The Pythagorean Theorem*. White Plains, NY: McGraw-Hill Glencoe, 2002.

- Lappan, Glenda, et al. *Samples and Populations: Data & Statistics*. Upper Saddle River, NJ: Pearson Prentice Hall, 2004.
- Larson, Ron, et al. *McDougal Littell: Algebra 1*. Boston, MA: Holt McDougal, 2006.
- Malloy, Carol, et al. *Glencoe Pre-Algebra*. New York: McGraw-Hill / Glencoe, 2005.
- Muschla, Judith A. and Gary Robert Muschla. *Math Starters! 5- to 10-Minute Activities That Make Kids Think, Grades 6-12*. West Nyack, NY: The Center for Applied Research in Education, 1998.
- Ripp, Eleanor, et al., eds. *Pacemaker Algebra 1, Second Edition*. Parsippany, NJ: Globe Fearon, 2001.
- Ripp, Eleanor, et al., eds. *Pacemaker Pre-Algebra, Second Edition*. Parsippany, NJ: Globe Fearon, 2001.
- Schultz, James E., et al. *Holt Algebra 1*. Austin, TX: Holt, Rinehart, and Winston, 2004.

## **Production Software**

- Adobe InDesign 3.0.1. San Jose, CA: Adobe Systems.
- Adobe Photoshop 8.0. Mountain View, CA: Adobe Systems.
- Macromedia Freehand 8.0. San Francisco: Macromedia.
- Microsoft Office 98. Redmond, WA: Microsoft.

# **Algebra I**

**Course No. 1200310**

**Bureau of Exceptional Education and Student Services  
Florida Department of Education**

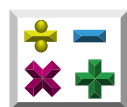
**2009**

This product was developed by Leon County Schools, Exceptional Student Education Department, through the Curriculum Improvement Project, a special project, funded by the State of Florida, Department of Education, Bureau of Exceptional Education and Student Services, through federal assistance under the Individuals with Disabilities Education Act (IDEA), Part B.

Copyright  
State of Florida  
Department of State  
2009

Authorization for reproduction is hereby granted to the State System of Public Education consistent with Section 1006.39(2), Florida Statutes. No authorization is granted for distribution or reproduction outside the State System of Public Education without prior approval in writing.





# Algebra I

## Course No. 1200310

content revised by  
**Sylvia Crews**  
**Sue Fresen**

developed and edited by  
**Sue Fresen**

graphics by  
**Jennifer Keele**  
**Rachel McAllister**

page layout by  
**Jennifer Keele**  
**Rachel McAllister**

**Curriculum Improvement Project**  
**IDEA, Part B, Special Project**



**Exceptional Student Education**

<http://www.leon.k12.fl.us/public/pass/>

**Curriculum Improvement Project**

Sue Fresen, Project Manager

**Leon County Exceptional Student Education (ESE)**

Ward Spisso, Executive Director of Exceptional Student Education

Pamela B. Hayman, Assistant Principal on Special Assignment

**Superintendent of Leon County Schools**

Jackie Pons

**School Board of Leon County**

Joy Bowen, Chair

Dee Crumpler

Maggie Lewis-Butler

Dee Dee Rasmussen

Forrest Van Camp

# Table of Contents

Acknowledgments .....	xv
<b>Unit 1: Are These Numbers Real?</b> .....	1
Unit Focus .....	1
Vocabulary .....	3
Introduction.....	11
<b>Lesson One Purpose</b> .....	11
The Set of Real Numbers .....	12
Practice .....	16
Practice .....	17
Practice .....	18
Practice .....	19
Practice .....	20
<b>Lesson Two Purpose</b> .....	21
The Order of Operations .....	22
Adding Numbers by Using a Number Line.....	25
Opposites and Absolute Value .....	28
Adding Positive and Negative Integers .....	30
Subtracting Integers .....	33
Practice .....	35
Practice .....	38
Practice .....	39
Multiplying Integers .....	40
Practice .....	42
Dividing Integers.....	43
Practice .....	46
Practice .....	47
Practice .....	49
Practice .....	50
Practice .....	52
<b>Lesson Three Purpose</b> .....	53
Algebraic Expressions.....	54
Practice .....	55
Practice .....	57
<b>Lesson Four Purpose</b> .....	58
Working with Absolute Value.....	59
Practice .....	60
Practice .....	61
Practice .....	63

Practice .....	65
Unit Review .....	66
<b>Unit 2: Algebraic Thinking</b> .....	71
Unit Focus .....	71
Vocabulary .....	73
Introduction .....	81
<b>Lesson One Purpose</b> .....	81
Solving Equations .....	83
Practice .....	85
Interpreting Words and Phrases .....	88
Practice .....	89
Solving Two-Step Equations .....	91
Practice .....	93
Special Cases .....	97
Practice .....	101
Practice .....	103
Practice .....	104
<b>Lesson Two Purpose</b> .....	105
The Distributive Property .....	106
Practice .....	110
Simplifying Expressions .....	112
Practice .....	114
Practice .....	116
Equations with Like Terms .....	117
Practice .....	119
Putting It All Together .....	124
Practice .....	127
Practice .....	130
<b>Lesson Three Purpose</b> .....	132
Solving Equations with Variables on Both Sides .....	133
Practice .....	136
Problems That Lead to Equations .....	140
Practice .....	142
Practice .....	148
Practice .....	157
<b>Lesson Four Purpose</b> .....	158
Graphing Inequalities on a Number Line .....	159
Practice .....	163
Solving Inequalities .....	165

Practice .....	168
Practice .....	170
Practice .....	174
Practice .....	175
Practice .....	176
<b>Lesson Five Purpose</b> .....	177
Formulas Using Variables .....	178
Practice .....	180
Unit Review .....	182
<b>Unit 3: Working with Polynomials</b> .....	189
Unit Focus .....	189
Vocabulary .....	191
Introduction .....	199
<b>Lesson One Purpose</b> .....	199
Polynomials .....	200
Practice .....	202
Practice .....	203
Practice .....	204
<b>Lesson Two Purpose</b> .....	205
Addition and Subtraction of Polynomials .....	206
Practice .....	209
Practice .....	212
<b>Lesson Three Purpose</b> .....	215
Multiplying Monomials .....	216
Practice .....	218
Practice .....	220
<b>Lesson Four Purpose</b> .....	222
Dividing Monomials .....	223
Practice .....	226
Practice .....	228
<b>Lesson Five Purpose</b> .....	230
Multiplying Polynomials .....	231
Practice .....	236
Practice .....	242
<b>Lesson Six Purpose</b> .....	245
Factoring Polynomials .....	246
Practice .....	249

Practice .....	252
Practice .....	255
<b>Lesson Seven Purpose</b> .....	256
Factoring Quadratic Polynomials .....	257
Practice .....	260
Practice .....	262
Practice .....	264
Practice .....	266
Practice .....	268
Practice .....	271
Unit Review .....	272
<b>Unit 4: Making Sense of Rational Expressions</b> .....	277
Unit Focus .....	277
Vocabulary .....	279
Introduction .....	287
<b>Lesson One Purpose</b> .....	287
Simplifying Rational Expressions .....	289
Practice .....	291
Practice .....	293
Additional Factoring .....	295
Practice .....	296
Practice .....	297
Practice .....	299
Practice .....	300
Practice .....	301
<b>Lesson Two Purpose</b> .....	303
Addition and Subtraction of Rational Expressions .....	304
Finding the Least Common Multiple (LCM) .....	304
Practice .....	307
Practice .....	308
Practice .....	310
Practice .....	312
<b>Lesson Three Purpose</b> .....	313
Multiplication and Division of Rational Expressions .....	314
Practice .....	315
Practice .....	316
Practice .....	318
Practice .....	319
Practice .....	321

<b>Lesson Four Purpose</b> .....	322
Solving Equations.....	323
Practice .....	324
Step-by-Step Process for Solving Equations .....	325
Practice .....	331
Practice .....	333
Practice .....	335
Practice .....	337
<b>Lesson Five Purpose</b> .....	339
Solving Inequalities .....	340
Practice .....	344
Practice .....	348
Practice .....	351
Unit Review .....	352
<b>Unit 5: How Radical Are You?</b> .....	359
Unit Focus .....	359
Vocabulary .....	361
Introduction.....	365
<b>Lesson One Purpose</b> .....	365
Simplifying Radical Expressions .....	366
Rule One .....	366
Practice .....	369
Practice .....	370
Rule Two .....	371
Practice .....	373
Practice .....	374
Practice .....	375
<b>Lesson Two Purpose</b> .....	376
Add and Subtract Radical Expressions .....	376
When Radical Expressions Don't Match or Are Not in Radical Form.....	378
Practice .....	379
Practice .....	381
<b>Lesson Three Purpose</b> .....	383
Multiply and Divide Radical Expressions .....	383
Practice .....	384
Working with a Coefficient for the Radical .....	386
Practice .....	387
Practice .....	389

<b>Lesson Four Purpose</b>	391
Multiple Terms and Conjugates	391
Practice	393
The FOIL Method	395
Practice	396
Two-Term Radical Expressions	399
Practice	402
Practice	404
Practice	406
Unit Review	407
<b>Unit 6: Extreme Fractions</b>	411
Unit Focus	411
Vocabulary	413
Introduction	419
<b>Lesson One Purpose</b>	419
Ratios and Proportions	420
Practice	422
Using Proportions Algebraically	424
Practice	425
Practice	427
<b>Lesson Two Purpose</b>	428
Similarity and Congruence	429
Practice	430
Using Proportions Geometrically	433
Practice	435
Using Proportions to Find Heights	438
Practice	440
Practice	442
Unit Review	443
<b>Unit 7: Exploring Relationships with Venn Diagrams</b>	449
Unit Focus	449
Vocabulary	451
Introduction	455
<b>Lesson One Purpose</b>	455
Sets	456
Practice	458
Practice	459



<b>Lesson Two Purpose</b> .....	461
Unions and Intersections .....	462
Practice .....	466
Practice .....	468
<b>Lesson Three Purpose</b> .....	472
Complements .....	473
Practice .....	474
Complements in Venn Diagrams .....	476
Practice .....	477
<b>Lesson Four Purpose</b> .....	478
Cartesian Cross Products .....	479
Practice .....	480
Practice .....	481
<b>Lesson Five Purpose</b> .....	482
Using Venn Diagrams for Three Categories .....	483
Practice .....	488
Practice .....	489
Unit Review .....	491
<b>Unit 8: Is There a Point to This?</b> .....	497
Unit Focus .....	497
Vocabulary .....	501
Introduction .....	509
<b>Lesson One Purpose</b> .....	509
Distance .....	511
Practice .....	517
Practice .....	527
Practice .....	528
Using the Distance Formula .....	529
Practice .....	532
Practice .....	535
<b>Lesson Two Purpose</b> .....	537
Midpoint .....	538
Method One Midpoint Formula .....	539
Practice .....	540
Method Two Midpoint Formula .....	542
Practice .....	544
Practice .....	546

<b>Lesson Three Purpose</b> .....	547
Slope .....	549
Practice .....	553
<b>Lesson Four Purpose</b> .....	557
Equations of Lines .....	558
Practice .....	562
Slope-Intercept Form .....	568
Practice .....	571
Transforming Equations into Slope-Intercept Form .....	577
Practice .....	578
Practice .....	584
<b>Lesson Five Purpose</b> .....	585
Parallel and Perpendicular Lines .....	586
Practice .....	587
Practice .....	590
Practice .....	592
Practice .....	593
<b>Lesson Six Purpose</b> .....	594
Point-Slope Form .....	595
Practice .....	600
Unit Review .....	606
<b>Unit 9: Having Fun with Functions</b> .....	617
Unit Focus .....	617
Vocabulary .....	619
Introduction .....	625
<b>Lesson One Purpose</b> .....	625
Functions .....	627
Practice .....	628
Practice .....	629
Graphs of Functions .....	631
Practice .....	632
Practice .....	634
<b>Lesson Two Purpose</b> .....	635
The Function of $X$ .....	636
Practice .....	637
<b>Lesson Three Purpose</b> .....	640
Graphing Functions .....	641

Practice .....	644
Practice .....	645
Linear Relations in the Real World .....	649
Practice .....	650
More about the Slope of a Line .....	658
Practice .....	659
Practice .....	661
Practice .....	663
<b>Lesson Four Purpose</b> .....	665
Graphing Quadratics .....	666
Practice .....	669
Solving Quadratic Equations .....	677
Practice .....	679
Practice .....	689
Unit Review .....	690
<b>Unit 10: X or (X, Y) Marks the Spot!</b> .....	701
Unit Focus .....	701
Vocabulary .....	705
Introduction .....	713
<b>Lesson One Purpose</b> .....	713
Quadratic Equations .....	715
Practice .....	717
Factoring to Solve Equations .....	719
Practice .....	721
Practice .....	724
Practice .....	725
Solving Word Problems .....	726
Practice .....	728
Practice .....	730
Using the Quadratic Formula .....	731
Practice .....	734
Practice .....	740
<b>Lesson Two Purpose</b> .....	746
Systems of Equations .....	748
Practice .....	753
Using Substitution to Solve Equations .....	759
Practice .....	761
Using Magic to Solve Equations .....	767
Practice .....	769

Solving More Word Problems.....	771
Practice .....	773
Practice .....	775
Practice .....	776
<b>Lesson Three Purpose</b> .....	777
Graphing Inequalities .....	778
Practice .....	782
Graphing Multiple Inequalities .....	790
Practice .....	795
Practice .....	801
Unit Review .....	802
<b>Appendices</b> .....	815
Appendix A: Table of Squares and Approximate Square Roots .....	817
Appendix B: Mathematical Symbols .....	819
Appendix C: Mathematics Reference Sheet.....	821
Appendix D: Graph Paper .....	823
Appendix E: Index.....	825
Appendix F: References .....	829

## Acknowledgments

The staff of the Curriculum Improvement Project wishes to express appreciation to the content revisor and reviewers for their assistance in the development of *Algebra I*. We also wish to express our gratitude to educators from Broward, Hillsborough, Indian River, Leon, Okeechobee, Orange, Pasco, Pinellas, Polk, Sarasota, St. Lucie, and Volusia county school districts for the initial *Parallel Alternative Strategies for Students (PASS)* Mathematics volumes.

### Content Revisor

Sylvia Crews, Mathematics Teacher  
Department Chair  
School Advisory Committee Chair  
Leon High School  
Tallahassee, FL

### Review Team

Lao Alovus, Exceptional Student Education (ESE) Teacher School for Arts and Innovative Learning (SAIL) High School Tallahassee, FL	Veronica Delucchi, English for Speakers of Other Languages (ESOL) Coordinator Pines Middle School Pembroke Pines, FL
Janet Brashear, Hospital/Homebound Program Coordinator Program Specialist Exceptional Student Education (ESE) Indian River County School District Vero Beach, FL	Heather Diamond, Program Specialist for Specific Learning Disabilities (SLD) Bureau of Exceptional Education and Student Services Florida Department of Education Tallahassee, FL
Todd Clark, Chief Bureau of Curriculum and Instruction Florida Department of Education Tallahassee, FL	Steven Friedlander, Mathematics Teacher Lawton Chiles High School 2007 Edyth May Sliffe Award President, Florida Association of Mu Alpha Theta Past Vice President, Leon County Council of Teachers of Mathematics (LCTM) Tallahassee, FL
Vivian Cooley, Assistant Principal Rickards High School Tallahassee, FL	
Kathy Taylor Dejoie, Program Director Clearinghouse Information Center Bureau of Exceptional Education and Student Services Florida Department of Education Tallahassee, FL	

Debbie Gillis, Assistant Principal  
Okeechobee High School  
Past Treasurer, Florida Council of  
Teachers of Mathematics (FCTM)  
Okeechobee, FL

Mark Goldman, Honors Program  
Chairman and Professor  
Tallahassee Community College  
2009 National Institute for Staff and  
Organizational Development  
(NISOD) Lifetime Teaching  
Excellence Award  
Past President, Leon Association for  
Children with Learning Disabilities  
(ACLD)  
Parent Representative, Leon County  
Exceptional Student Education  
(ESE) Advisory Committee  
Tallahassee, FL

Kathy Kneapler, Home School Parent  
Palm Bay, FL

Edythe M. MacMurdo, Mathematics  
Teacher  
Department Chair  
Seminole Middle School  
Plantation, FL

Daniel Michalak, Mathematics Teacher  
Timber Creek High School  
Orlando, FL

Jeff Miller, Mathematics Teacher  
Gulf High School  
New Port Richey, FL

William J. Montford, Chief Executive  
Officer  
Florida Association of School District  
Superintendents  
Superintendent of Leon County  
Schools 1996-2006  
Tallahassee, FL

Marilyn Ruiz-Santiago, Multi-Cultural  
Specialist/ Training Director  
Parents Educating Parents in the  
Community (PEP)  
Family Network on Disabilities of  
Florida, Inc.  
Clearwater, FL

Teresa D. Sweet, Mathematics  
Curriculum Specialist  
Bureau of Curriculum and Instruction  
Florida Department of Education  
Tallahassee, FL

Joyce Wiley, Mathematics Teacher  
Osceola Middle School  
Past President, Pinellas Council of  
Teachers of Mathematics (PCTM)  
Seminole, FL

Ronnie Youngblood, Division  
Director  
Facilities Systems Management  
Leon County Schools  
Tallahassee, FL

### **Production Staff**

Sue Fresen, Project Manager  
Jennifer Keele, Media Production Specialist  
Rachel McAllister, Media Production Specialist  
Curriculum Improvement Project  
Tallahassee, FL